Radiative Transfer

Lecture 03

The Physics of Star Formation

Les Houches School of Physics February 22, 2024

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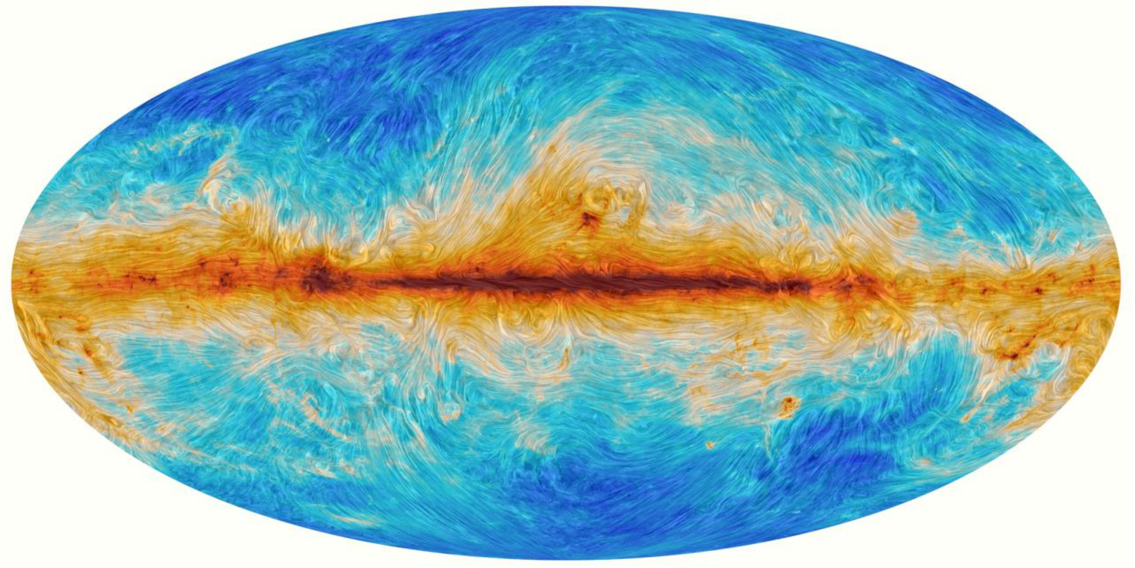
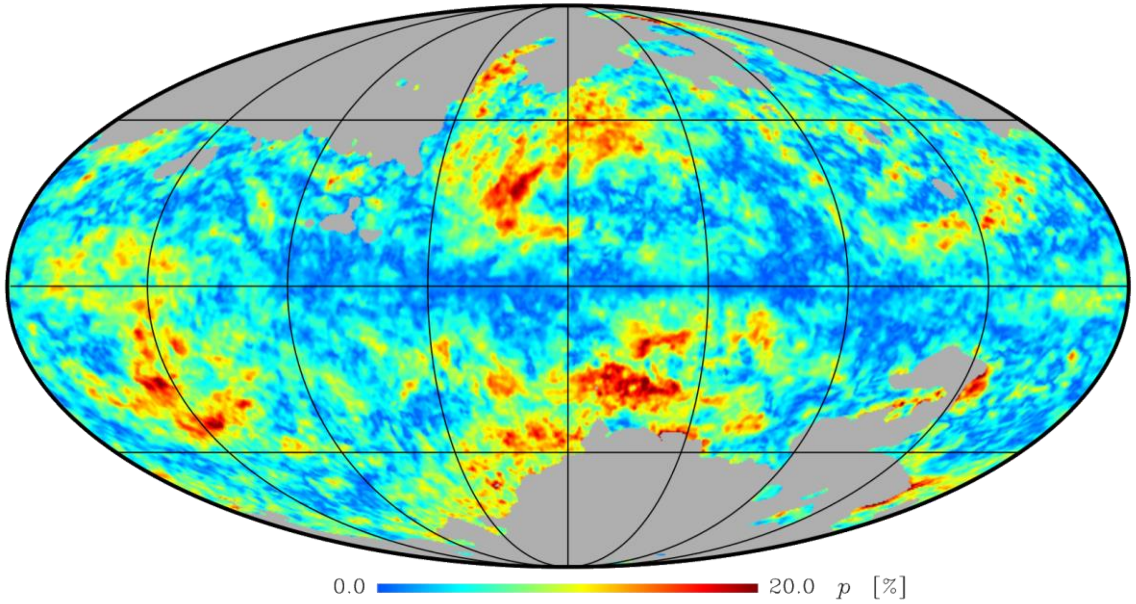
Magnetic field and ISM structure

polarized thermal 850 µm emission from Galactic dust



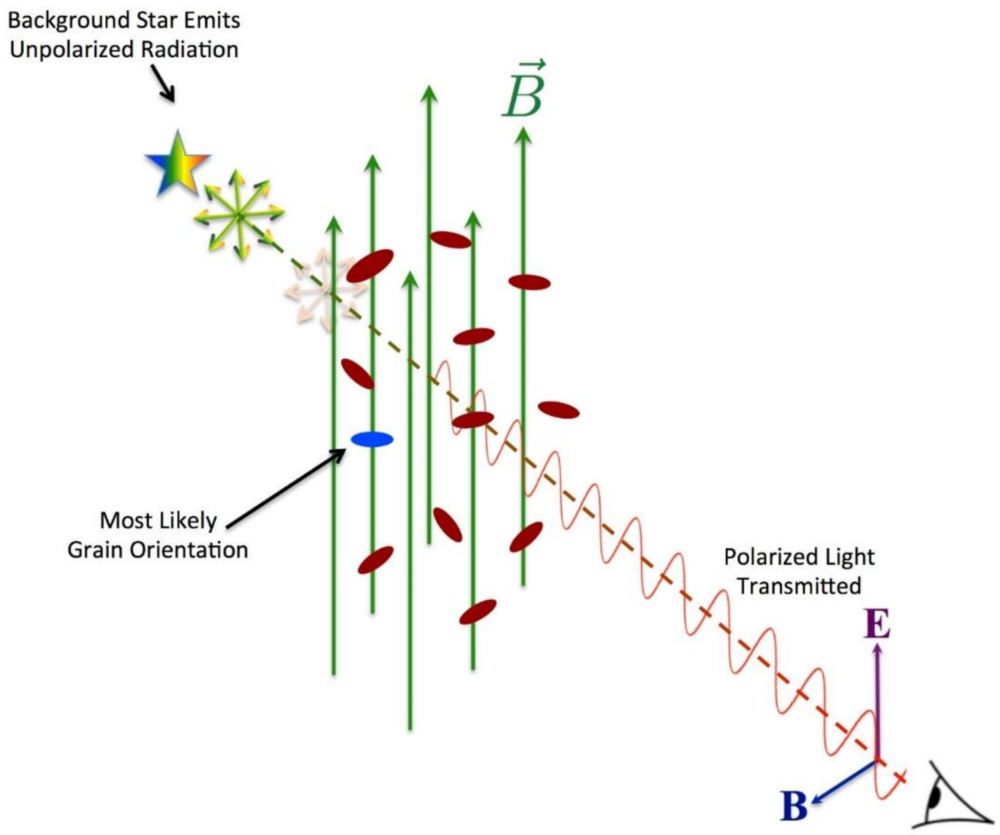


- Large degree of linear polarization (<20%) suggests effective grain alignment
- Depolarized regions correspond to high column density
- Interpretation of observed polarized dust emission depend heavily on our understanding of grain dynamics

Grain Alignment Theory

(as far as we know about it)

Dust polarization

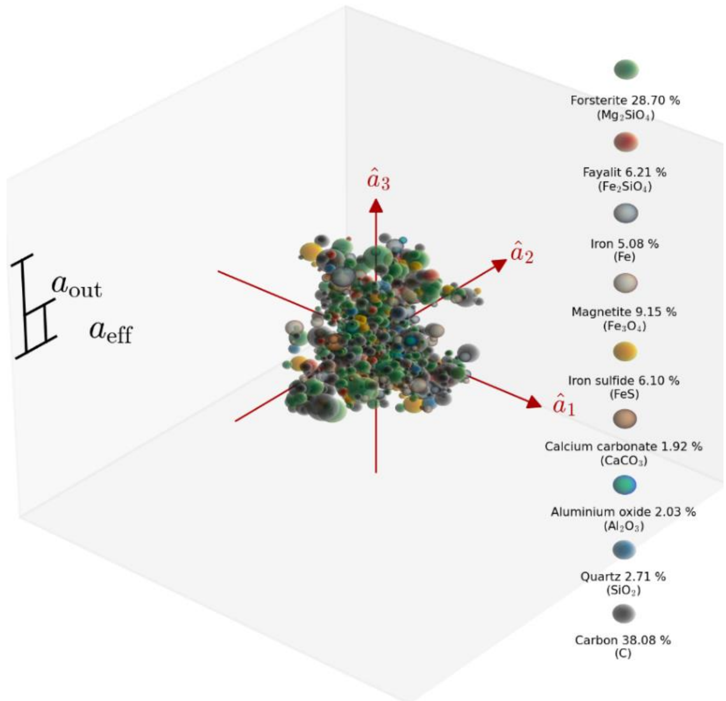


Non-spherical rotating dust grains are aligned with their shorter axis with the magnetic field orientation

Dichroic extinction:
Polarized (star)light gets preferentially blocked along the longer grain axis $\Rightarrow$ Polarization is parallel to $\vec{B}$.

Thermal emission:
Warm dust grains emit thermal radiation preferentially along tier longer axis $\Rightarrow$ Polarization is perpendicular to $\vec{B}$.

Grain rotation axis



The inertia tensor of each object can be diagonalized

$\Rightarrow$ unique orthonormal coordinate system

$\{\hat{a}_1, \hat{a}_2, \hat{a}_3\}$

with moments of inertia

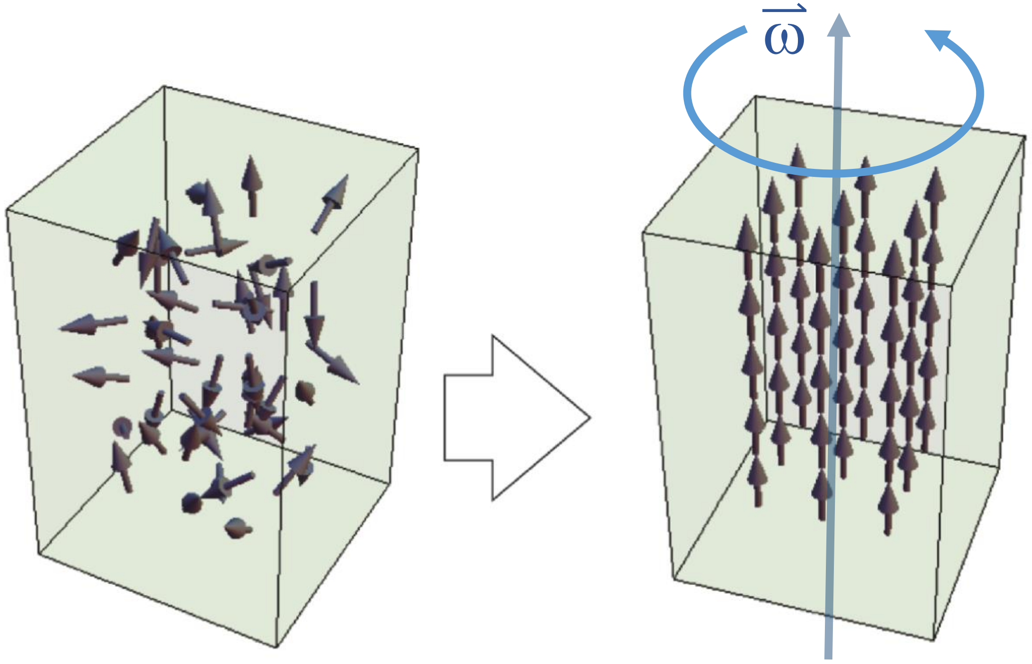
$\hat{I}_1 \geq \hat{I}_2 \geq \hat{I}_3$

Dust grains are not rigid bodies $\Rightarrow$ internal dissipation leads to rotation preferentially around $\hat{a}_1$

https://www.youtube.com/watch?v=BPMjcN-sBJ4

Internal relaxation processes: Barnett relaxation (Purcell 1979; Lazarian & Roberge 1997), nuclear relaxation (Lazarian & Draine 1999b), and inelastic relaxation (Purcell 1979; Lazarian & Efroimsky 1999)

Barnett effect



$$\vec{L}_{\text{net}} = \vec{L}_{\text{body}} + \sum \vec{L}_{\text{spin}} = 0$$

=0 =0

$$\vec{L}_{\text{net}} = \vec{L}_{\text{body}} + \sum \vec{L}_{\text{spin}} = 0$$

>0 >0

$$\Rightarrow \vec{L}_{\text{body}} = -\sum \vec{L}_{\text{spin}} = \int \frac{e}{2\, m_e c} \vec{\mu}_{el} dV = \frac{e}{2g\, m_e c} \vec{M}$$

Grain magnetic moment:

$$\vec{\mu}_{Ba} = \frac{\chi V h}{g \vec{\mu}_B} \vec{\omega}$$

with magnetic susceptibility $\chi = \chi' + i\, \chi''$

Torque in the presence of an external B-field

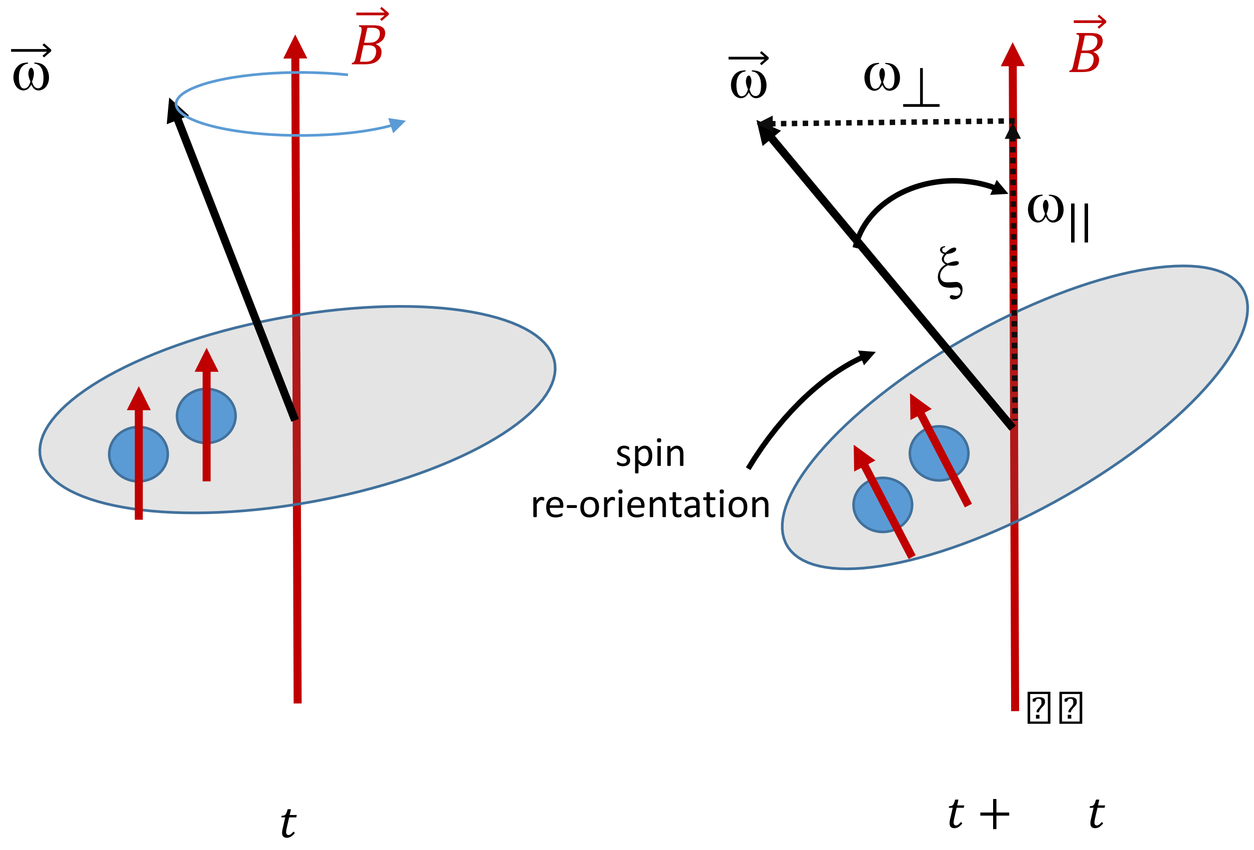
$$\vec{\Gamma}_{\text{Ba}} = \vec{B} \times \vec{\mu}_{Ba}$$

$\Rightarrow$ Barnett effect causes grain precession around $\vec{B}$ on a timescale τ_{Ba}

Note: alignment is not an effect of grain charge

$\frac{\vec{\mu}_{Ba}}{\vec{\mu}_e} \approx 10^5$ (for typical ISM conditions)

Davis Greenstein (DG) Effect



Spin re-orientation induces lattice vibrations (phonons) $\Rightarrow$ dissipation of $\omega_\perp$ into heat via

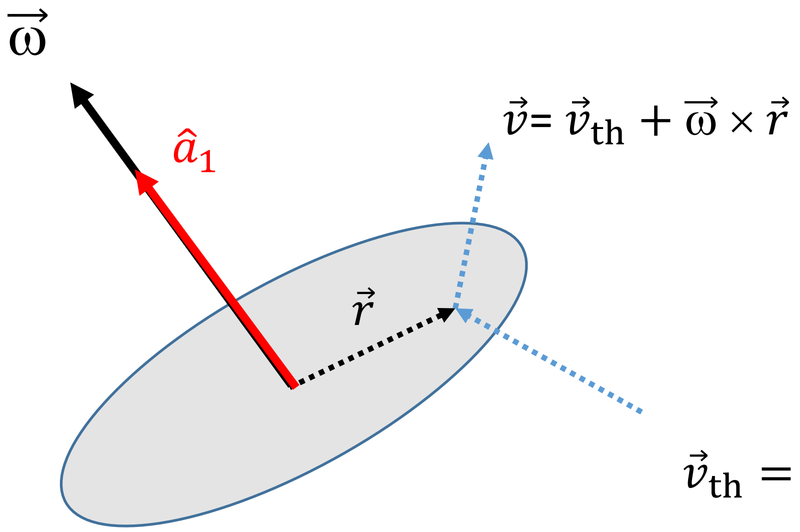
$$I_1\omega_\perp^2 \rightarrow k_B T_{\mathrm{dust}}$$

The DG torque

$$\vec{\Gamma}_{\mathrm{DG}} = \frac{\chi'' V}{2\mu_0\omega_\perp}(\vec{\omega} \times \vec{B}) \times \vec{B}$$

minimizes the alignment angle $\xi \rightarrow 0$ over a characteristic time scale τ_{DG}

(rotational) gas drag



$$\vec{v}_{\rm th} = \sqrt{\frac{8k_{\rm B}T_{\rm g}}{\pi m_{\rm g}}}$$

Change in angular momentum per collision

$$\Delta\vec{L} = -m_{\rm g}\vec{r}\times(\vec{\omega}\times\vec{r})$$

Gas-dust collision rate

$$dR_{\rm gas} = n_{\rm g}v_{\rm th}\,{\rm dA}$$

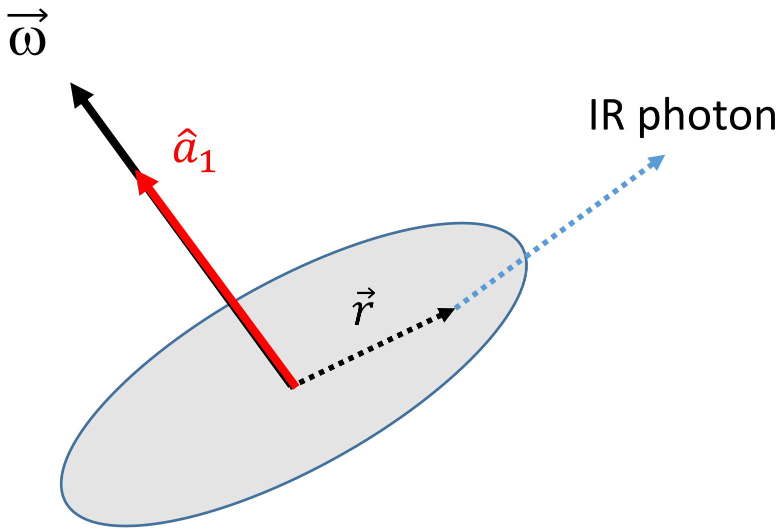
Gas drag

$$\vec{\Gamma}_{\rm gas} = \int \Delta\vec{L}\, dR_{\rm gas} = n_{\rm g}m_{\rm g}v_{\rm th}a_{\rm eff}^4\,\omega\vec{Q}_{\rm gas} = \hat{a}_1\frac{I_1\omega}{\tau_{\rm gas}}$$

with time scale $\tau_{\rm gas}$

Note: For a sphere $\vec{Q}_{\rm gas}=-\frac{4\sqrt{\pi}}{3}\hat{a}_1$

Infrared (IR) drag



Infrared drag

$$\vec{\Gamma}_{\mathrm{IR}} = \int \Delta\vec{L}\, dR_{\mathrm{em}} = -\hat{a}_1 \frac{a_{\mathrm{eff}}^2 f_{\mathrm{IR}}}{c} = -\hat{a}_1 \frac{I_1 \omega}{\tau_{\mathrm{IR}}}$$

with

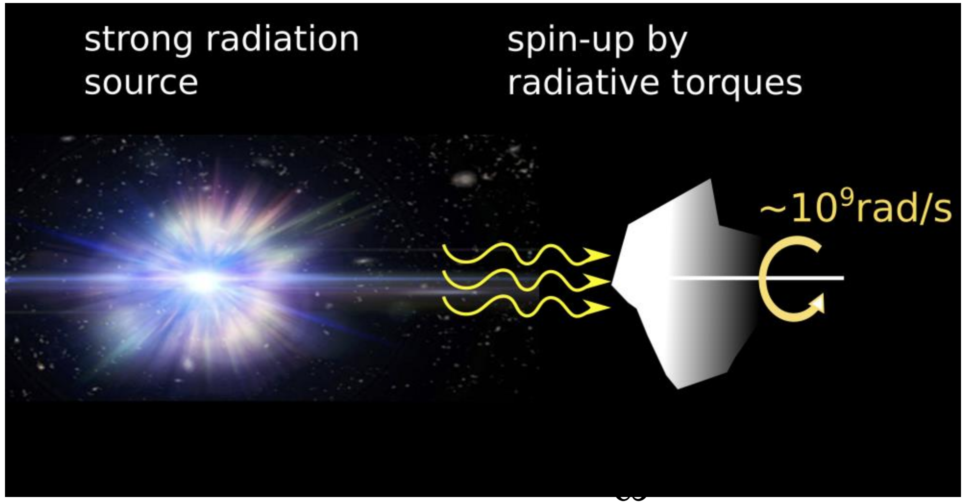
$$f_{\mathrm{IR}} = 4\pi \int \lambda^2\, Q_{\mathrm{abs}} B_\lambda(T_{\mathrm{dust}})\, \mathrm{d}\lambda$$

$\Rightarrow$ Total rotational drag torque

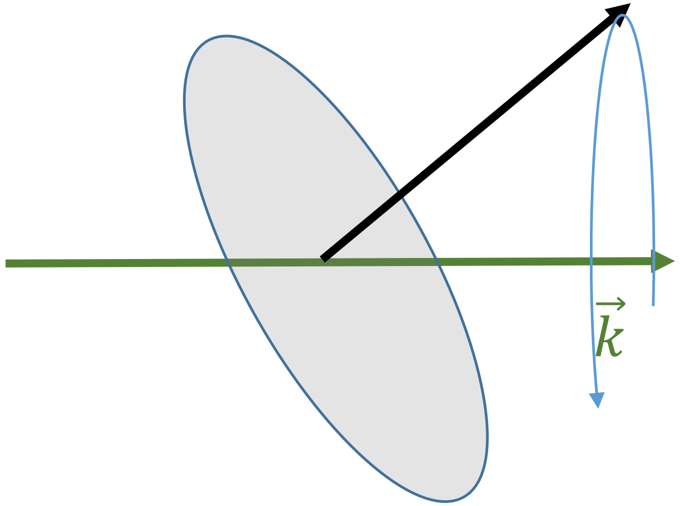
$$\vec{\Gamma}_{\mathrm{drag}} = -\hat{a}_1 \frac{I_1 \omega}{\tau_{\mathrm{drag}}}$$

$$\tau_{\mathrm{drag}}{}^{-1} = \left(\tau_{\mathrm{gas}}{}^{-1} + \tau_{\mathrm{IR}}{}^{-1}\right)^{-1}$$

Radiative torque (RAT)



Credit: Thiem Hoang



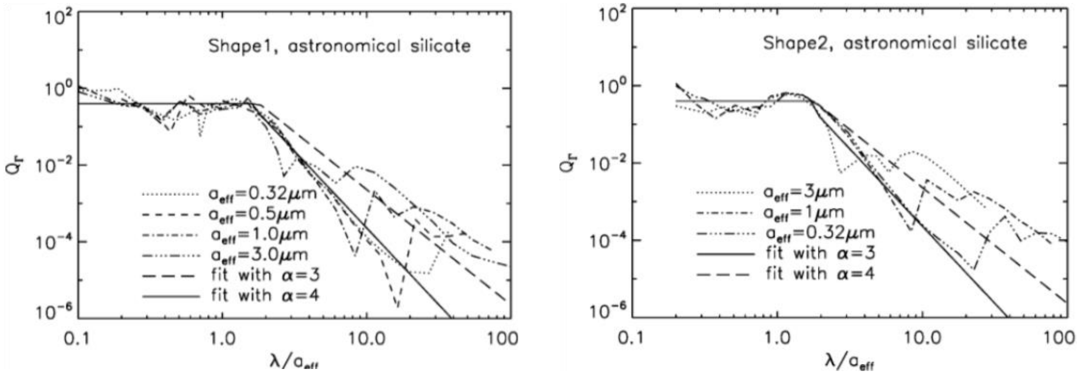
Differential scattering on non-spherical grains leads to a net torque

$$\vec{\Gamma}_{\mathrm{RAT}} = \frac{1}{2\pi}\hat{a}_1 \int \gamma_\lambda\, a_{\mathrm{eff}}^2 u_\lambda \lambda \vec{Q}_{\mathrm{RAT}}\, \mathrm{d}\lambda$$

Parametrization of the RAT efficiency

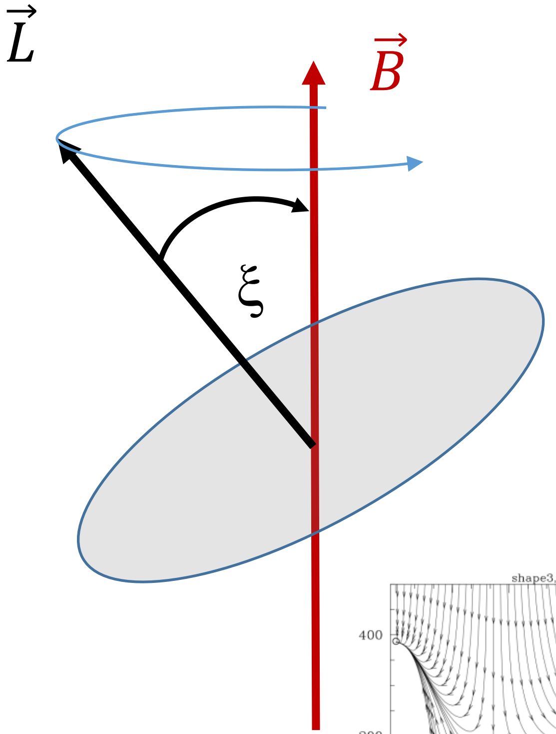
$$Q_{\mathrm{RAT}} = 0.4 \begin{cases} 1 & \text{if } \lambda < a_{\mathrm{eff}} \\ \left(\frac{\lambda}{a_{\mathrm{eff}}}\right)^{-q} & \text{otherwise} \end{cases}$$

Precession timescale around the wave vector $\vec{k}$ on a timescale τ_{k}



Lazarian & Hoang 2007

Alignment dynamics

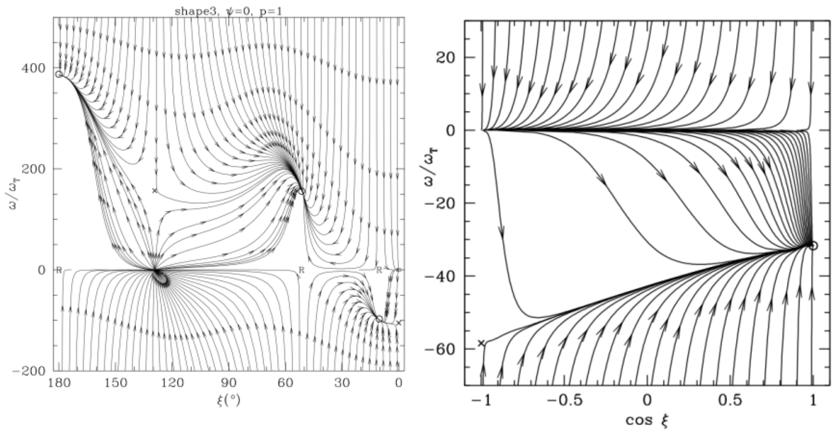


spin-down, precession

$$\frac{d\vec{L}}{dt} = \vec{\Gamma}_{\mathrm{RAT}} + \vec{\Gamma}_{\mathrm{drag}} + \vec{\Gamma}_{\mathrm{DG}} + \vec{\Gamma}_{\mathrm{Ba}}$$

spin-up, alignment

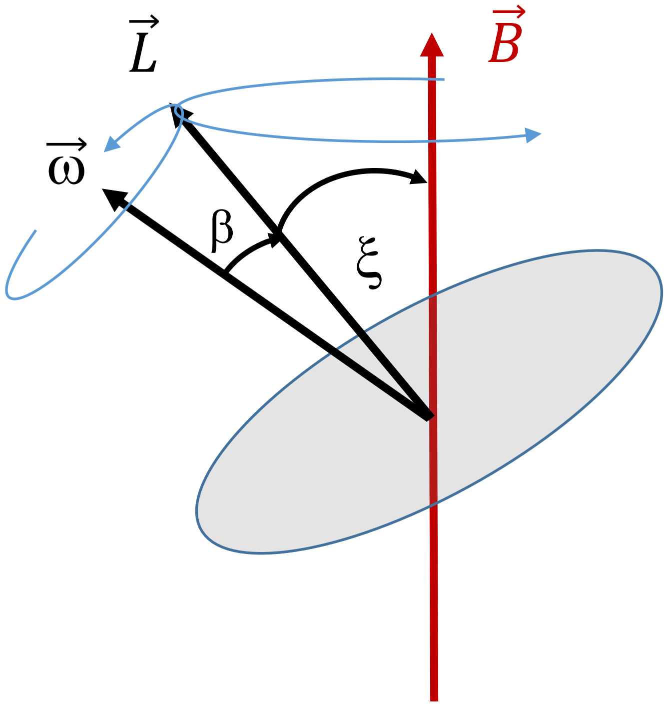
Analyze the appearance of attractor points for an grain ensemble and compare the characteristic time scales for the static solution $\frac{d\vec{L}}{dt} = 0$ to derive characteristic size limits



Draine & Weingartner 1998

- RAT spin-up vs. randomization timescales
  $\tau_{\mathrm{k}}/\tau_{gas} \Rightarrow a_{\mathrm{alig}}$
- Magnetic field vs. radiation field timescales
  $\tau_{\mathrm{Ba}}/\tau_{\mathrm{k}} \Rightarrow a_{\mathrm{krat}}$
- Barnett precession vs. randomization timescales
  $\tau_{\mathrm{Ba}}/\tau_{gas} \Rightarrow a_{\mathrm{Larm}}$

Internal alignment (IA)



If the grain undergoes strong thermal fluctuations the exchange of vibrational and rotational energy may disalign $\vec{L}$ and $\vec{\omega}$. The angular distribution of the angle between and follows

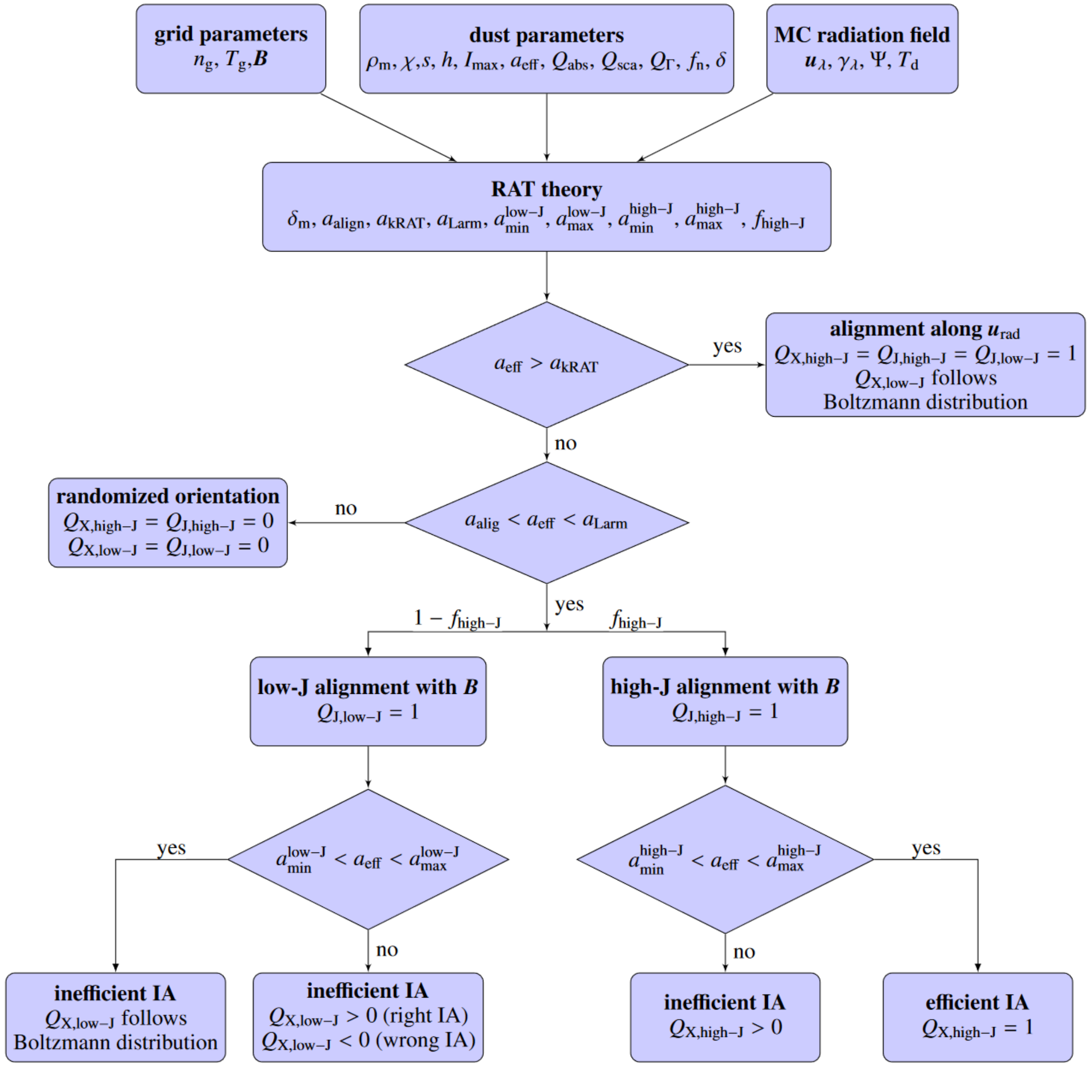
$$f(\beta) \propto \sin\left(-\frac{L^2}{2I_1 k_B T_B}\left[1-\left(\frac{I_1}{I_2+I_3}-1\right)\sin^2\beta\right]\right)$$

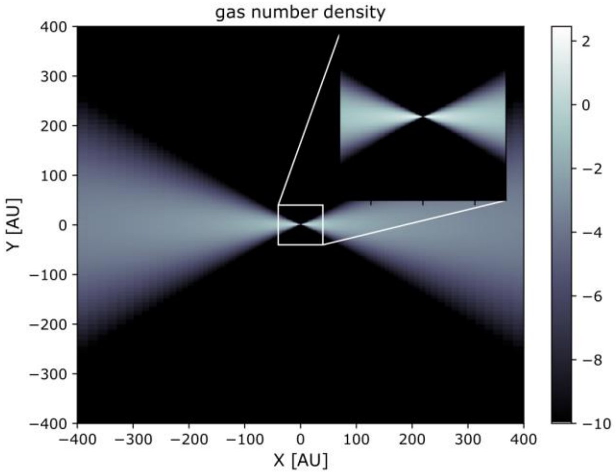
with

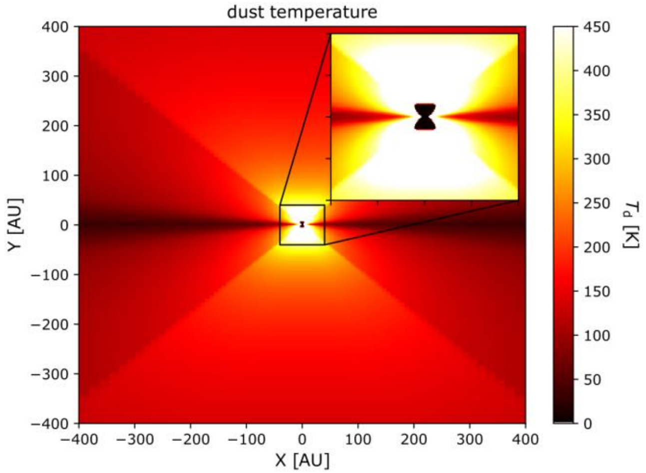
$$\int_0^{\pi} f(\beta)\sin\beta \; d\beta = 1$$

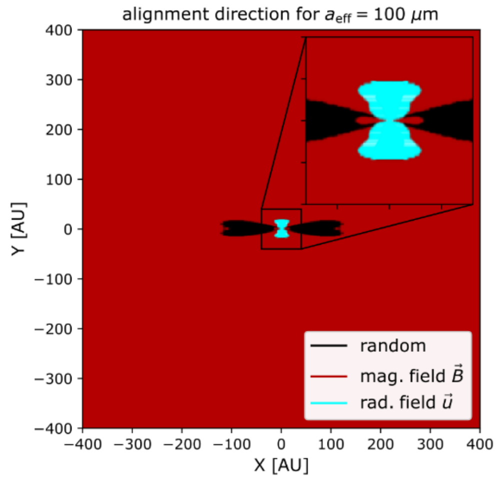
Lazarian & Robergee 1997

The grain alignment direction







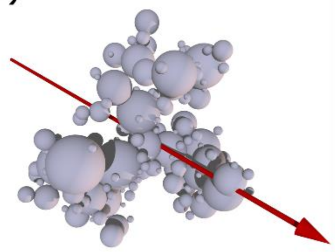


Mechanical grain alignment

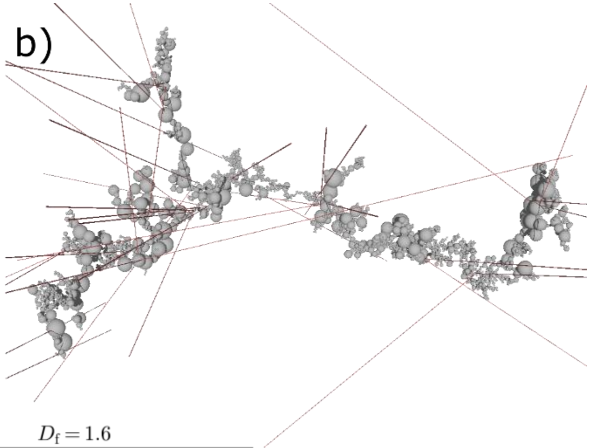
Direct Monte Carlo Simulation in the free molecular flow regime

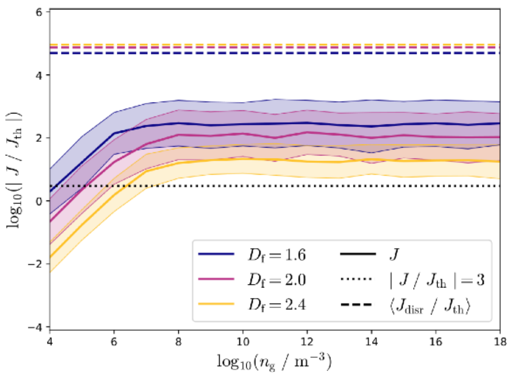
- Dimensionless gas velocity $s = v/v_{\mathrm{th}}$ and drift: $\Delta s = (v_{\mathrm{gas}} - v_{\mathrm{dust}})/v_{\mathrm{th}}$
- Skewed velocity distribution: $f(s, \Delta s, \theta)$
- Gas trajectories: $P_{\Delta s}(\theta) = \int_0^\infty f(s, \Delta s, \theta)\, ds$

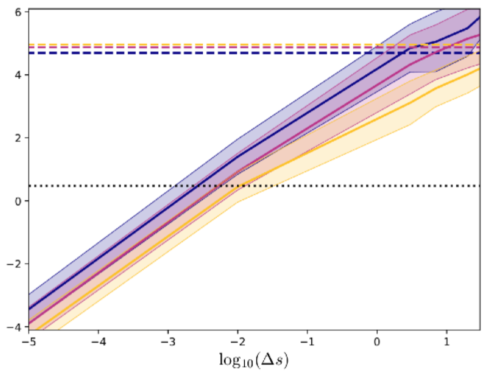
a)

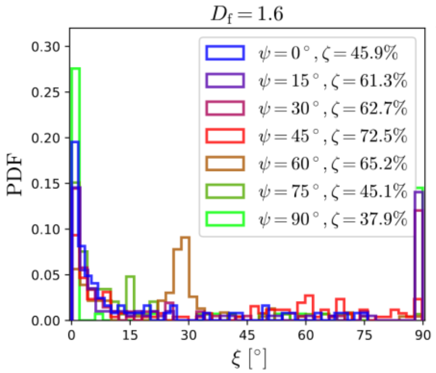


b)





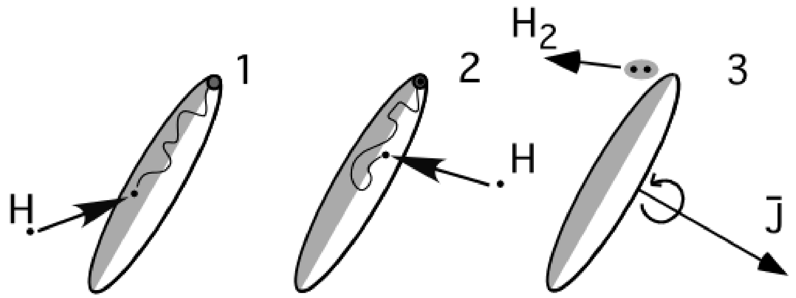




- Fractal like dust aggregate experience a mechanical torque $\vec{\Gamma}_{\mathrm{mech}}$
- Mechanical torque predicts alignment with magnetic field
- It is yet unclear how $\vec{\Gamma}_{\mathrm{mech}}$ interacts with $\vec{\Gamma}_{\mathrm{RAT}}$

Hydrogen formation

The formation of molecular hydrogen in occurs on of grains where the surface acts as an catalyst.



Additional change in angular momentum J may induce a torque $\vec{\Gamma}_{\mathrm{H2}}$ (Purcell torque, Pinwheel torque, rocket thrusters) :

- Formation energy (4.5 eV) is much large than any other process
- How much formation energy is transferred onto the grain surface?
- Is the $\vec{\Gamma}_{\mathrm{H2}}$ a spin-up or a spin-down torque?

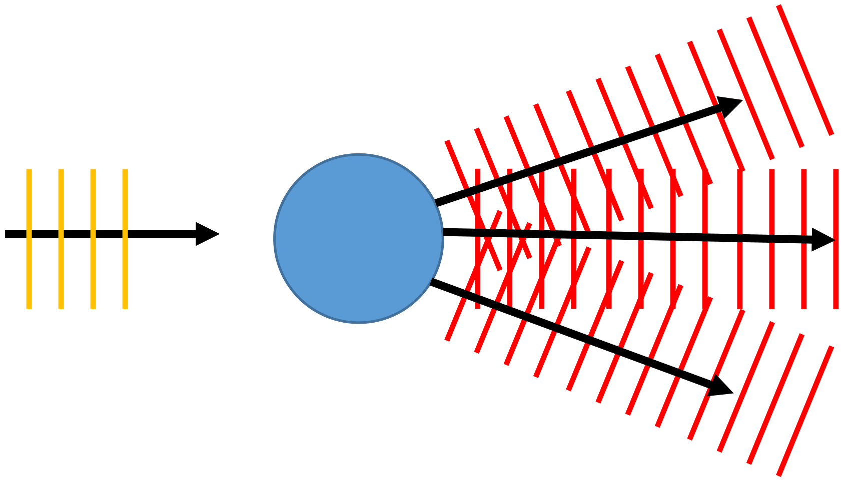
The incomplete picture

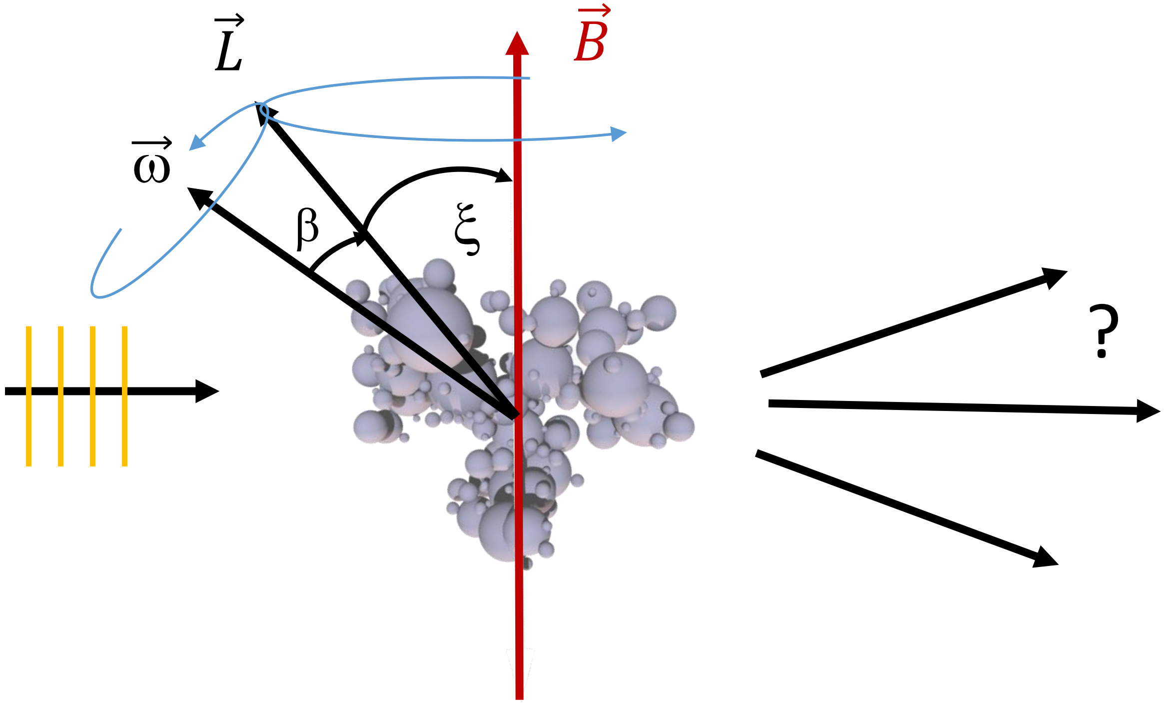
Observation	Larger grains are better aligned	General alignment only active for $a > 0.045\ \mu m$	H_2 formation enhances alignment	H_2 formation not required for alignment	Alignment seen when $T_{gas} = T_{dust}$	Alignment is not correlated with ferromag-netic inclusions	Alignment is lost at $A_V \sim 20$ mag	Alignment depends on angle between radiation and magnetic fields	Carbon grains are unaligned
Theory									
Davis-Greenstein	−				−				
Super-paramagnetic	+				−	−			
Suprathermal			+	−					
Mechanical			−				−		−
Radiative alignment torque	+	+	+				+	+	+

Table taken from B-G Anderson 2013: Interstellar Grain Alignment - Observational Status

Scattering on non-spherical grains

The scattering on spherical grains can be well modelled
(see 2nd lecture)

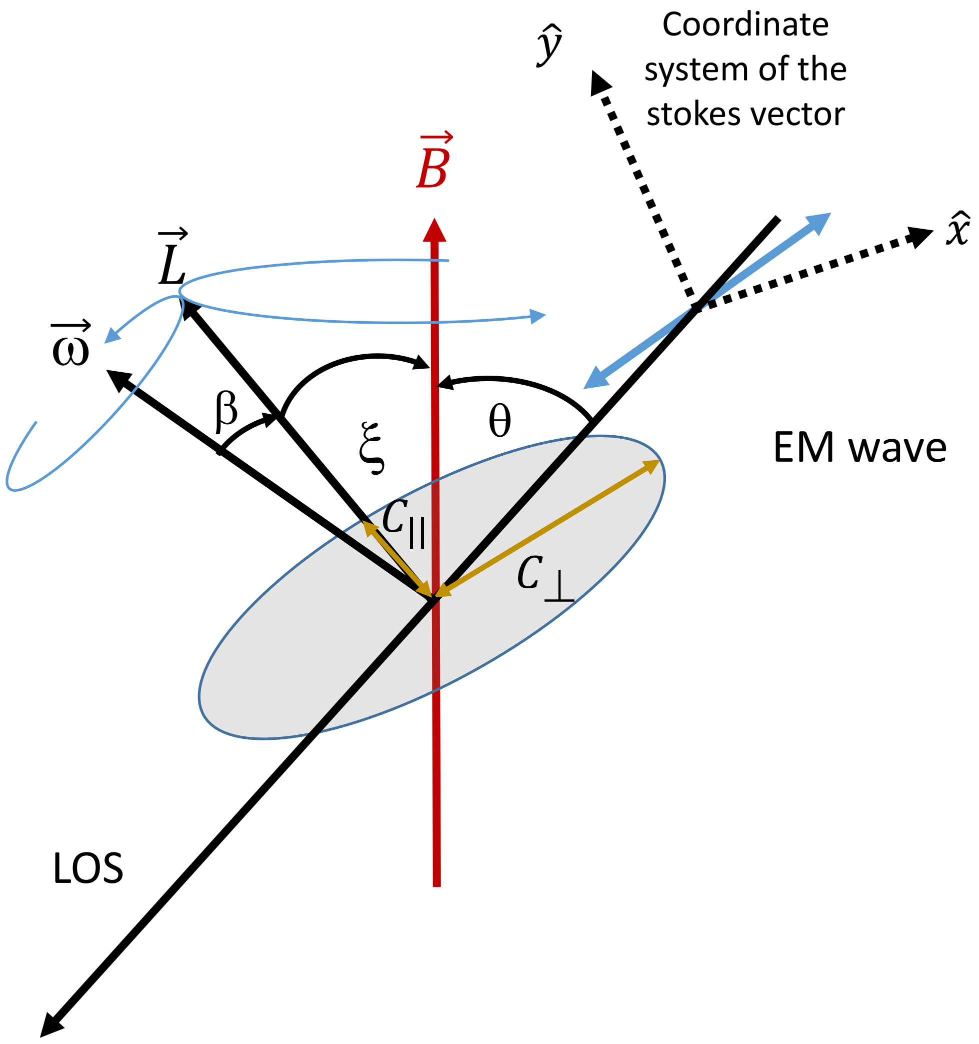




- What are the observational implications of non-spherical rotating dust grains partially aligned with the magnetic field?
- Despite the complexity of the problem, individual theories and tools to model such observations are already available but need to be applied in tandem

Dust Polarization

The Rayleigh reduction factor (RRF)



The RRF quantifies the reduction of polarization by grain precession

$$R=\frac{3}{2}\left(\langle\cos^2\xi\rangle-\frac{1}{3}\right)\left(\langle\cos^2\beta\rangle-\frac{1}{3}\right)$$

where $\langle\cos^2\xi\rangle$ in the ensemble average

In the reference frame of the dust the polarization is determined by the cross sections parallel $C_{||}$and perpendicular $C_{\perp}$ to the grain rotation

In the reference frame of the polarized light the cross sections are

- $C_x = C_R + \frac{1}{3}R\left(C_{||} - C_{\perp}\right)$
- $C_y = C_R + \frac{1}{3}R(C_{||} + C_{\perp})(1 - 3\sin^2\theta)$
- $C_R = \frac{2C_{||}+C_{\perp}}{3}$

where C_R represents a randomized grain

Dichroic extinction

In the reference frame of the Stokes vector:

- Total extinction: $C_{ext} = \frac{1}{2}(C_{ext,x} + C_{ext,y})$
- Dichroic extinction $\Delta C_{ext} = \frac{1}{2}(C_{ext,x} - C_{ext,y})$

<u>no grain alignment R=0</u>

- $C_{ext} = C_R$
- $\Delta C_{ext} = 0$

<u>perfect grain alignment R=1</u>

LOS perpendicular to $\vec{B}$ i.e θ=0°

- $C_{ext} = \frac{1}{2}(C_{\perp} + C_{||})$
- $\Delta C_{ext} = \frac{1}{2}(C_{\perp} - C_{||})$

LOS parallel to $\vec{B}$ i.e θ=0°

- $C_{ext} = C_{||}$
- $\Delta C_{ext} = 0$ $\Rightarrow$ No dust along the LOS

The same procedure for the absorption cross sections C_{abs} and ΔC_{abs}

The RT equation for aligned dust

The cross sections incorporate now the grain alignment physics

Dust RT equation:

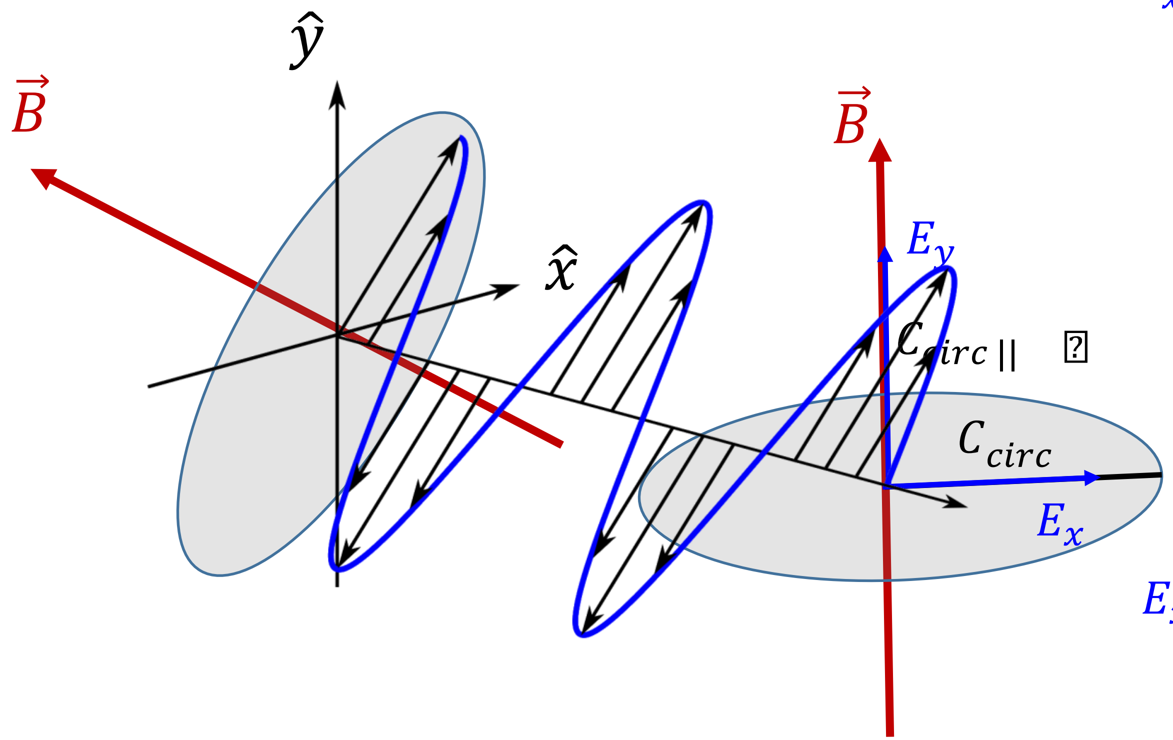
$$\frac{d}{n_{\mathrm{d}}dr}\begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = -\begin{pmatrix} \bar{C}_{\mathrm{ext}} & \Delta\bar{C}_{\mathrm{ext}} & 0 & 0 \\ \Delta\bar{C}_{\mathrm{ext}} & \bar{C}_{\mathrm{ext}} & 0 & 0 \\ 0 & 0 & \bar{C}_{\mathrm{ext}} & \Delta\bar{C}_{\mathrm{circ}} \\ 0 & 0 & -\Delta\bar{C}_{\mathrm{circ}} & \bar{C}_{\mathrm{ext}} \end{pmatrix}\begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} + B_{\lambda}(T_{\mathrm{d}})\begin{pmatrix} \bar{C}_{\mathrm{abs}} \\ \Delta\bar{C}_{\mathrm{abs}} \\ 0 \\ 0 \end{pmatrix}$$

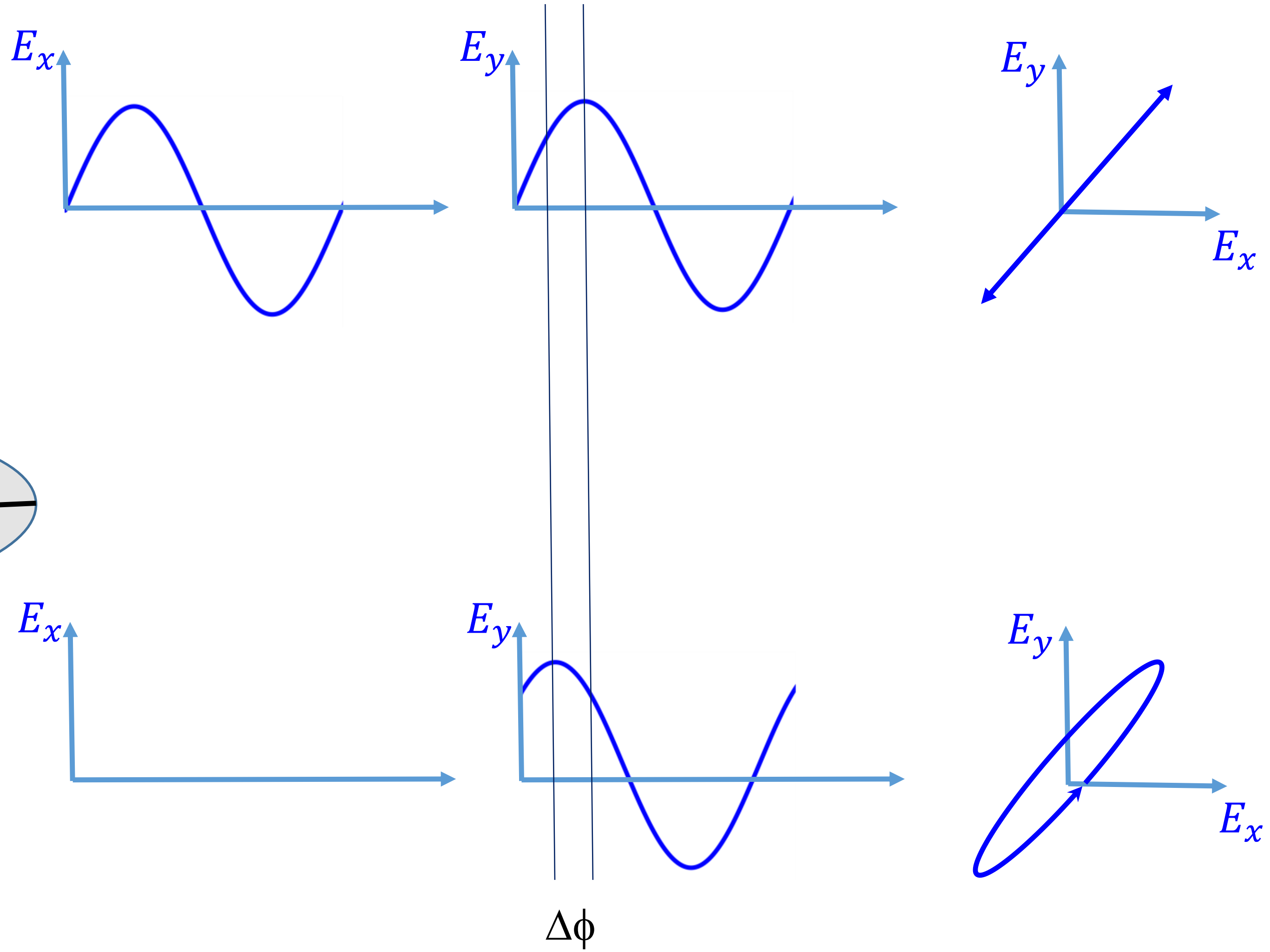
Stokes I → Q

Stokes U → V

Where does circular polarization come from in the first place?

Circular polarization





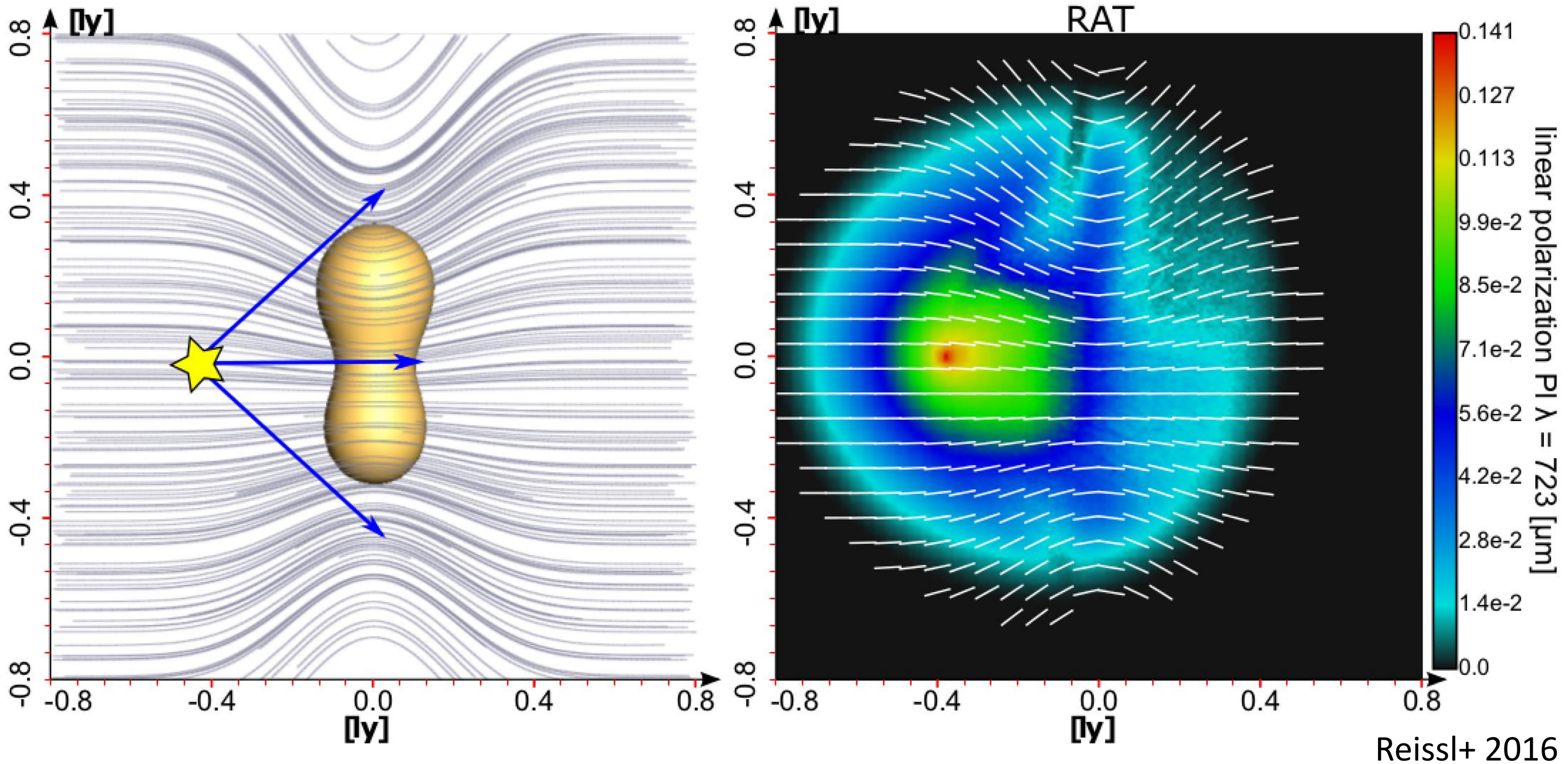
Phase lag $\Delta\phi = n_d \mathrm{L}\, \Delta C_{circ}$
where L is the path length through the dust grain

Observational Implications

RAT alignment on small scales

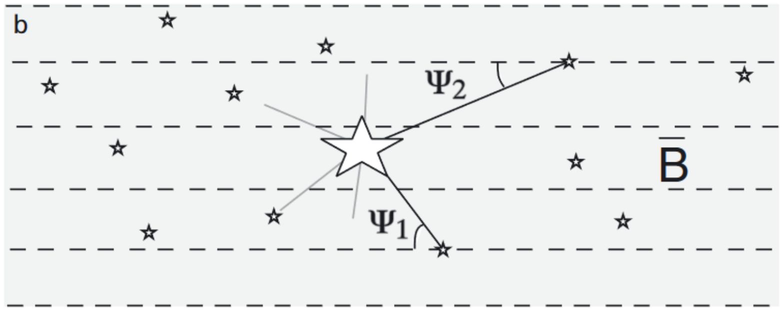
Predictions of RAT alignment theory:

- Polarization scales with the radiation field
- Angular dependence of $\vec{k}$ and $\vec{B}$ - field



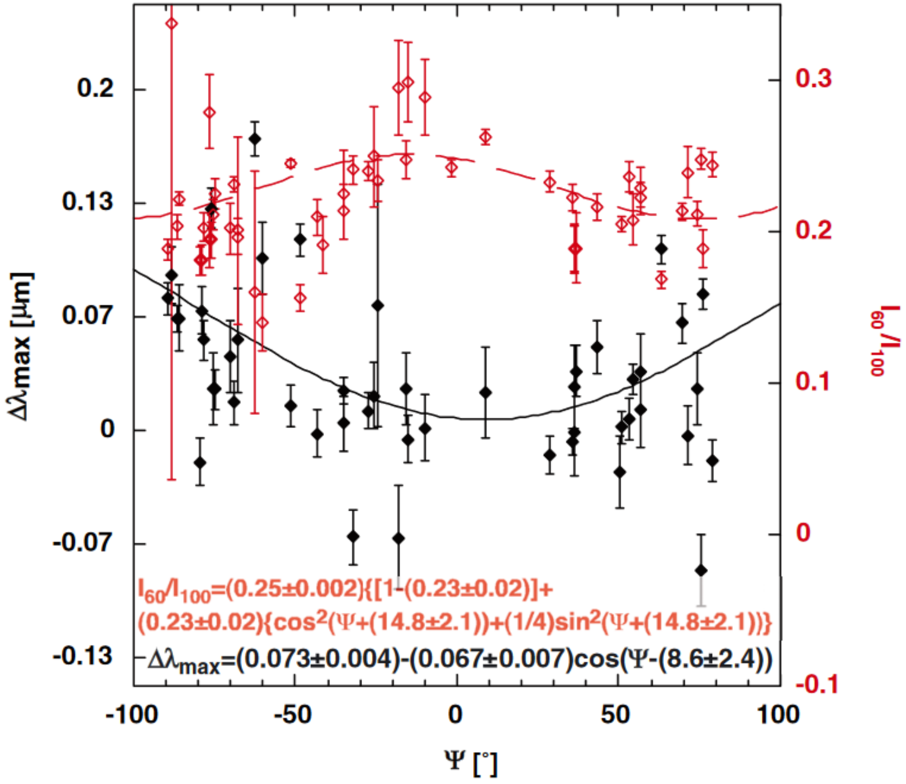
Reissl+ 2016

Observational confirmation



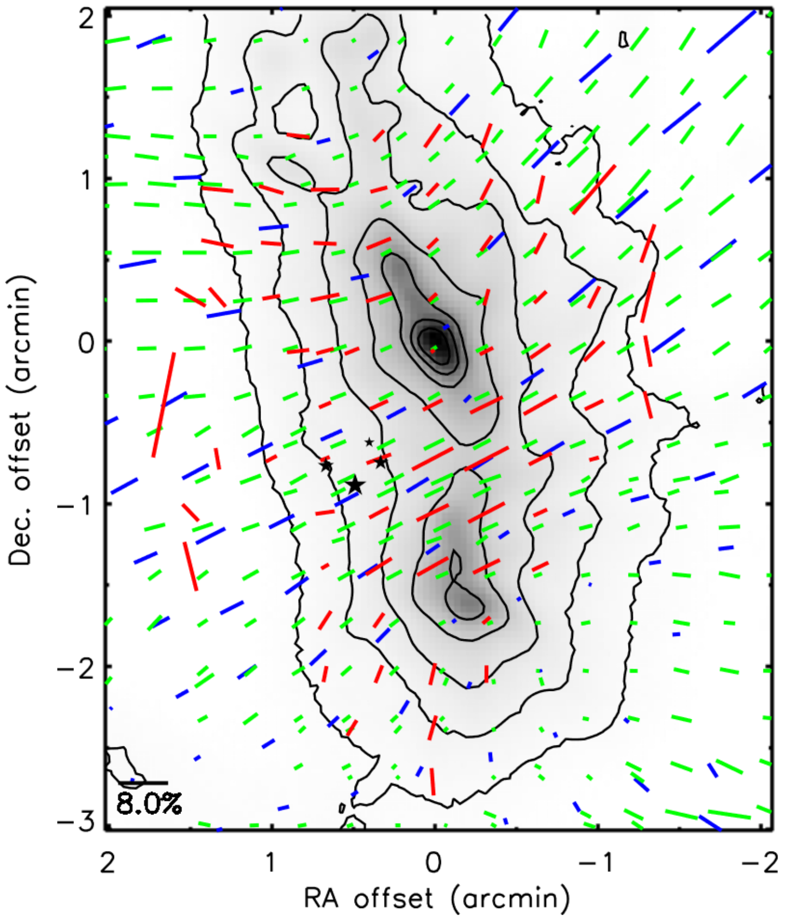
Heating of aligned grains dependent on orientation (?)

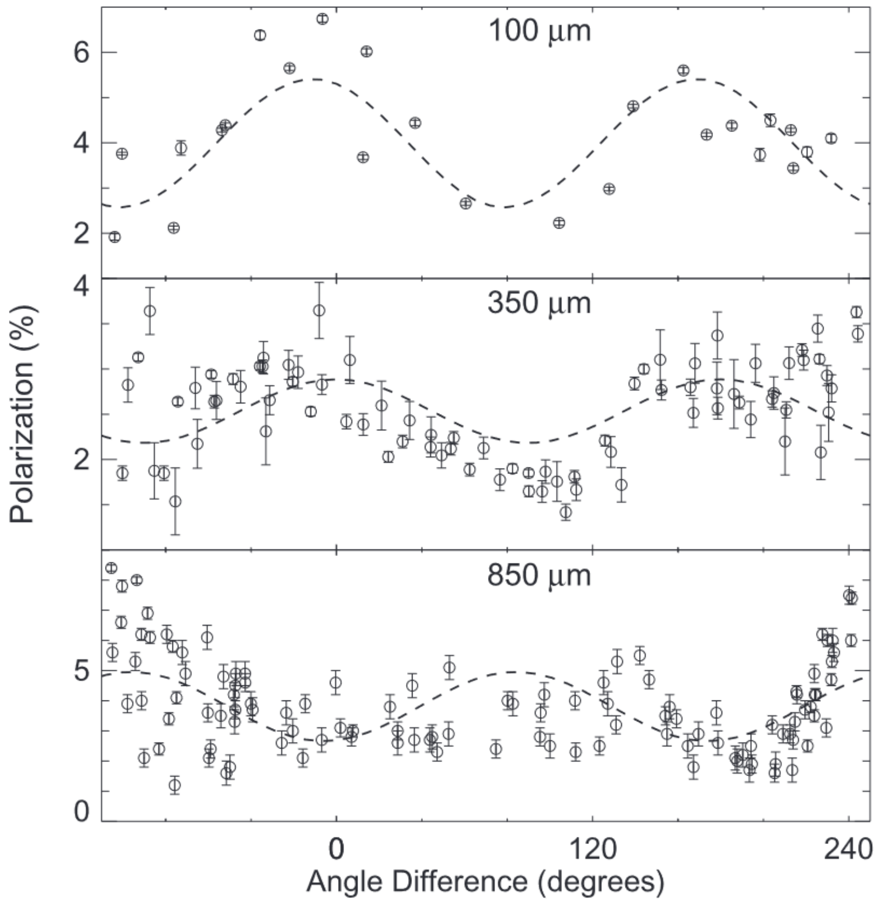
The heating effect may not be efficient enough to account for the flux!



Andersson+ 2011

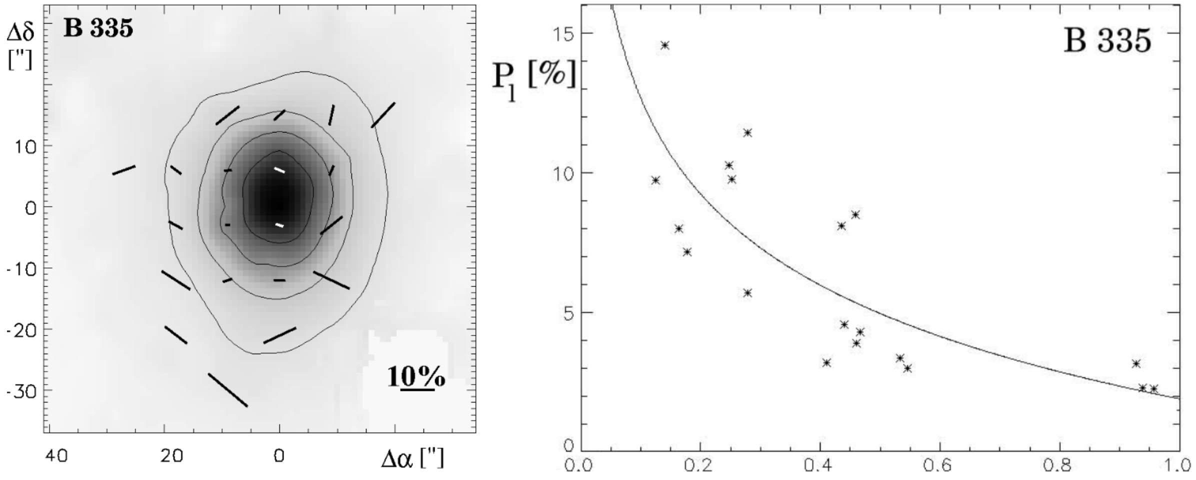
Observational confirmation





Vaillancourt & Andersson 2015

Polarization holes in Bok Globules



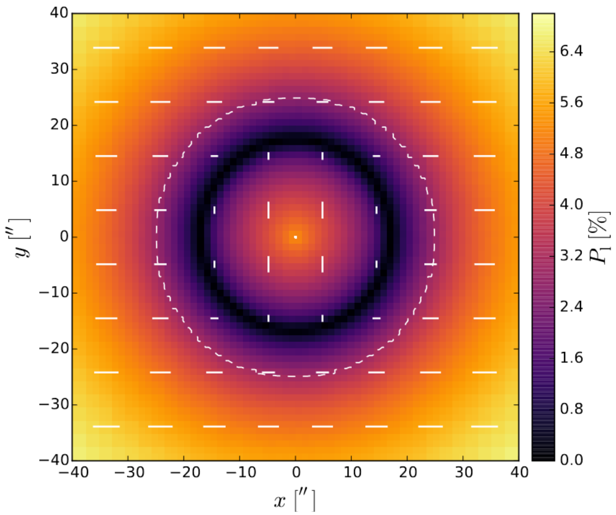
Wolf+ 2003

Solve RT equation for Q=0 & U=0

Critical distance $l_{crit} = \frac{1}{n_d(C_{ext\perp} - C_{ext||})} ln\left(\frac{C_{abs\perp}}{C_{abs||}}\right)$

$l < l_{crit}$: emission dominates
$l = l_{crit}$: extinction and emission cancel each other out
$l > l_{crit}$: extinction dominates

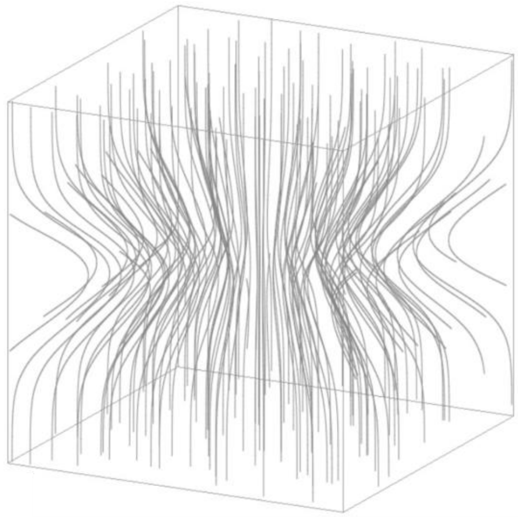
$\Rightarrow$ Holes can also be explained by extinction and emission

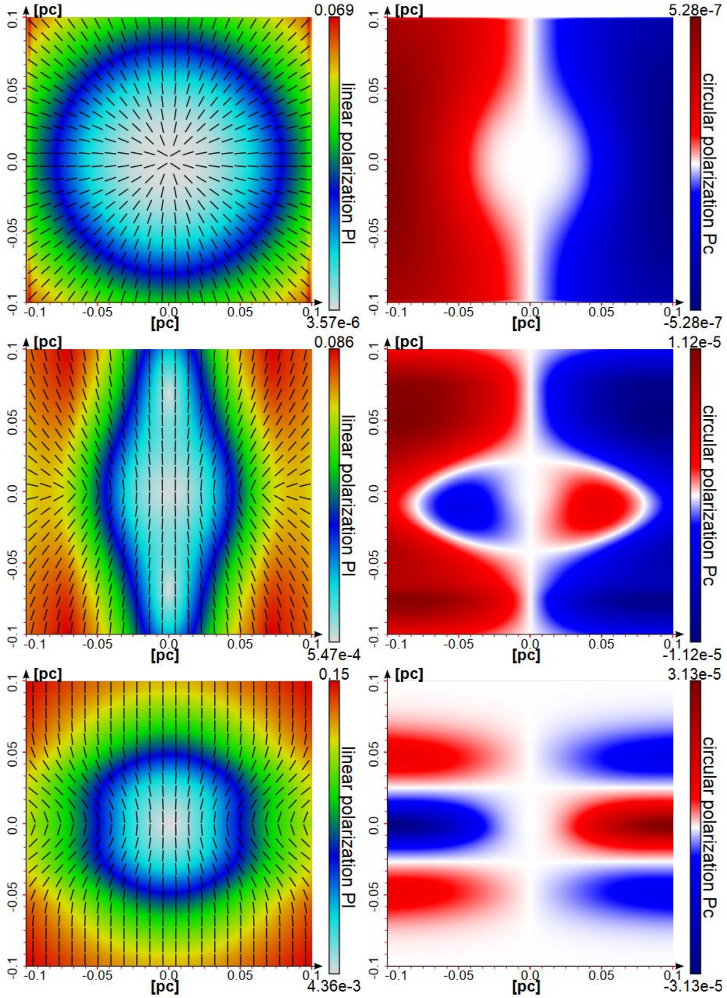


Reissl+ 2014, Brauer+ 2016

Circular polarization as a magnetic field tracer

Analytic representation of an hourglass field



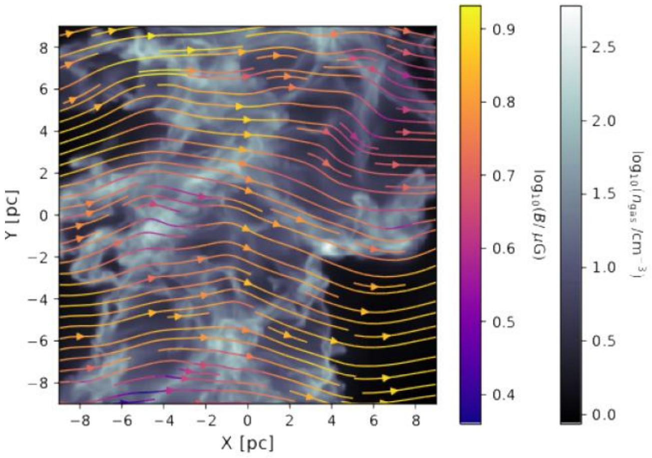


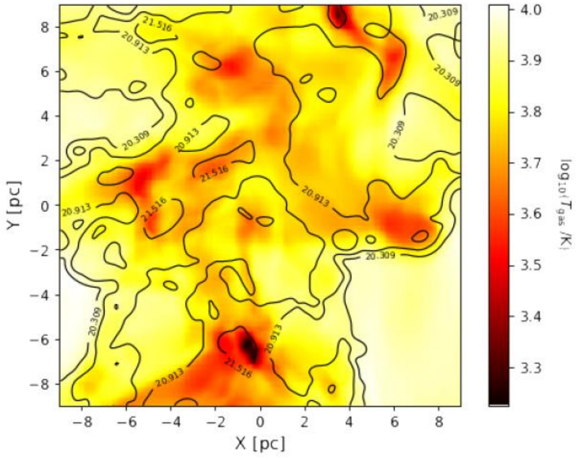
Reissl+ 2014

- Orientation of field lines along the LOS lead to characteristic polarization pattern
- Circular polarization is very low (beyond observation)
- Center is severely depolarized

Dust polarization on large scales

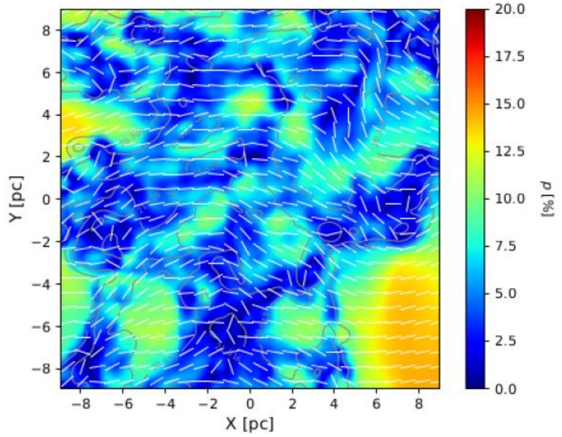
MHD simulation of the diffuse ISM

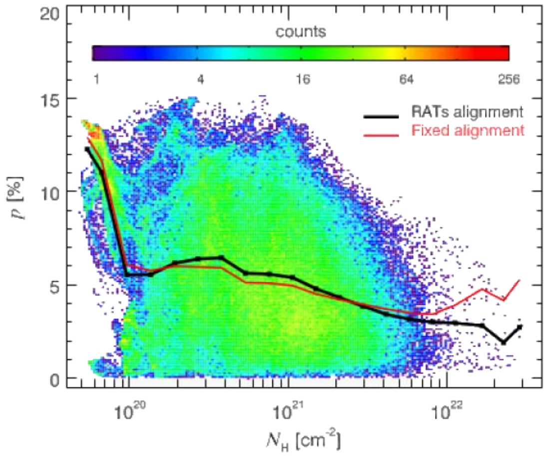




(Hennebelle+ 2008, Planck Collaboration XX 2015)

Synthetic observations





Reissl+ 2020

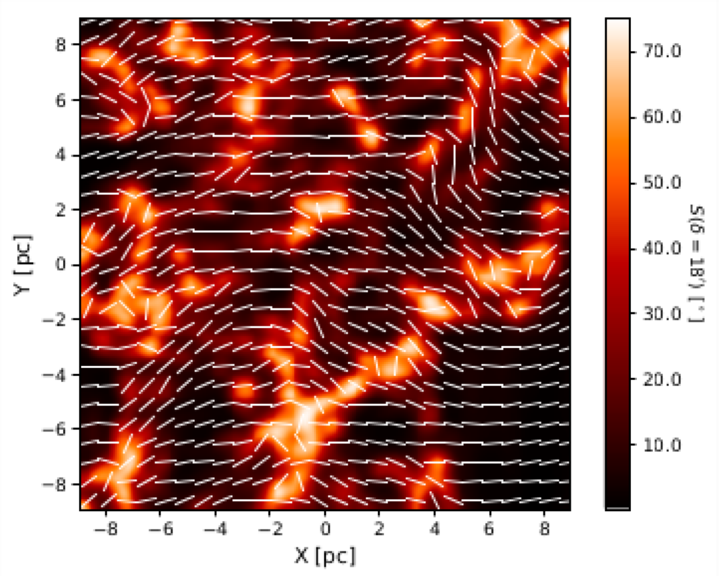
Polarization dispersion analysis

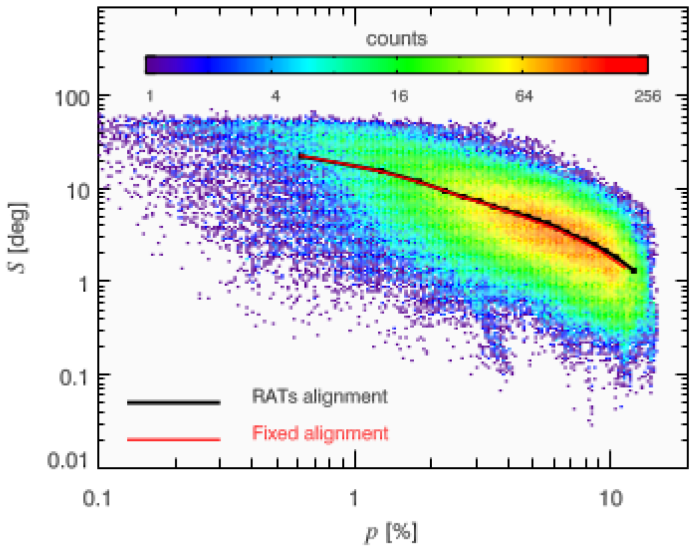
Polarization angle dispersion function:

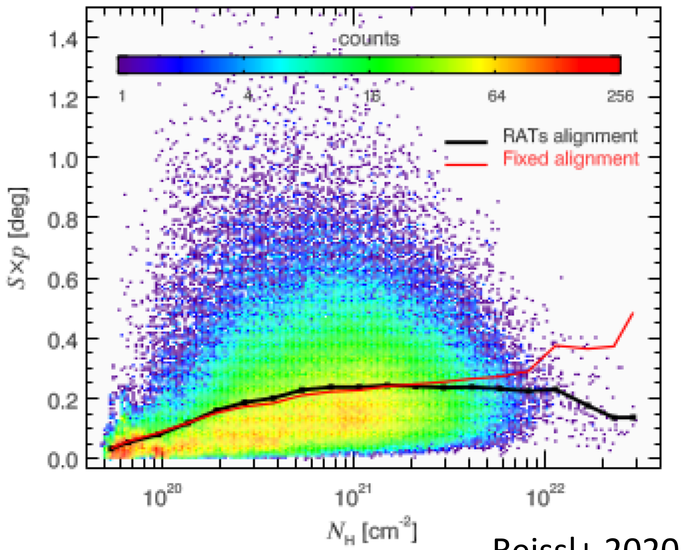
$$S(\vec{r},\delta) = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left[\chi(\vec{r}) - \chi(\vec{r}+\vec{\delta_i})\right]}$$

Grain alignment:

1. RAT theory
2. With fixed radius $a_{\mathrm{eff}} = 100\ \mu m$

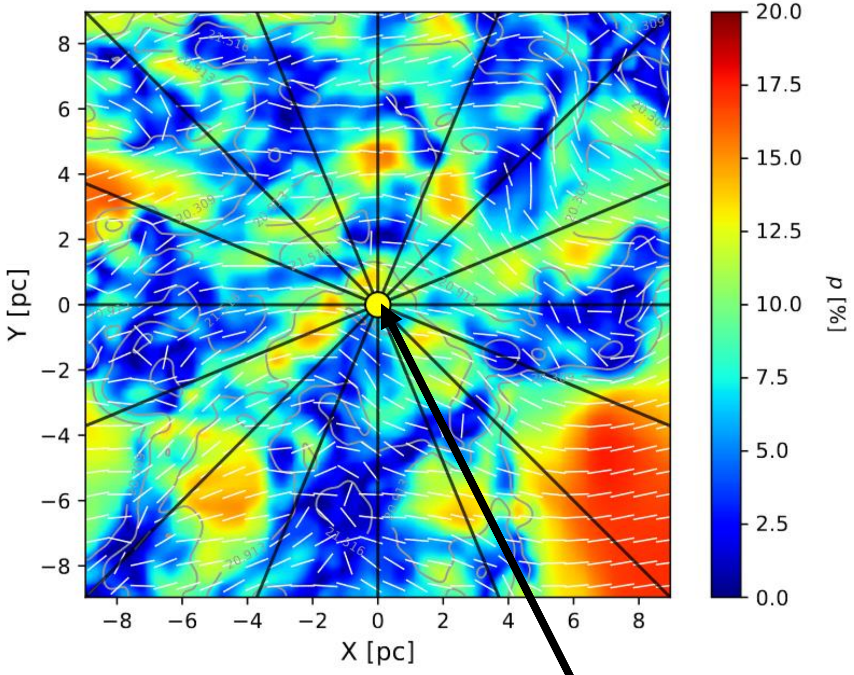


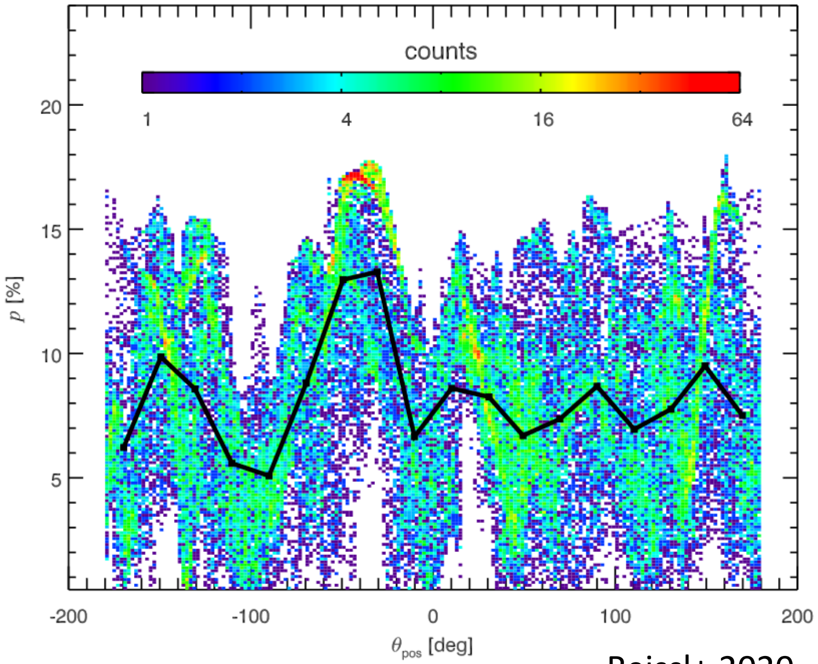




Reissl+ 2020

Angular dependency



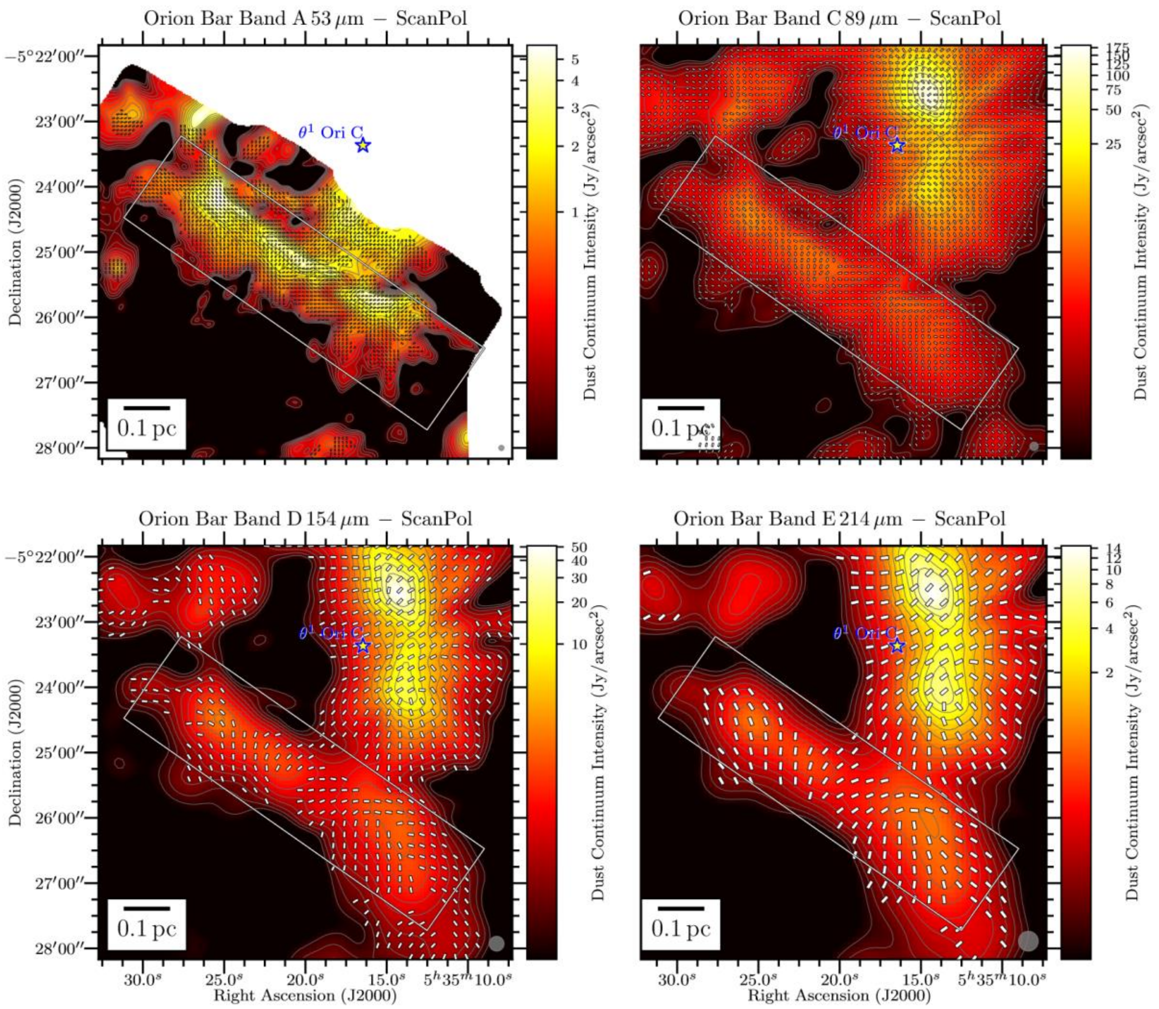


Reissl+ 2020

Introducing a single star reveals the predicted angular dependency

On the direction of alignment

If dust grains would align with $\vec{k}$ and not $\vec{B}$ on large scales, this would have serious consequences for magnetic field observations.

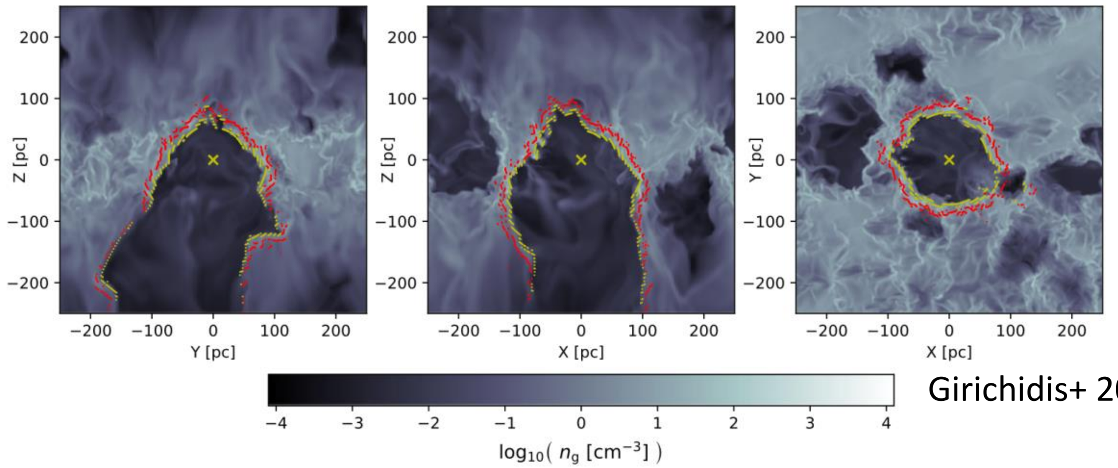


Le Gouellec+ 2023

Modelling shows that the grain sizes that could align with $\vec{k}$ would have been destroyed by rotational disruption in the first place.

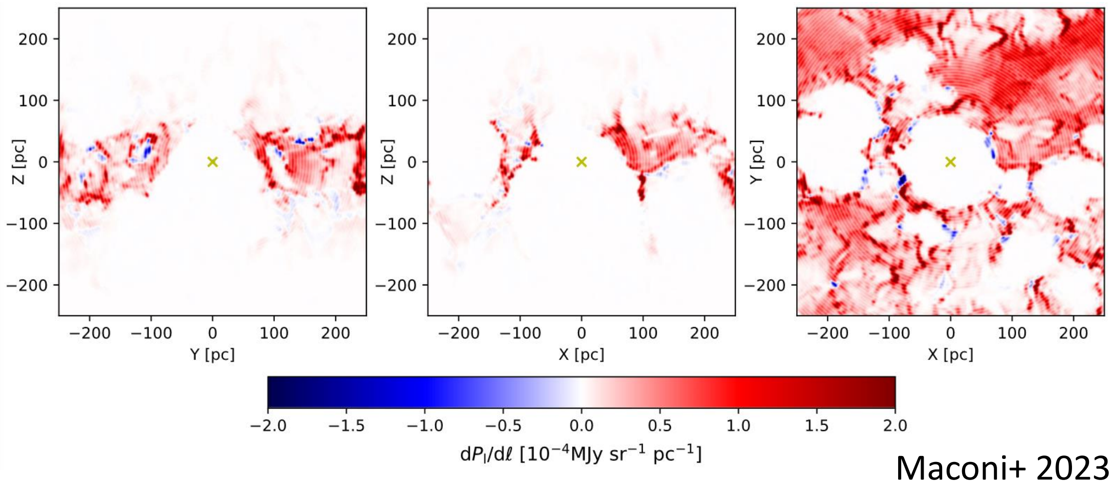
The origin of dust polarization in the MW

SILCC MHD simulation

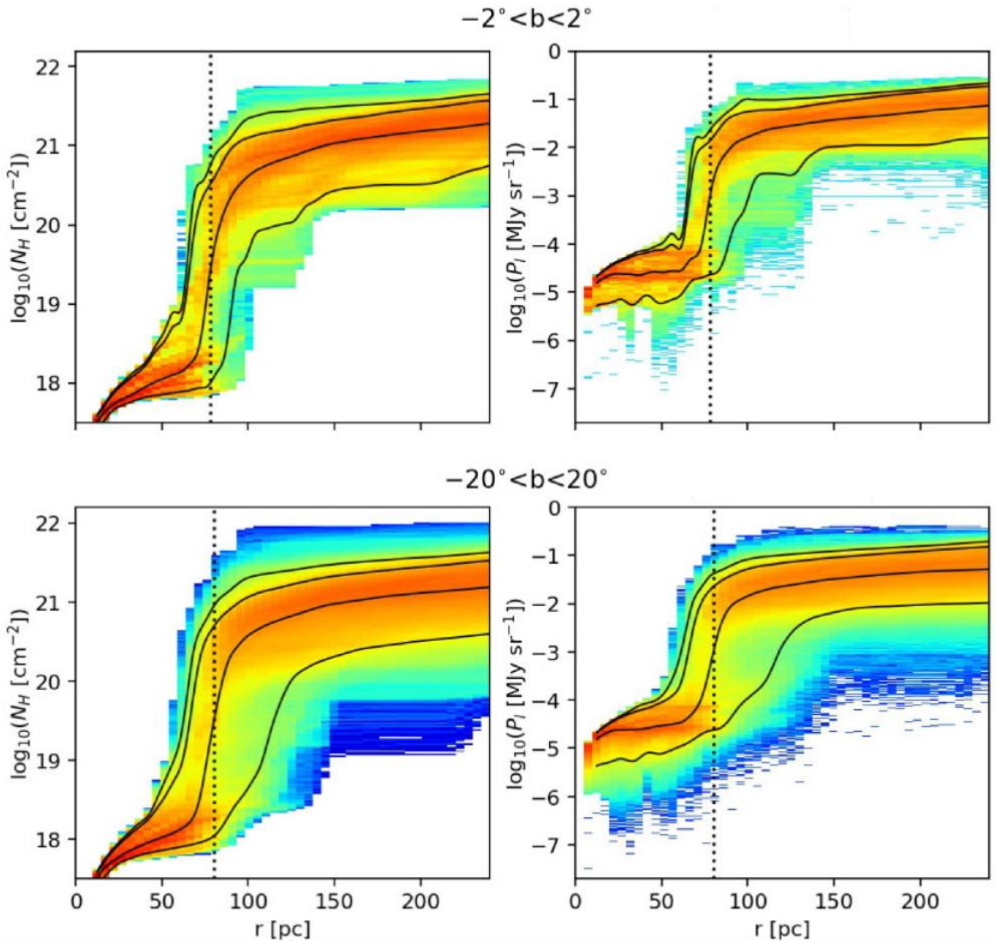


Girichidis+ 2018

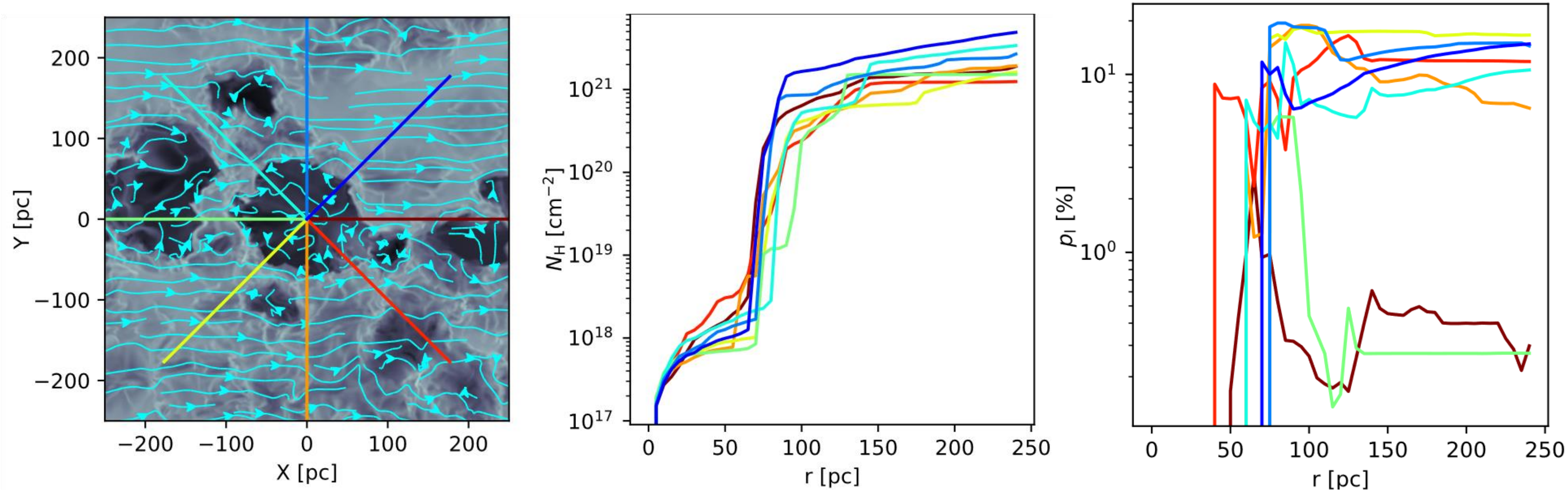
Change in polarization along the LOS



Maconi+ 2023



The origin of dust polarization in the MW



Maconi+ 2023

- The dust polarization is reduced along distinct directions (green, brown)
- This is due to both the turbulent medium as well as grain alignment