Houches School - The Physics of Star Formation
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Dust dynamics and dust evolution during star formation

~~(no) more equations~~

Less equations

Short summary of yesterday

- Dust dynamics is important and can be very different from the gas during star formation and disk evolution.
- The dynamics of grains is controlled by the Stokes number which depend on the grain and gas properties.
- Dust grains settle fast in protoplanetary disks and drift radially.
- The radial drift is fast compared to the disk lifetime. To understand for planet formation it is key to find mechanisms to overcome the radial drift barrier

Short summary of yesterday

$$\underbrace{\frac{\partial \vec{B}}{\partial t} + \nabla \times \left(\vec{v} \times \vec{B}\right)}_{\text{Advection}} = \underbrace{\nabla \times \left[\eta_o \nabla \times \vec{B}\right]}_{\text{Ohmic dissipation}} \underbrace{- \nabla \times \left[\frac{\eta_A}{|\vec{B}|^2}\left(\left(\nabla \times \vec{B}\right) \times \vec{B}\right) \times \vec{B}\right]}_{\text{Ambipolar "diffusion" Or ion-neutral drift}} \underbrace{+ \nabla \times \left[\frac{\eta_H}{|\vec{B}|^2}\left(\nabla \times \vec{B}\right) \times \vec{B}\right]}_{\text{Hall effect}}$$

Advection

Ohmic dissipation

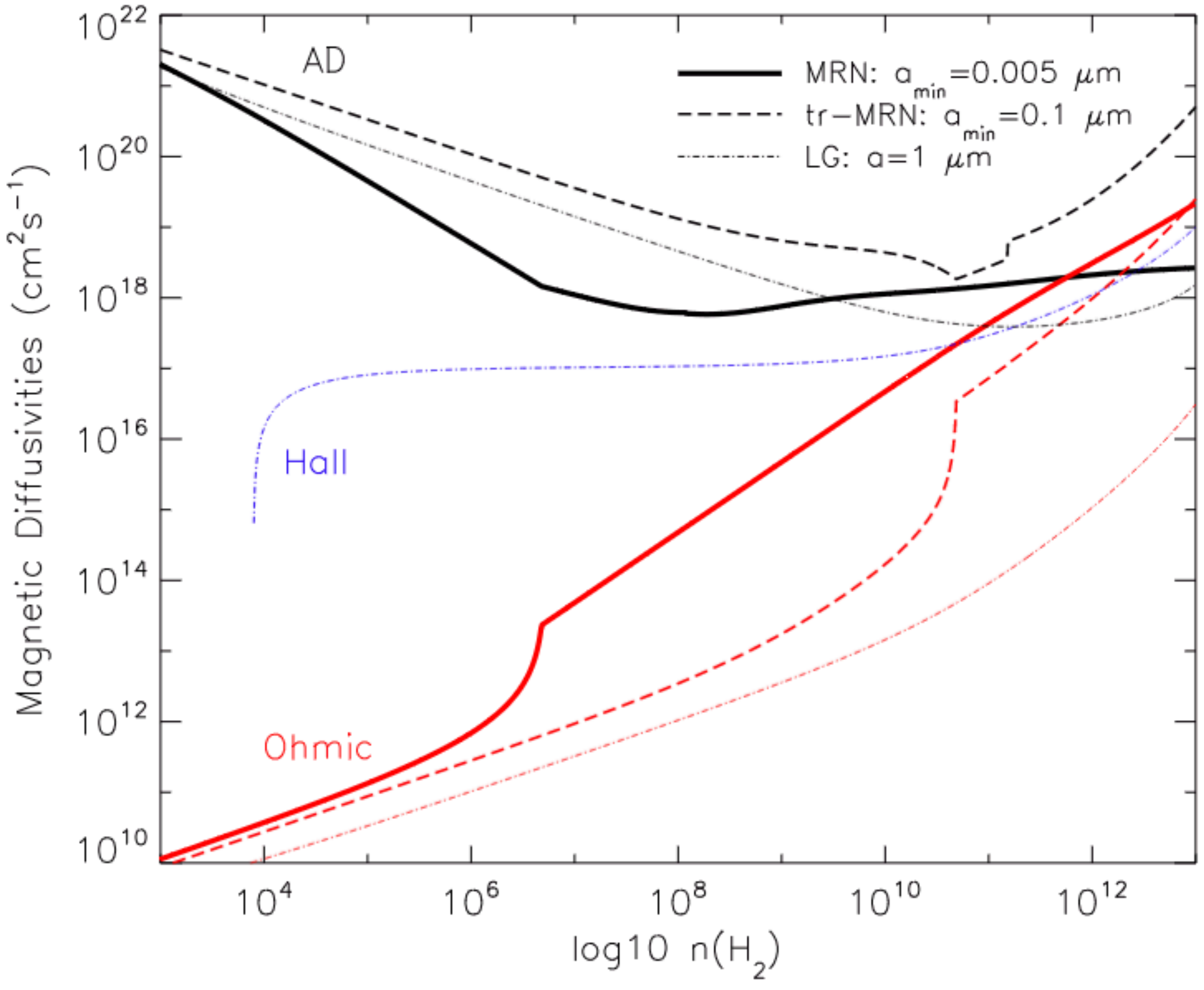
Ambipolar “diffusion”
Or ion-neutral drift

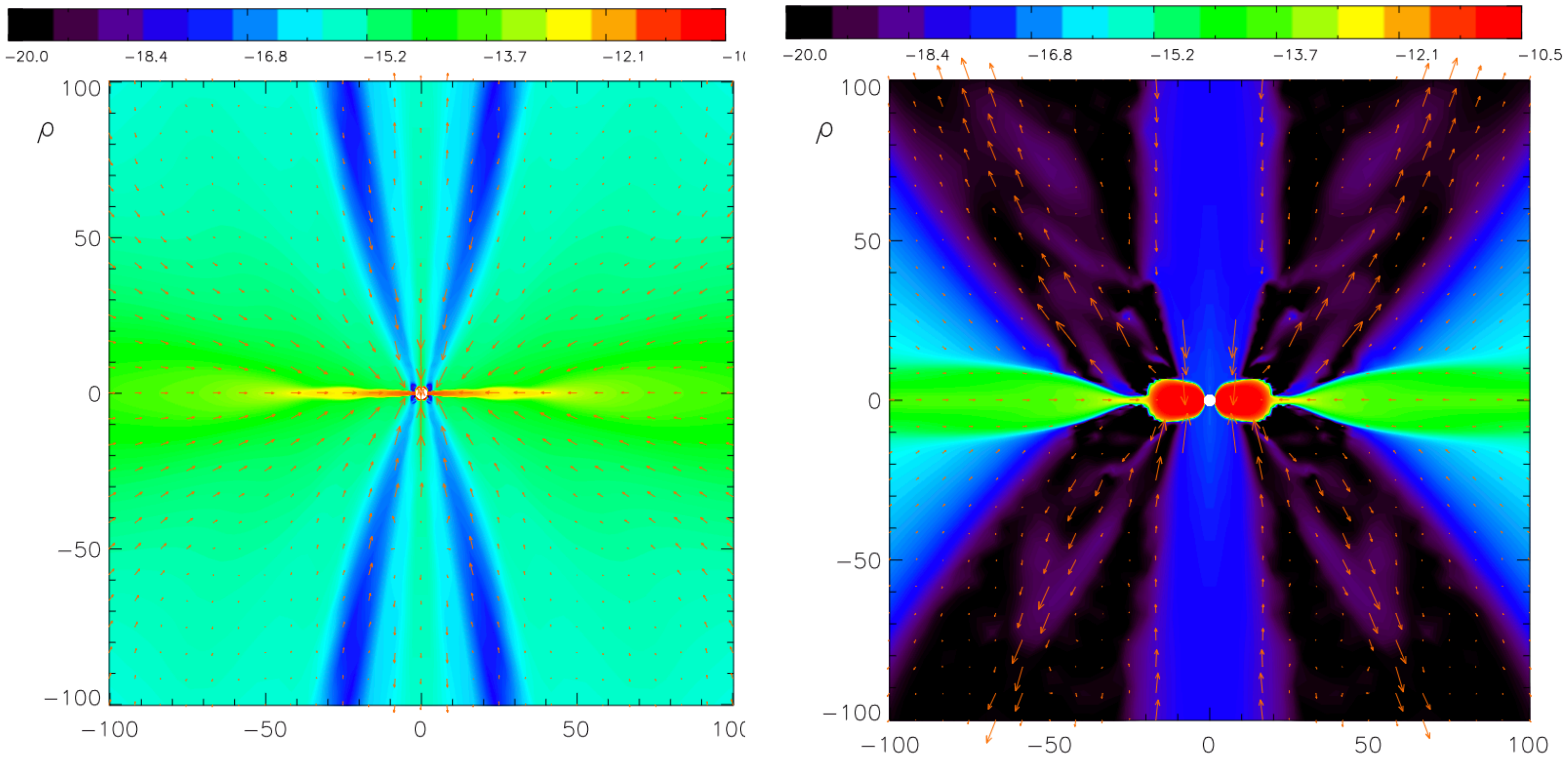
Hall effect

The resistivities depends on the abundance of charged particles, their charge, their Hall factor, the magnetic field strength

Role of grains in the MHD equations

Changing the dust size distribution (by for example removing the very small grains) can have a dramatic influence on the resistivities and therefore on the disk formation





Zhao, Caselli et al. (2016) : See also *Marchand et al. (2020)*

$$r_{\rm d,AD} \simeq 18 \text{ au}$$
$$\times\ \delta^{2/9} \left(\frac{\eta_{\rm AD}}{0.1 \text{ s}}\right)^{2/9} \left(\frac{B_z}{0.1 \text{ G}}\right)^{-4/9} \left(\frac{M_{\rm d} + M_*}{0.1\ M_\odot}\right)^{1/3}$$

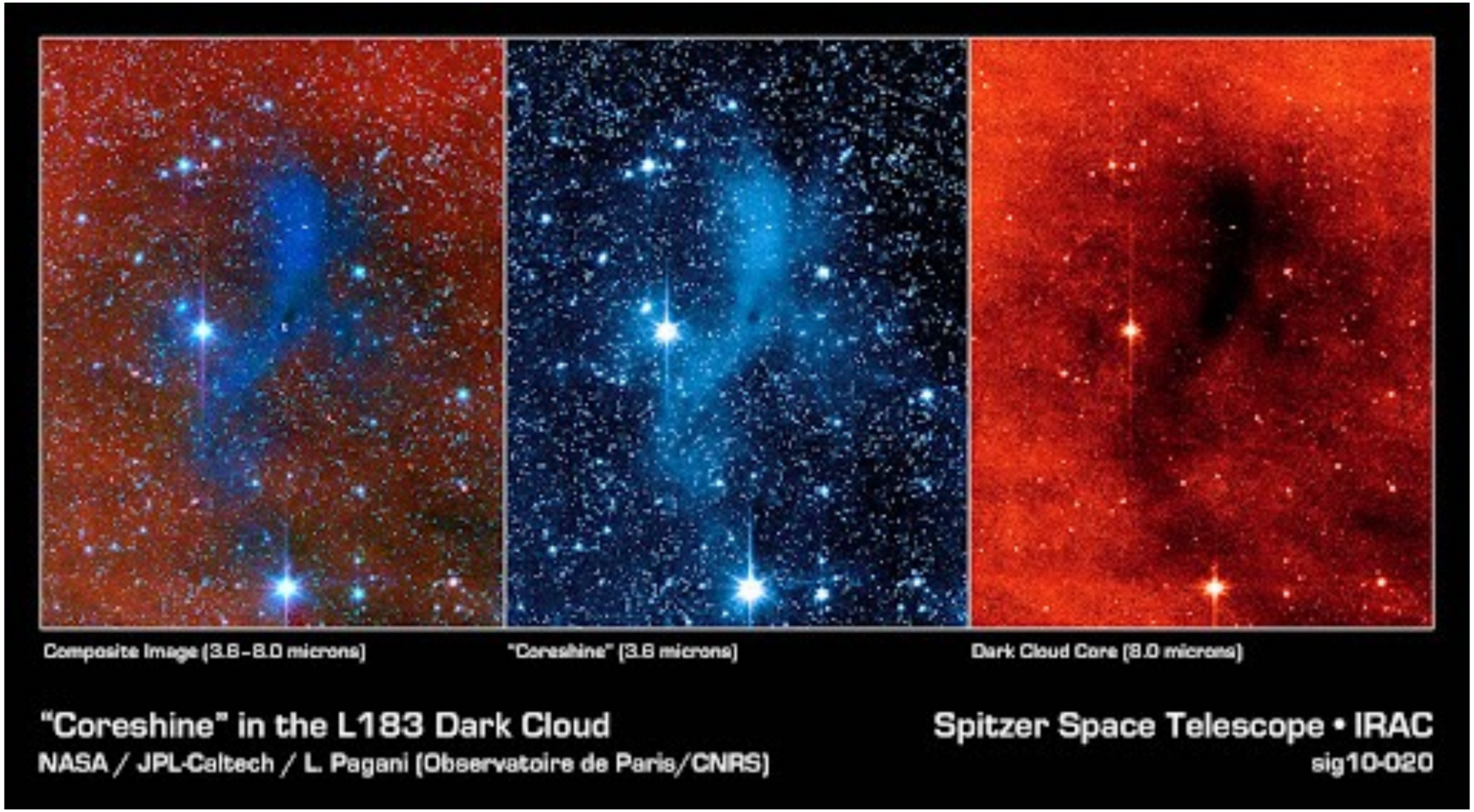
Hennebelle et al. (2016)

Dust evolution

Observational evidence of dust evolution

In prestellar cores :

IR excess (coreshine) could be explained by micron sized grains. (Pagani 2011)

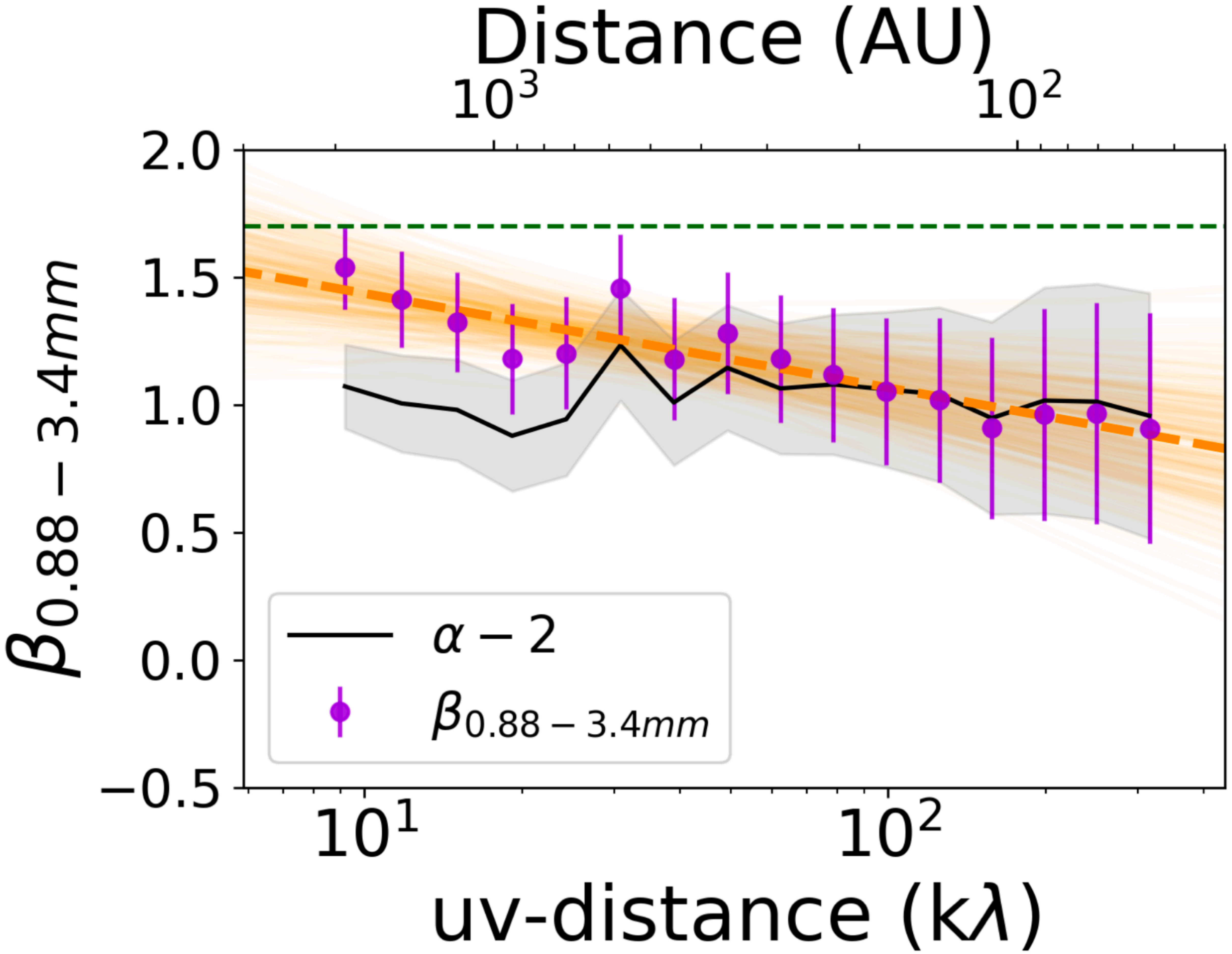


Observational evidence of dust evolution

In protostars envelopes :

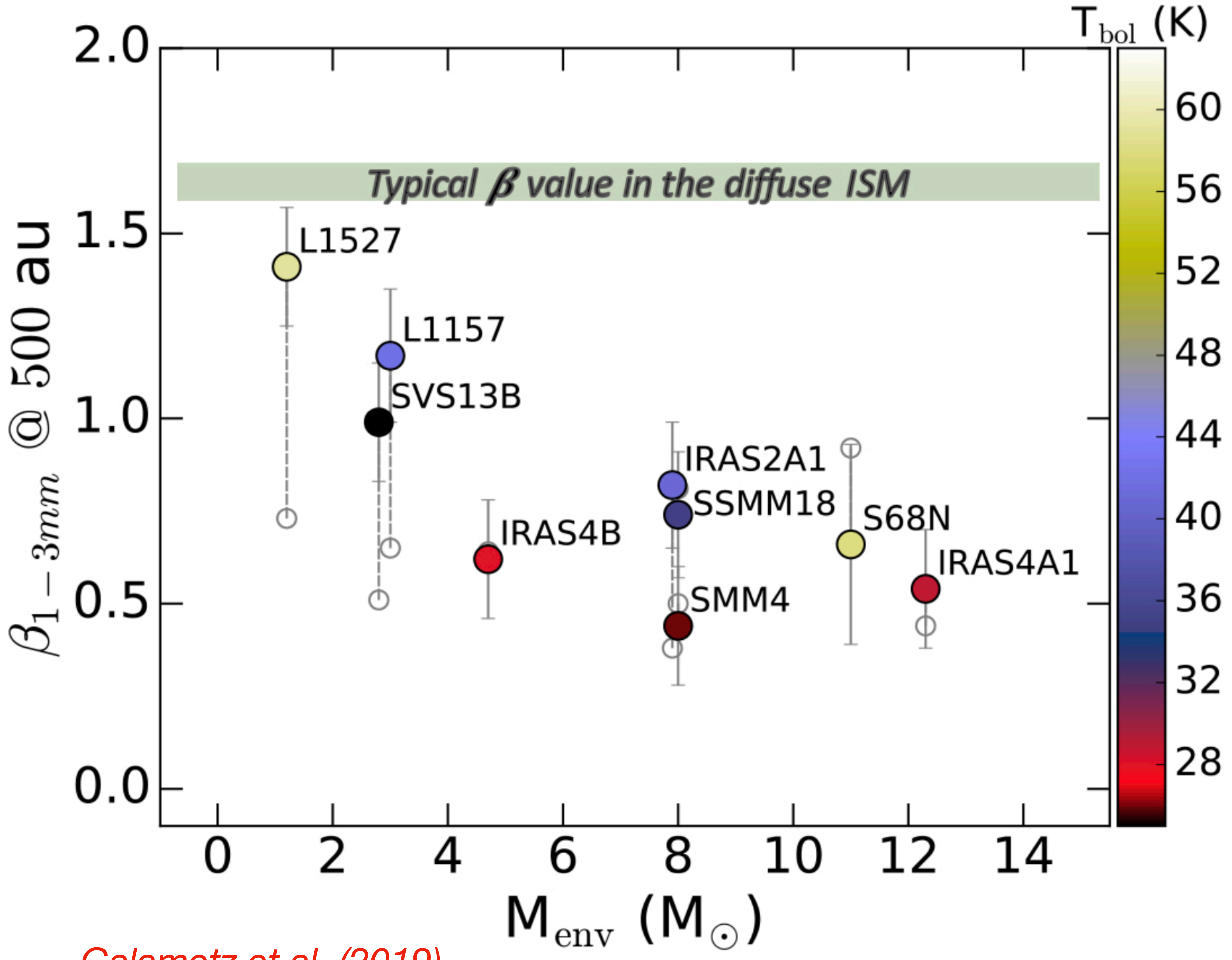
- Slope of the SED <u>might</u> be explained by grown ~100 micron grains. (e.g. *Galametz et al. 2019; Cacciapuoti et al. 2023,2024a*)
- Same goes with polarisation fraction (*Valdivia et al. 2019*)

Note : this is tentative because large grain optical properties are not very well constrained (yet).

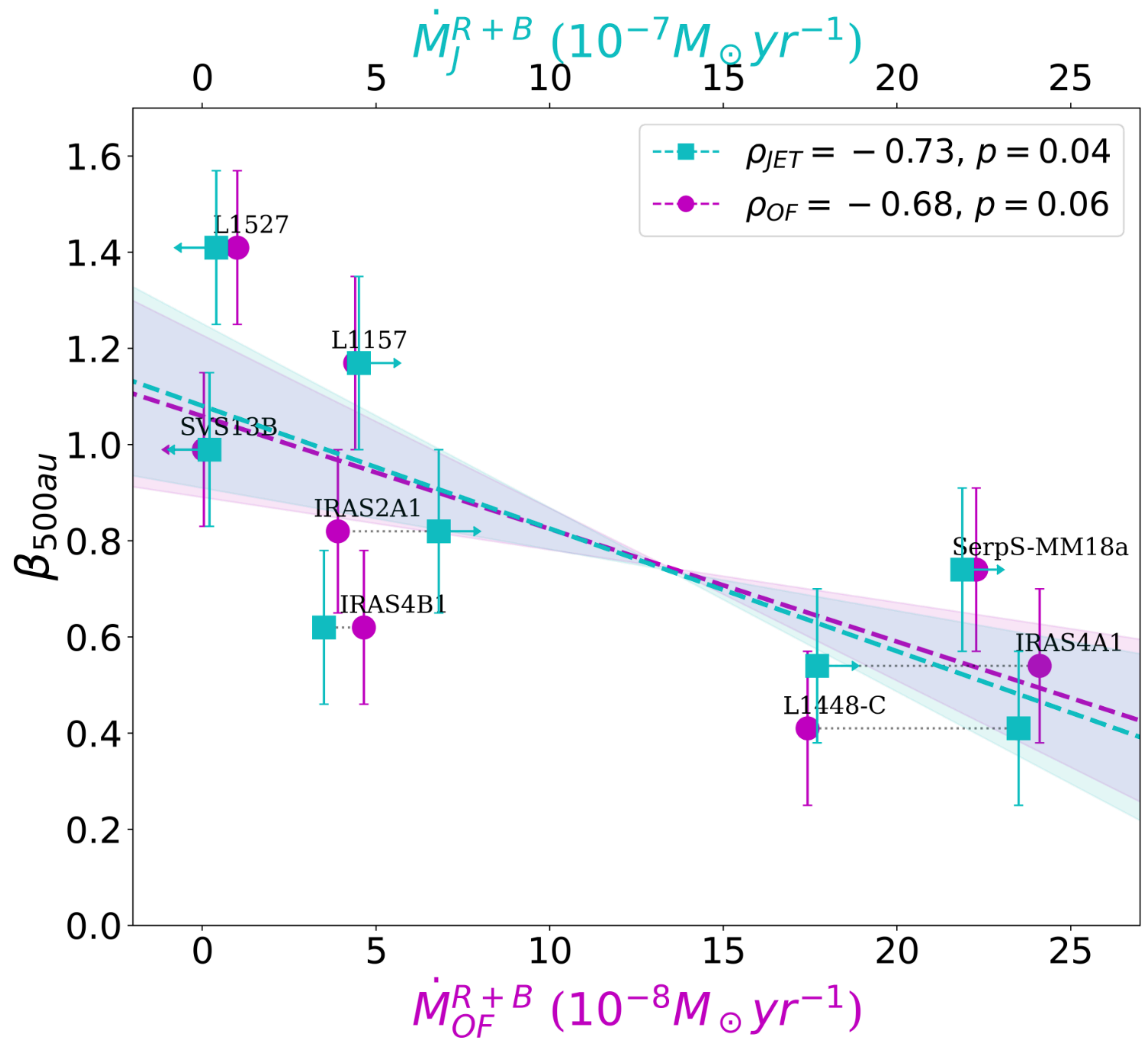


Cacciapuoti et al. 2023

Observational evidence of dust evolution



Galametz et al. (2019)

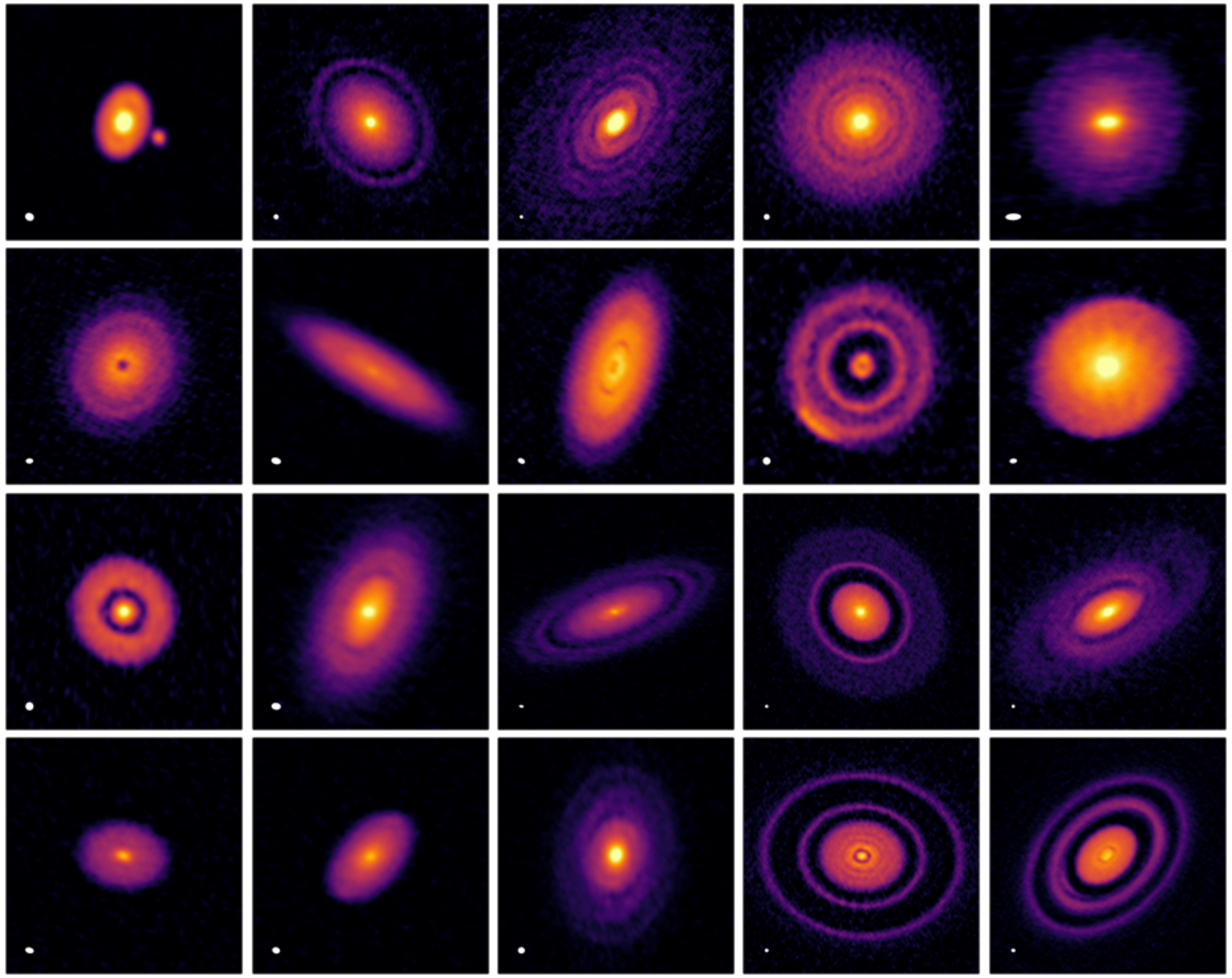


Cacciapuoti et al. 2024a

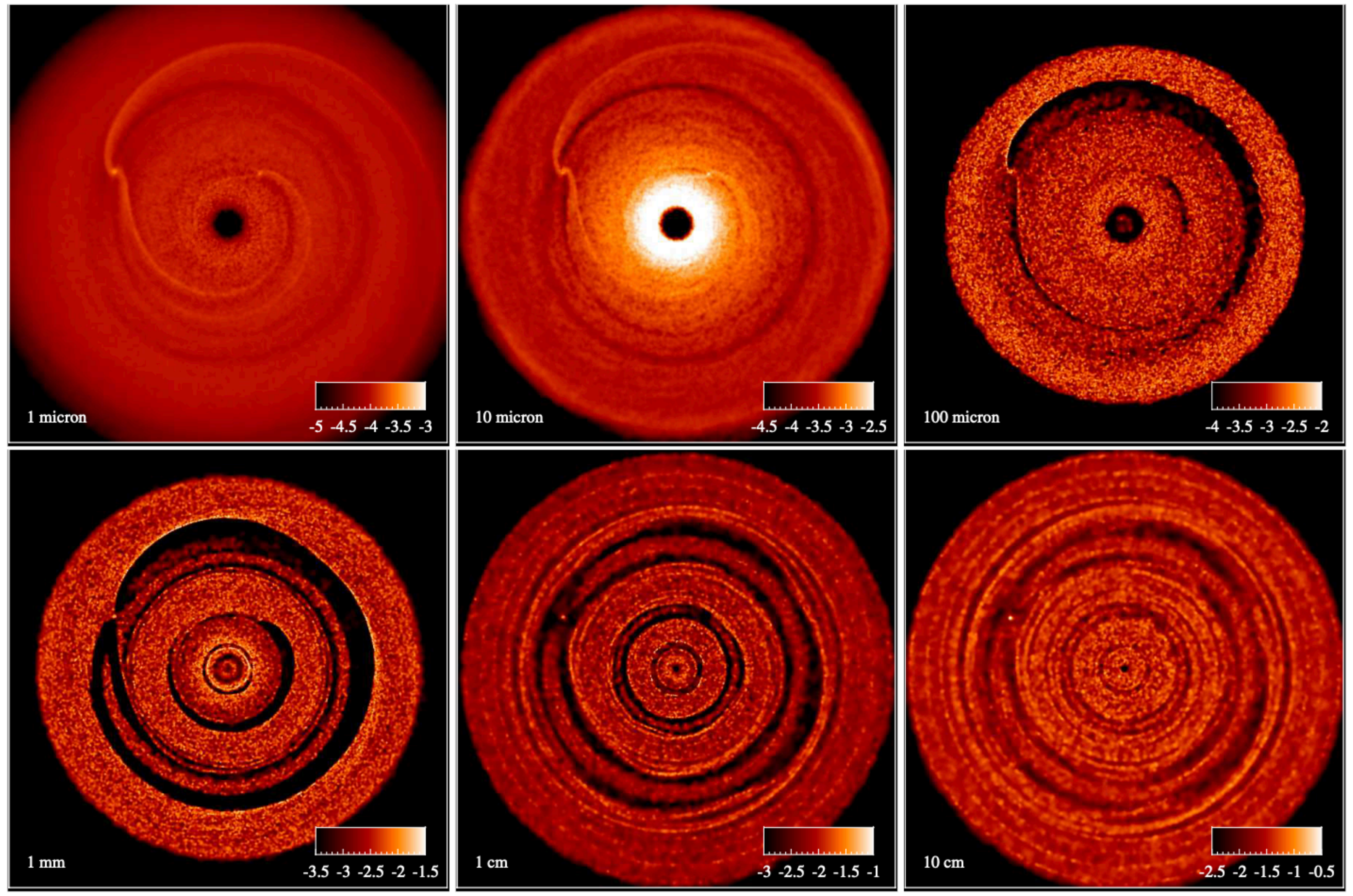
Observational evidence of dust evolution

In protoplanetary disks :

Dust emission in the mm in disks indicate grains of similar sizes.
—> Also, sub-structures and vertical settling could be indication that large St~1 are reached



ALMA observations of disks (DSHARP).
Andrews et al. (2018)



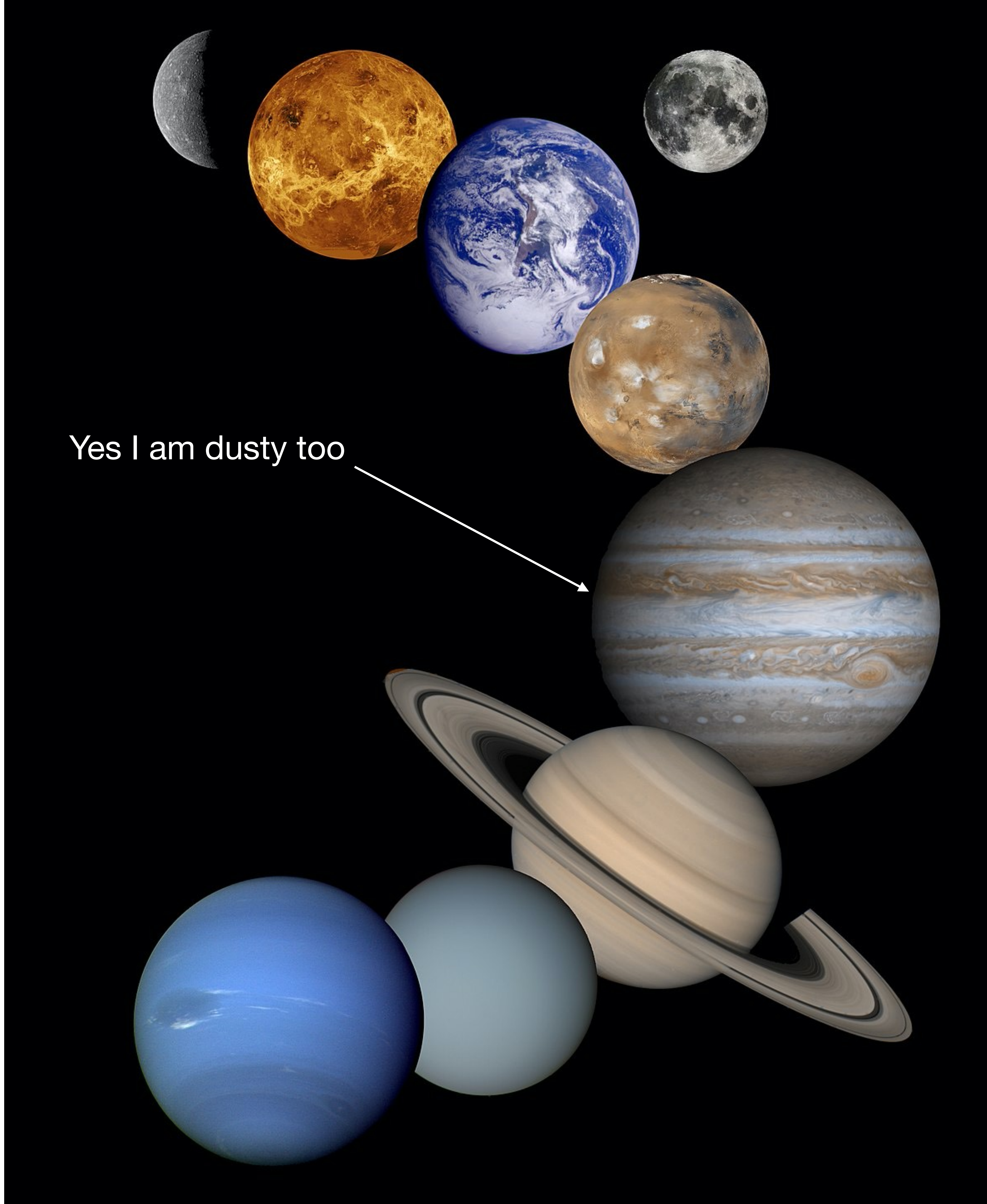
Dipierro et al. (2015)

Last evidence

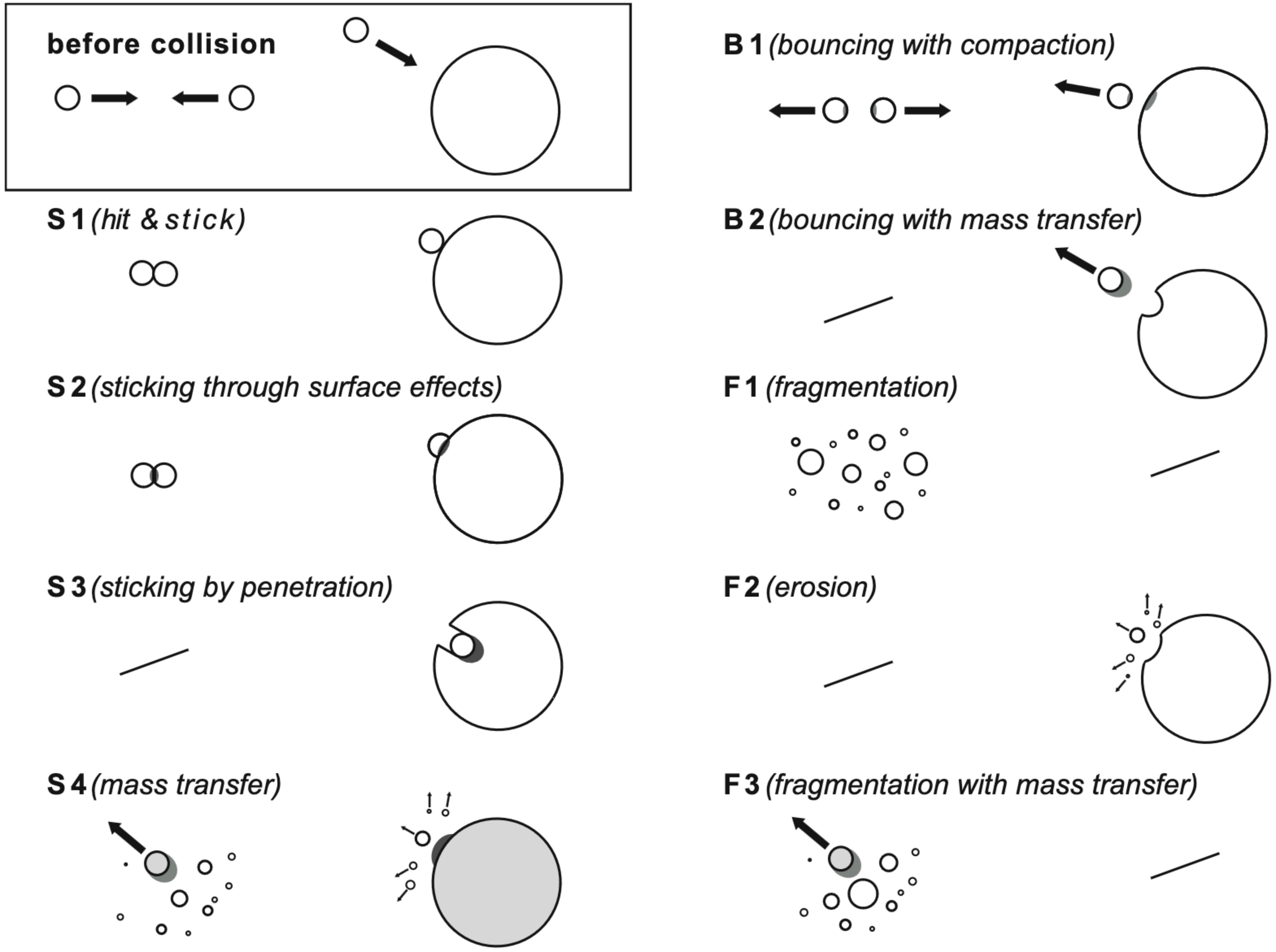
I wonder what could that be.

The dust under my bed ?

Thank you Sherlock

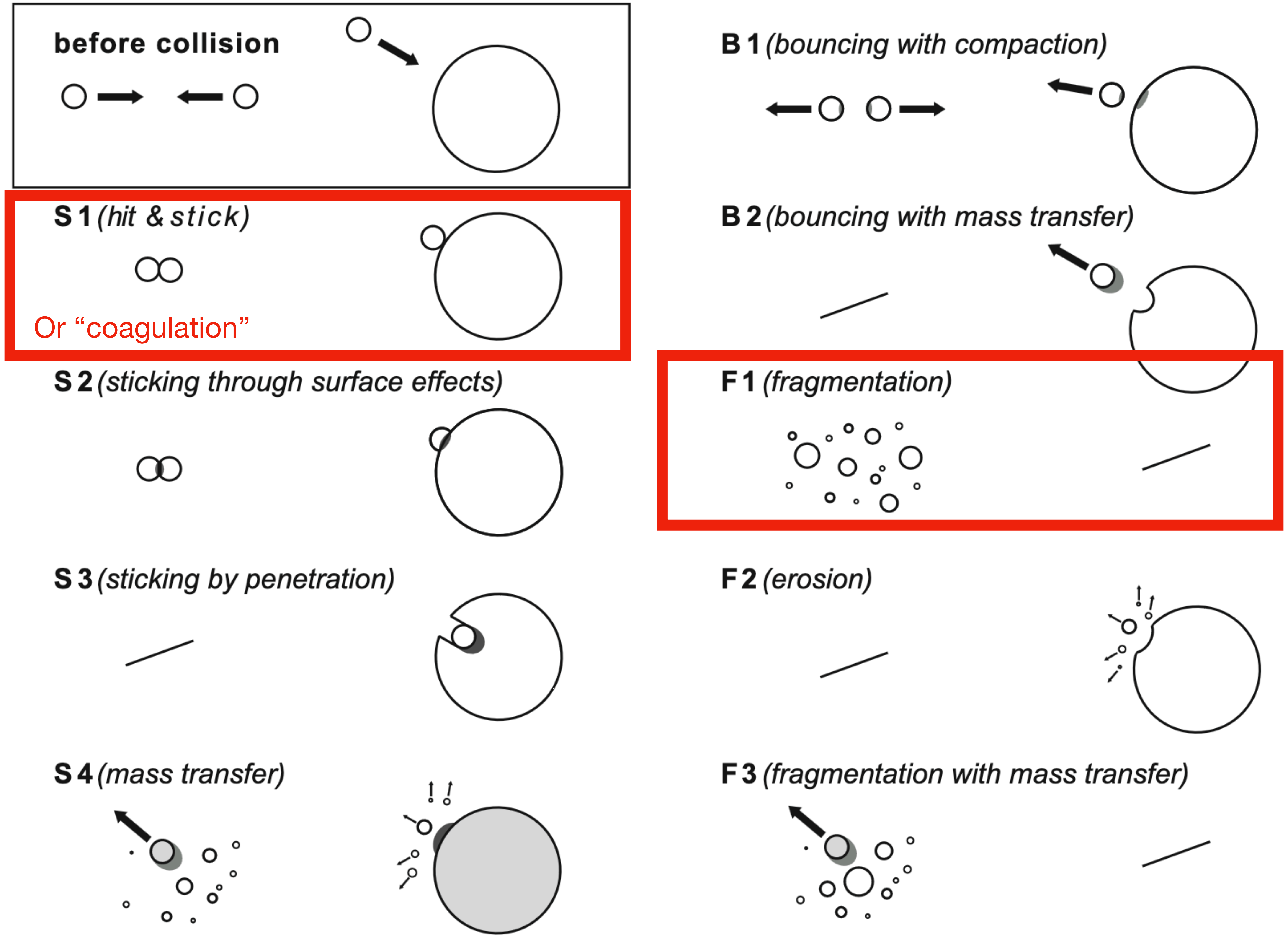


The many outcomes of grain-grain collisions



Güttler et al. 2010

The many outcomes of grain-grain collisions



Güttler et al. 2010

Dust evolution

1. How to model the dust coagulation ?

Smoluchowski equation(s)

Discrete (original) form *(Smoluchowski 1916)* **:**

$$\frac{dn_k}{dt} = \frac{1}{2}\sum_{j=1}^{k-1} K_{j,k-j} n_j n_{k-j} - n_k \sum_{j=1}^{\cancel{\infty}\ \mathcal{N}} K_{k,j} n_j$$

Gain :
Grain k formed from the collisions between smaller grains

Loss : k sticking with the other grain sizes (including k)

In the discrete form you solve $\mathcal{N}$ (the larger it is the better) coupled equations.

Coagulation Kernel (where you put the physics it's a volume per unit time) **:** $K_{i,j} = \pi(a_i + a_j)^2 \Delta v_{i,j}$

Continuous form (*Müller 1928*) **:**

$$\frac{\partial \tilde{n}}{\partial t} = \frac{1}{2}\int_0^m K(m', m - m')\,\tilde{n}(m')\,\tilde{n}(m - m')dm' - \int_0^\infty K(m', m')\,\tilde{n}(m')\,\tilde{n}(m')dm', \ \ \tilde{n} = \frac{dn}{dm}$$

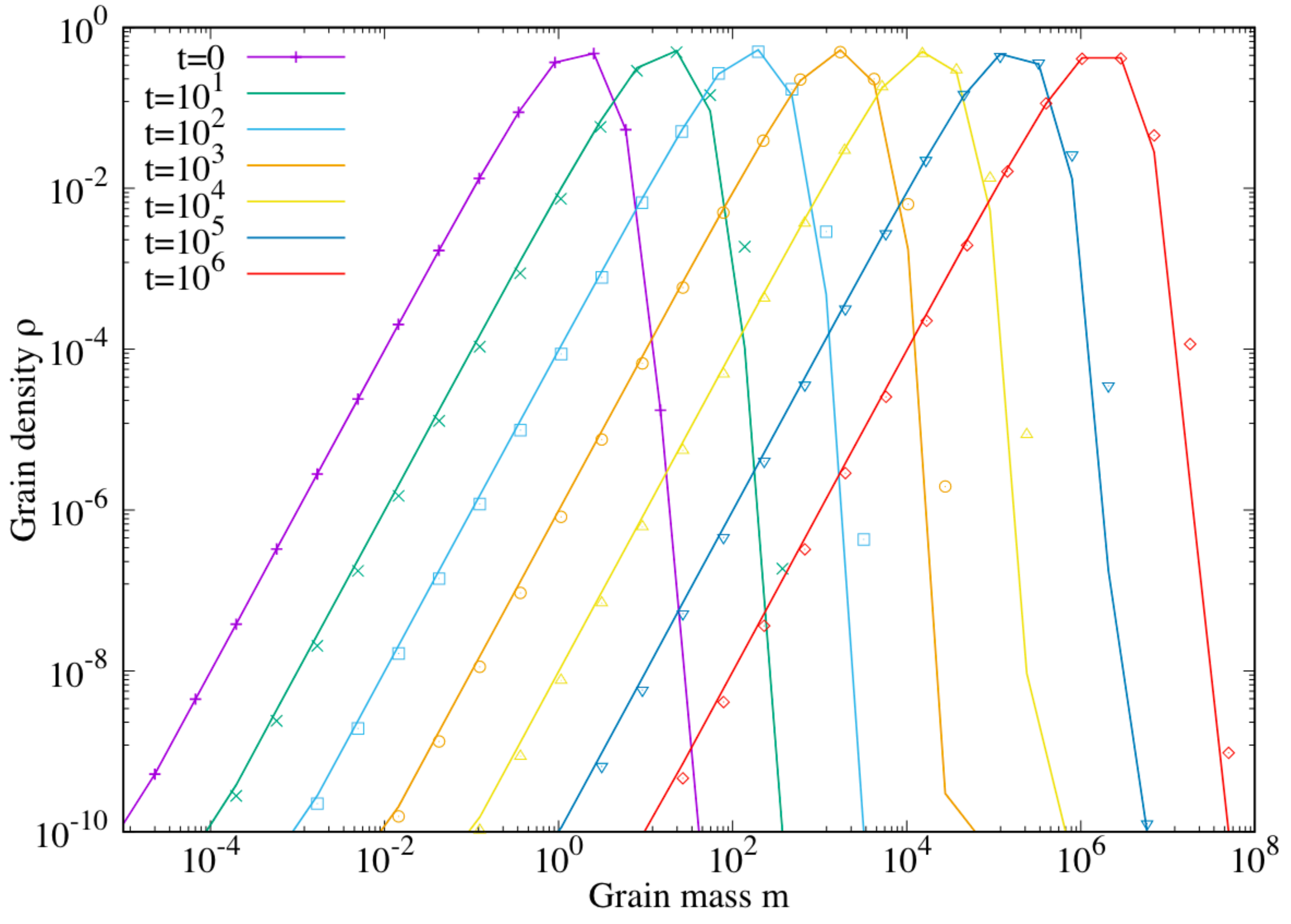
Smoluchowski equation

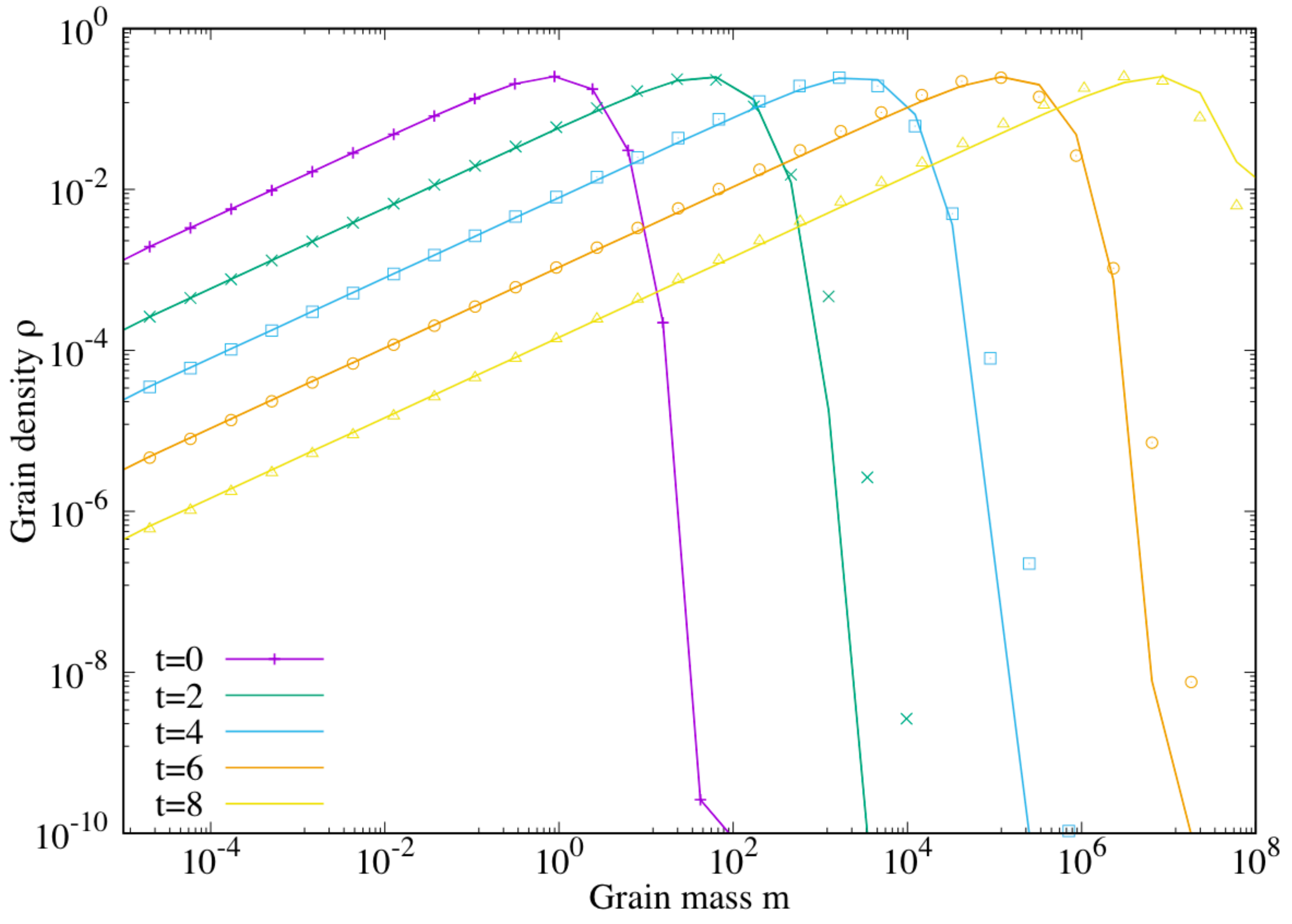
No analytical solutions expect for :

- Constant kernels $K_{i,j} = C$
- Additive kernel $K_{i,j} = m_i + m_j$
- Multiplicative kernels $K_{i,j} = m_i m_j$

Self-similar solutions derived in *Menon & Pego (2004)*

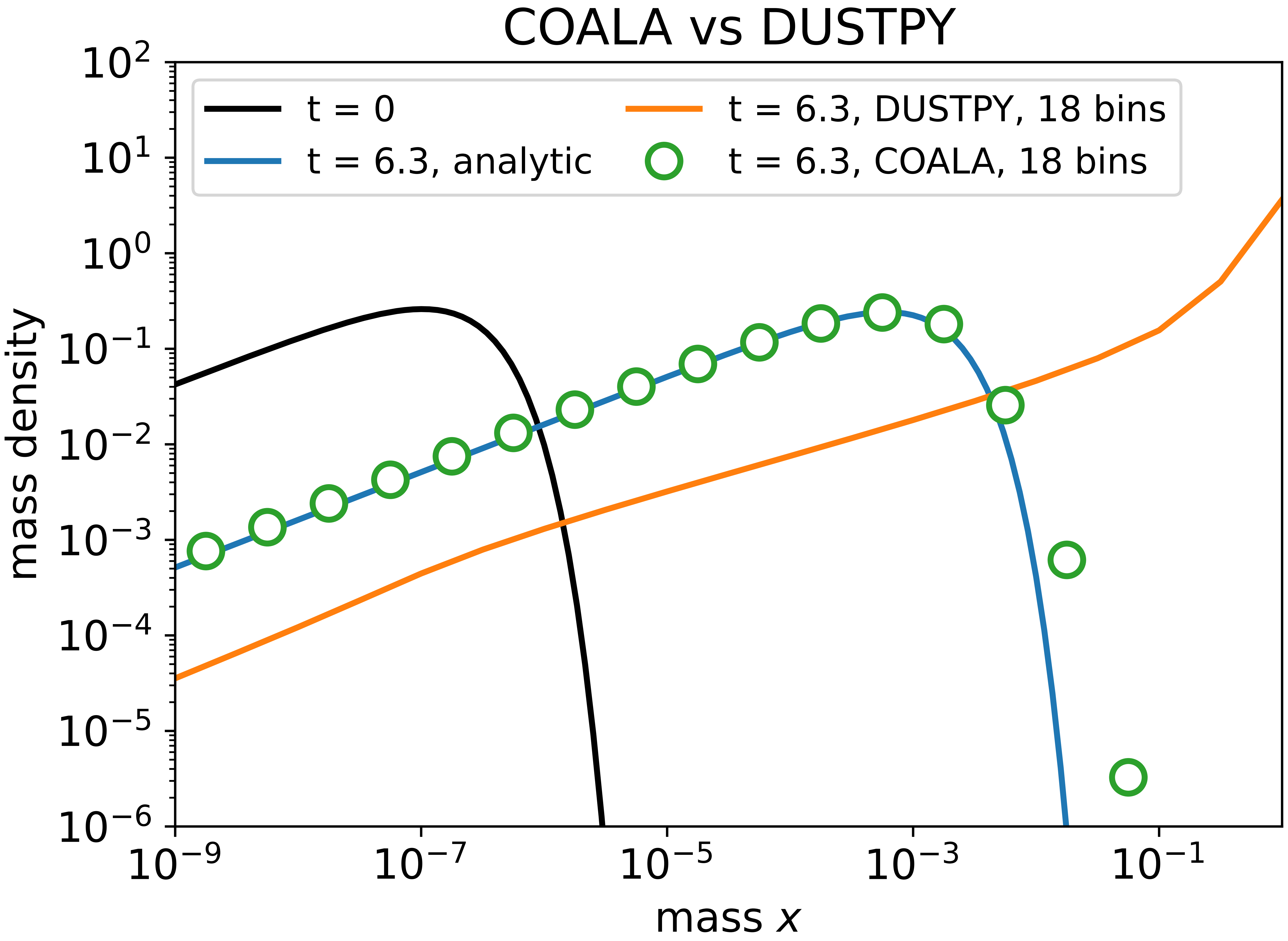
==> *Can be used to test dust coagulation algorithms*





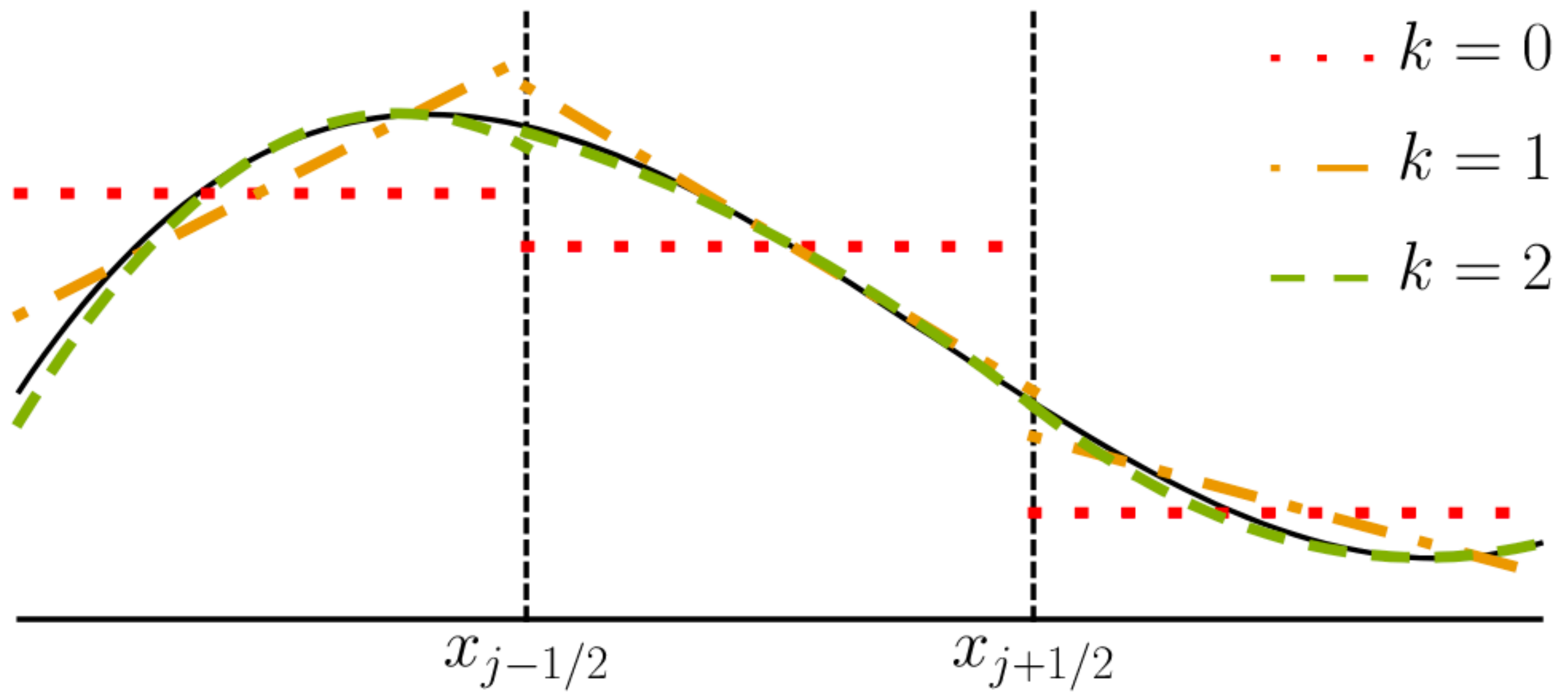
e.g. Marchand et al. (2021)

Problem with solving the Smoluchowski equation numerically



Coala : Lombart & Laibe 2021

Dustpy : Stammler et al. 2023



Idea : polynomial approx of the distribution in a bin

Most coagulation numerical methods need a lot of dust bins to converge : this is a problem for 3D simulations

New algorithm by *Lombart & Laibe (2021)* uses a Galerkin discontinuous methods and can get a good agreement with a low number of bins

An interesting way to overcome the cost of the Smoluchowski equation

If you assume a coagulation kernel that has the form : $K(m, m') = f_{\text{gas}}(n_H, T, \text{etc.})\, h(m, m')$

You can remove the gas properties from the integral (X -> dust to gas ratio) :

$$\frac{dX}{dt} = f_{\text{gas}} n_H I(X, a, t)$$

$$I(X, a, t) = \frac{1}{2}\int_0^m h(m - m', m')X(m - m', t)X(m', t)dm' - \int_0^\infty h(m, m')X(m, t)X(m', t)dm'$$

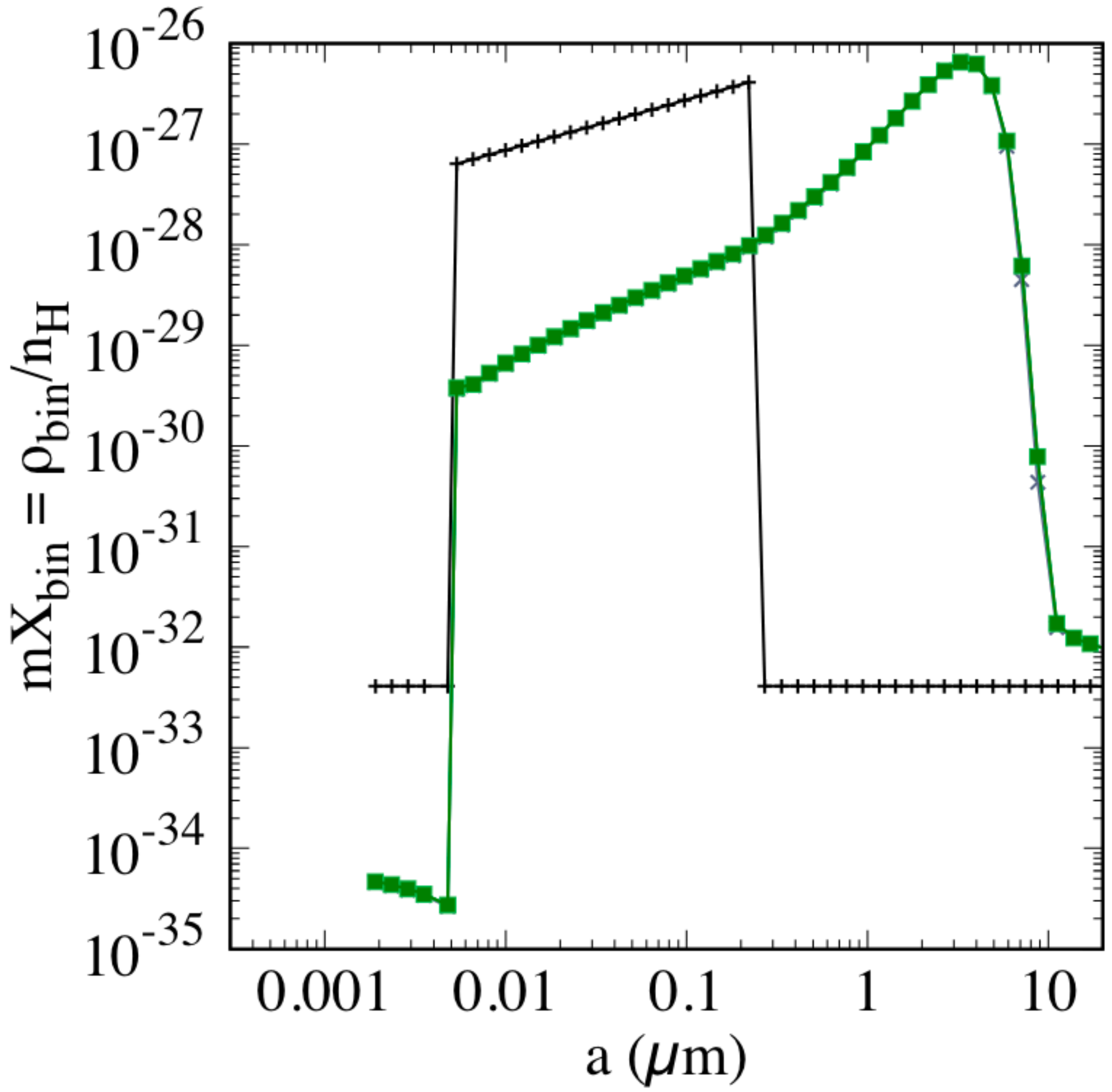
Coagulation becomes a 1D process :

$$d\chi = f_{\text{gas}} n_H dt \quad ---> \quad \frac{dX}{d\chi} = I(X, a, t)$$

Caveats :

- Only works for certain kernels (e.g turbulence)

- Incompatible with fragmentation



Marchand et al. 2021, 2022, 2023

Stepinski (or monodisperse) approach

$$\frac{da_{\rm grain}}{dt} = \frac{a_{\rm grain}}{3t_{\rm coag}}$$ *(Stepinski & Valageas 1996)*

Timescale for coagulating from a grain : $t_{\rm coag} = \frac{1}{\pi n_{\rm d}\, a_{\rm grain}^2 \Delta v}$ (*Draine 1985*)

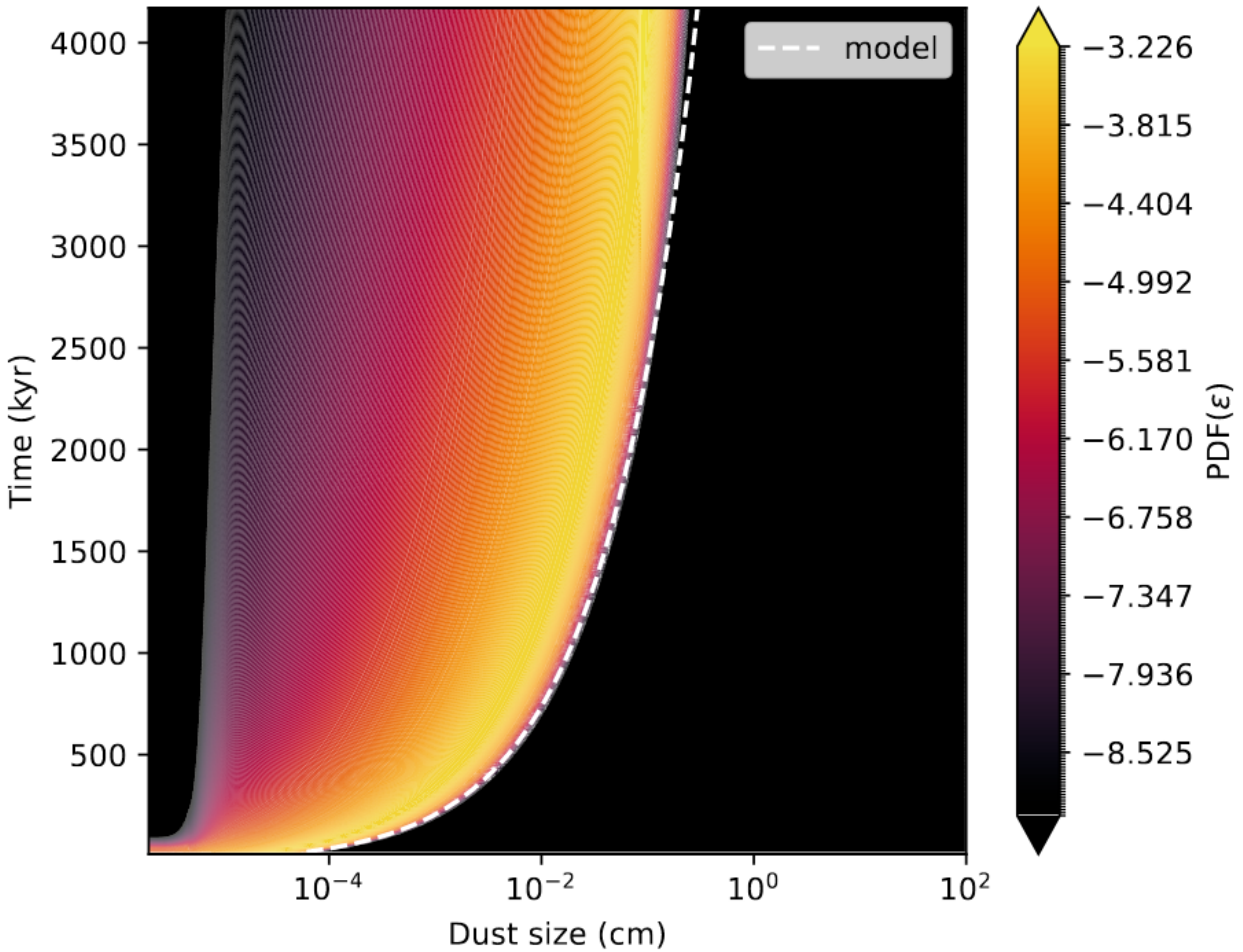
Hypothesis :

—> Grain collisions happen only for grain of similar sizes (which is wrong of course)

—> Only growth by turbulence or brownian motions (which are the only velocity source able to coagulate grain of similar sizes) — No systematic drift included (hydro drift, ambipolar drift)

Simplified but powerful to get the peak of the distribution

Stepinski approach



Vallucci-Goy et al. (in prep)

Dust evolution

2. Sources of grain-grain collisions

Sources of grain-grain collisions (velocities)

“Hydrodynamical” drift :

$$\Delta \overrightarrow{v} = \overrightarrow{v}_g - \overrightarrow{v}_d$$

this one comes from solving the fluid equations. (remember the 1st part of the course)

“**Ambipolar” drift or** **magnetic drift :** Same, but for charged grains $\Delta \overrightarrow{v} = \frac{c}{|\overrightarrow{B}|}\left(\frac{\Gamma_d^2}{1+\Gamma_d^2}\overrightarrow{E}_b \times \overrightarrow{b} + \frac{\Gamma_d}{1+\Gamma_d^2}\overrightarrow{E}_{b,\perp} + \Gamma_d \overrightarrow{E}_{b,\parallel}\right)$

$$c\overrightarrow{E}_b = \eta_o \nabla \times \overrightarrow{B} - \frac{\eta_A}{|\overrightarrow{B}|^2}\left(\left(\nabla \times \overrightarrow{B}\right) \times \overrightarrow{B}\right) \times \overrightarrow{B} + \frac{\eta_H}{|\overrightarrow{B}|^2}\left(\nabla \times \overrightarrow{B}\right) \times \overrightarrow{B}$$

—-> Grains of different sizes have a different coupling with the magnetic field.

—-> Small grains stick to the field and ‘big’ grains follow(-ish) the gas.

In principle, these two velocities are parts of the same systematic drift between the gas and the dust that should be obtained by solving the full multifluid-MHD (RT, gravo, etc.) gas and dust (and electrons and ions.. CRs?) equations

So far we sum the two contributions (with a simplified formula for the ambipolar drift)

Guillet et al. 2020; Silsee et al. 2020, Lebreuilly et al. 2023a, Vallucci-Goy et al. sub

Sources of grain-grain collisions (velocities)

Brownian motions : $\Delta v_{B,i,j} = \sqrt{\frac{8k_B T}{\pi}}\sqrt{\frac{m_i + m_j}{m_i m_j}}$

If we fix m_j, $\Delta v_{B,i,j}$ diverges with a decreasing m_i. Brownian motions are efficient at sticking small grains to big grains. (So not really to make large grains)

Turbulent motions *(Völk et al. 1980; Ormel & Cuzzi 2007) more general derivation in Gong et al. (2020;2021)* **:**

Grains of different sizes couple with eddies of sizes comparable to their stopping length. So the dynamics of the grains depends on how the stopping times compare with the injection timescale of the turbulence and its dissipation timescales.

Expression complicated (you can find in *Ormel & Cuzzi 2007).*

Important points :

- Turbulent can make grains of similar sizes collide so it is efficient at making large grains

- Expression of *Ormel & Cuzzi (2007)* widely used but remember that these are analytical expressions of something very complicated and neglect some key effects : the dust back-reaction (through collisions or magnetic fields) and changes of dust concentration (but we know dust can concentrate in the turbulence)

Dust evolution

Any questions ?

Dust evolution

3. Coagulation/fragmentation in astrophysical environments

Dust evolution

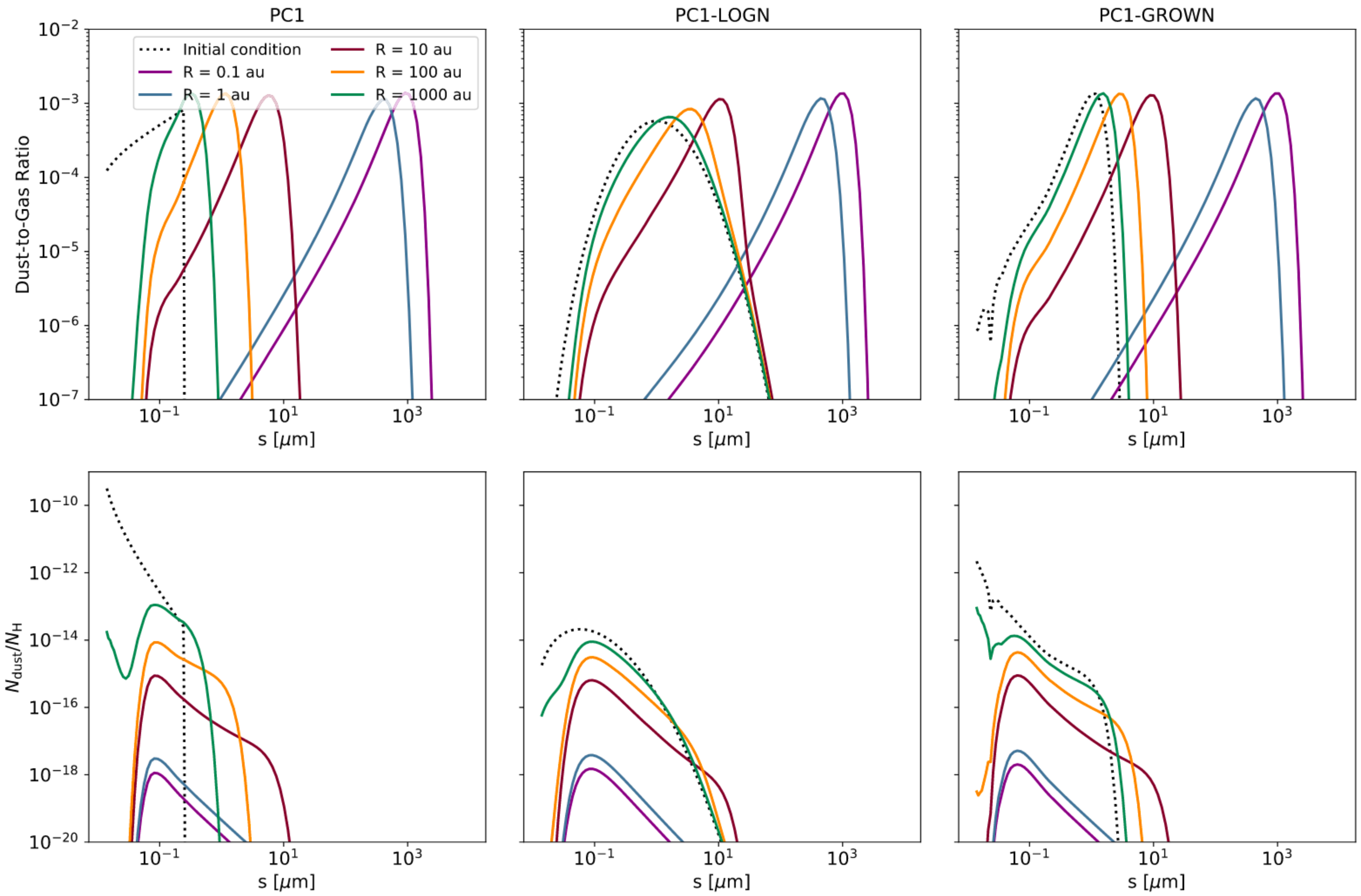
Protostellar collapse

Sources of grain-grain collisions (velocities)

Lebreuilly et al. 2023a *see also :*

Guillet et al. 2020; Silsee et al. 2020, Vallucci-Goy et al. sub

Memory loss of coagulation



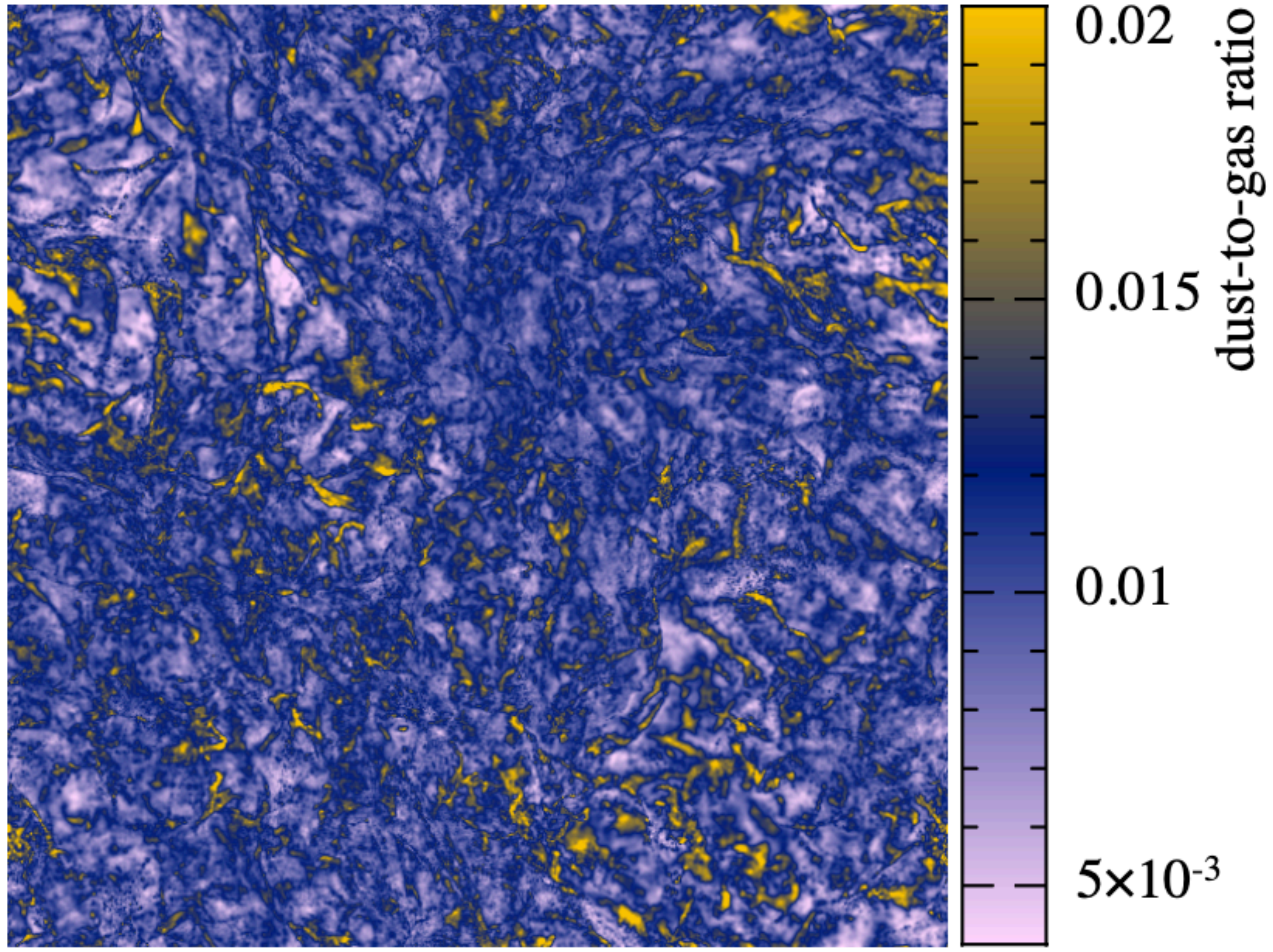
Lebreuilly et al. 2023a

Is there a timescale issue for coagulation ?

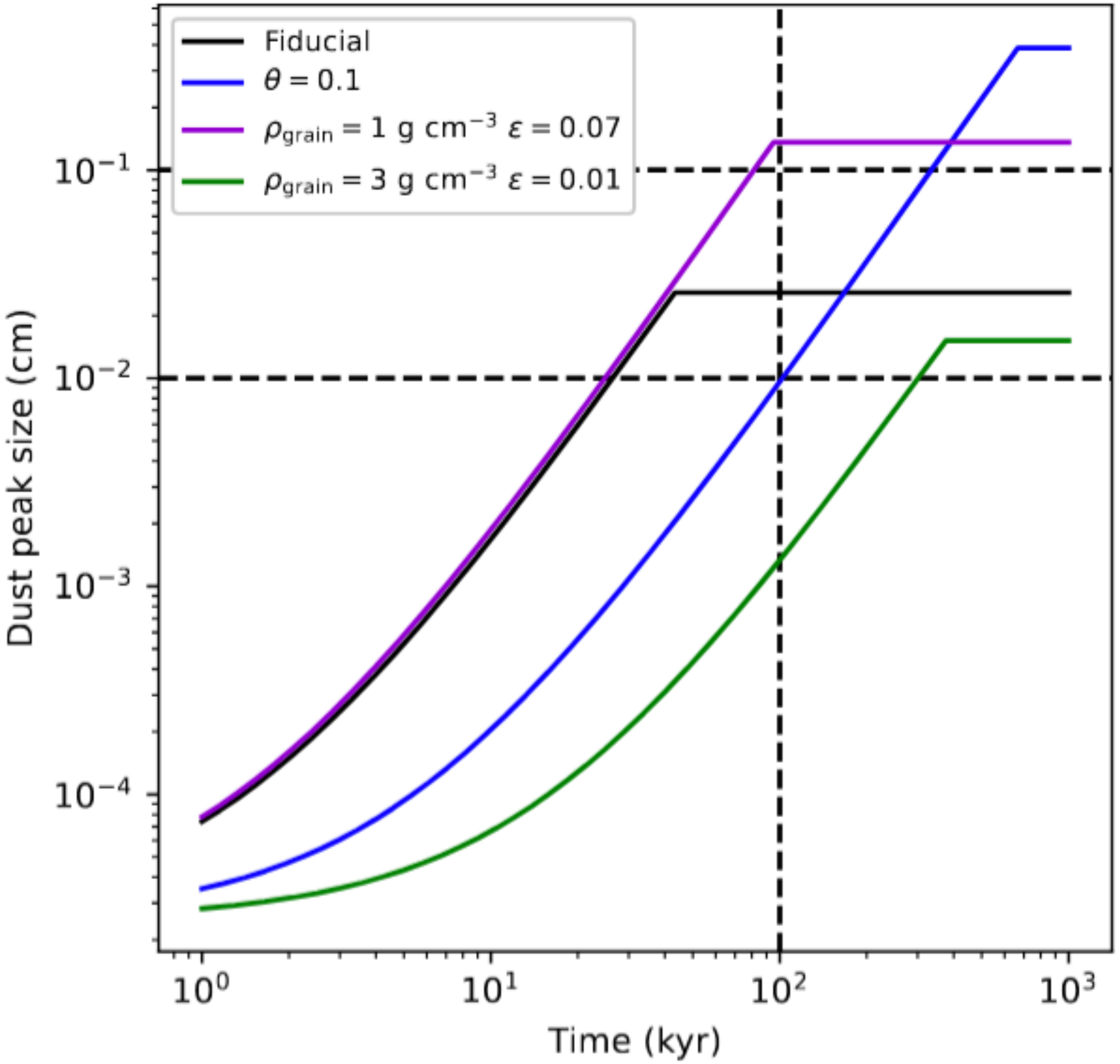
Growing millimetre grains in protostellar enveloppes is hard : but not impossible

3 ingredients are needed :

Low velocities (or high fragmentation threshold)
Porous grains (which accelerates the coagulation)
Some dust clumping (small scale dust clumping is fine)

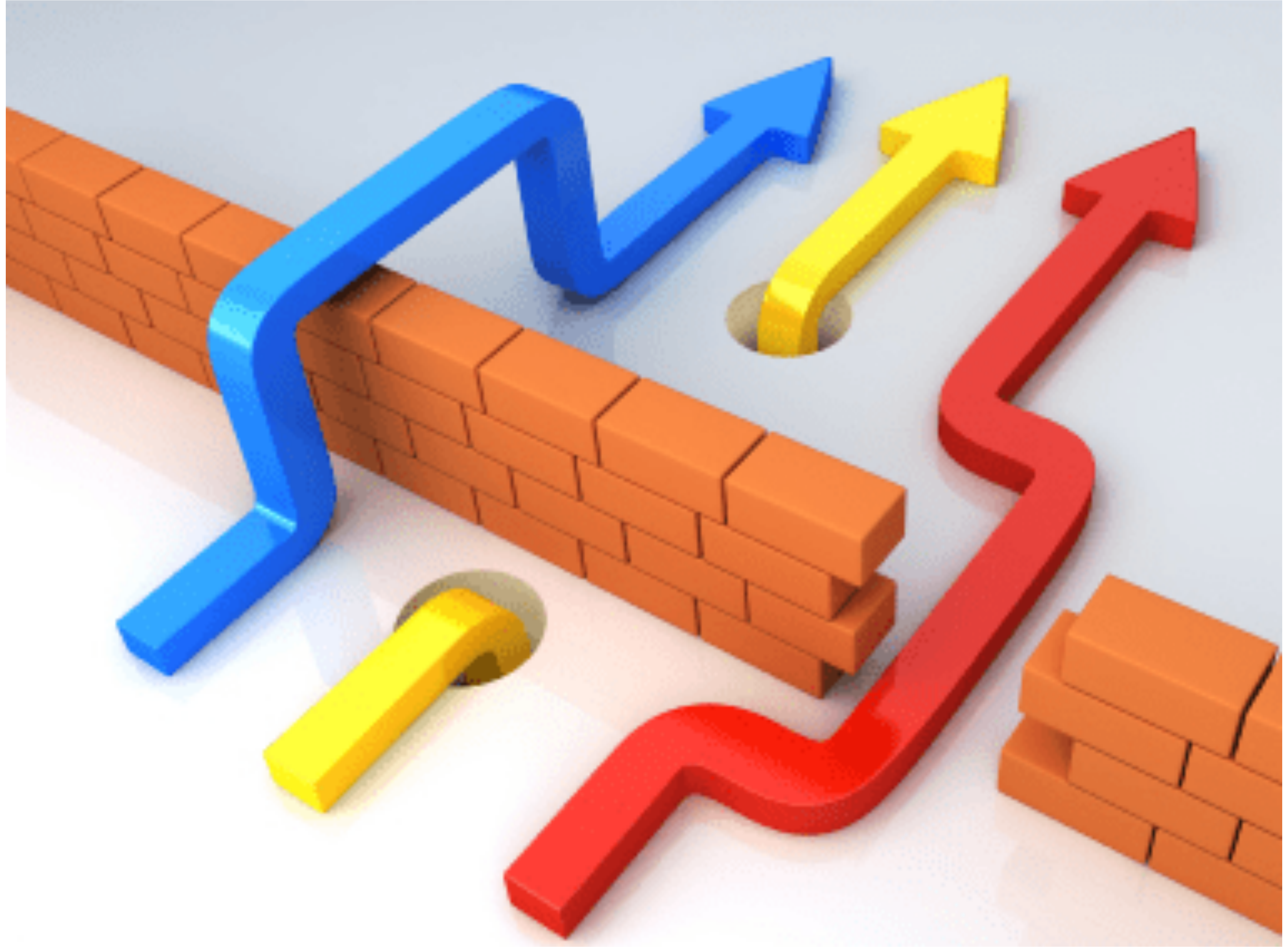


Tricco et al. 2016; Hopkins & Lee (2016), Lee, Hopkins & Squire (2017) ; Commerçon et al. 2023

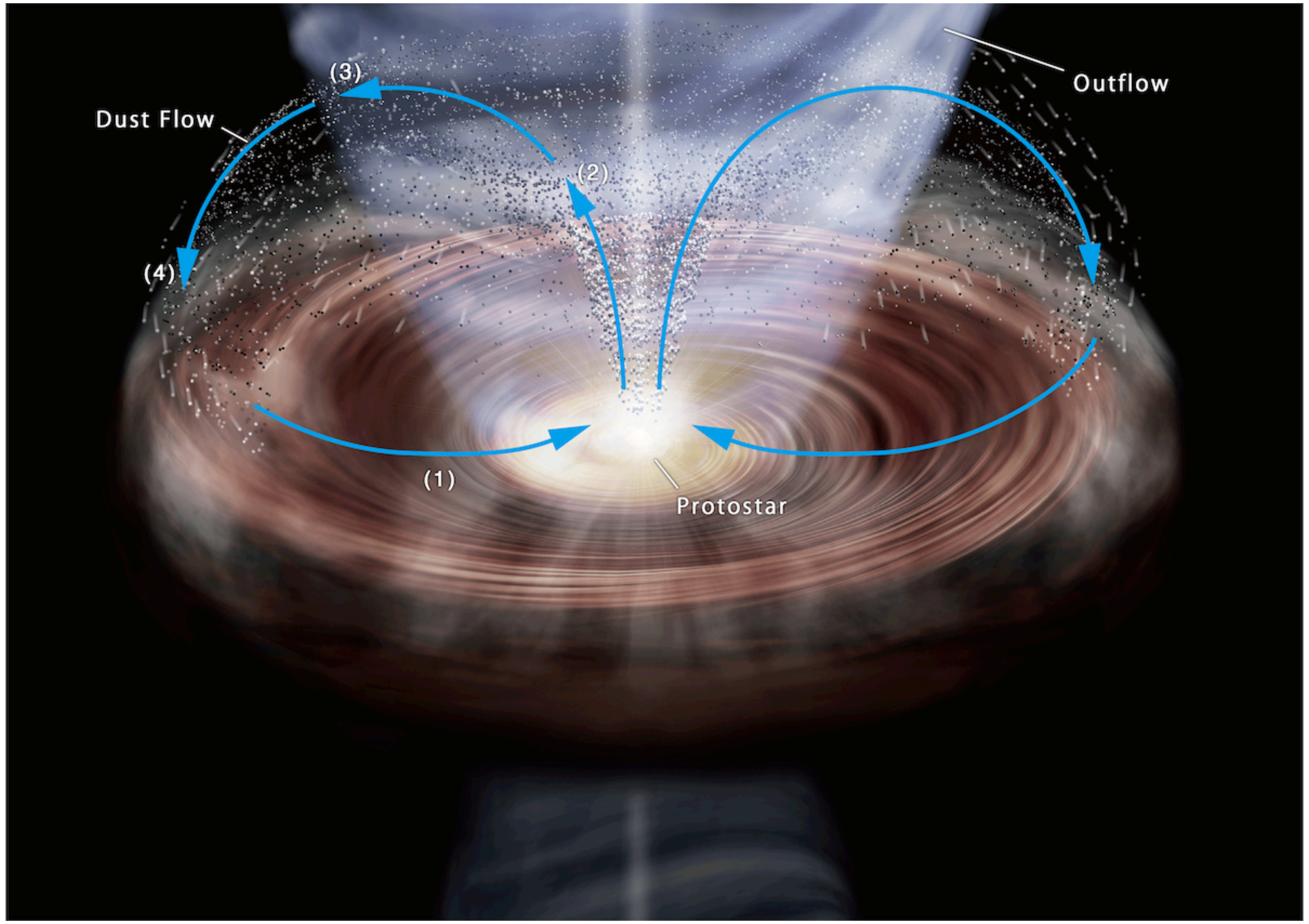


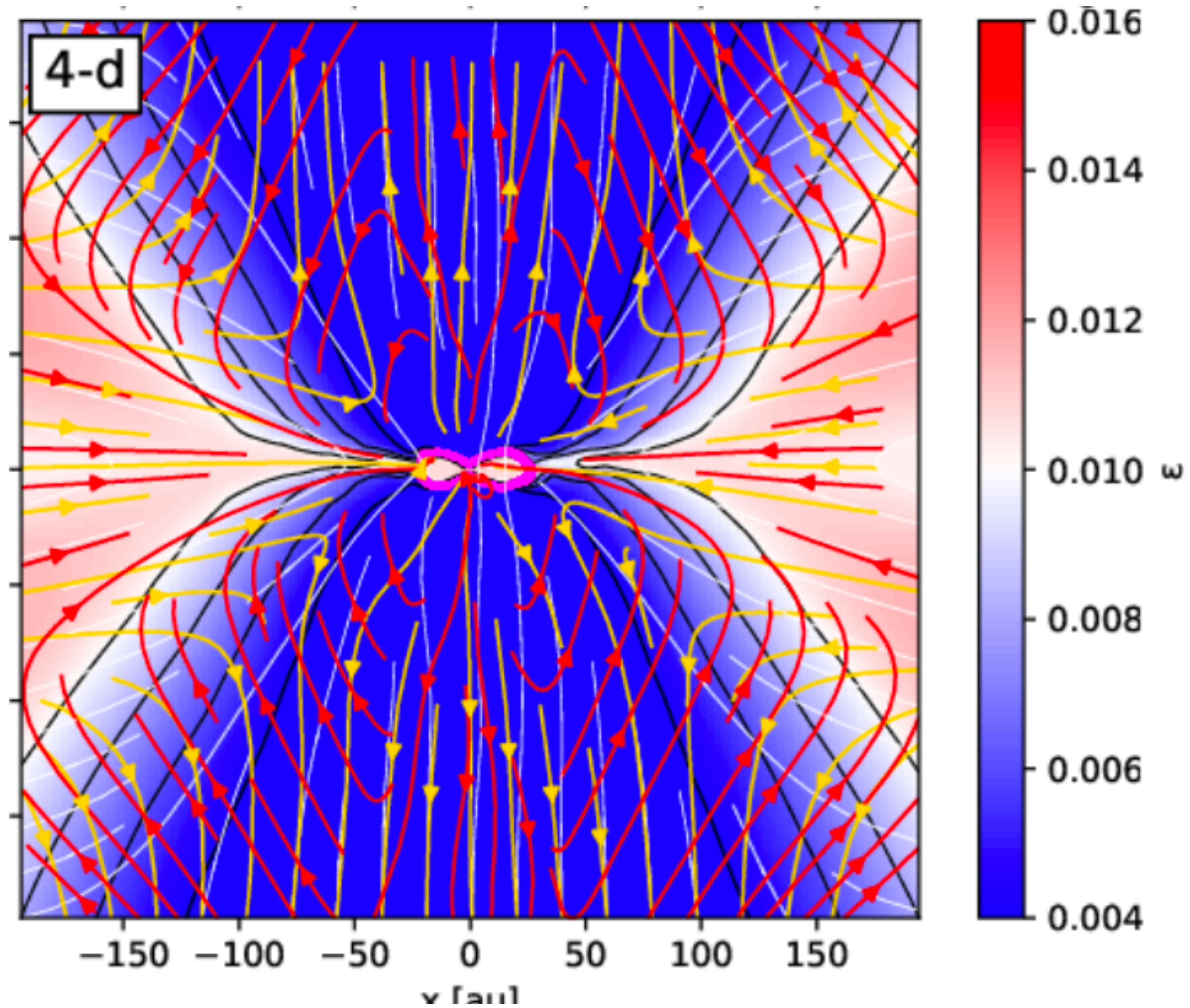
Vallucci-Goy et al. (in prep)

Protoplanetary disks
Overcoming the radial drift barrier



“Ash-fall” phenomenon

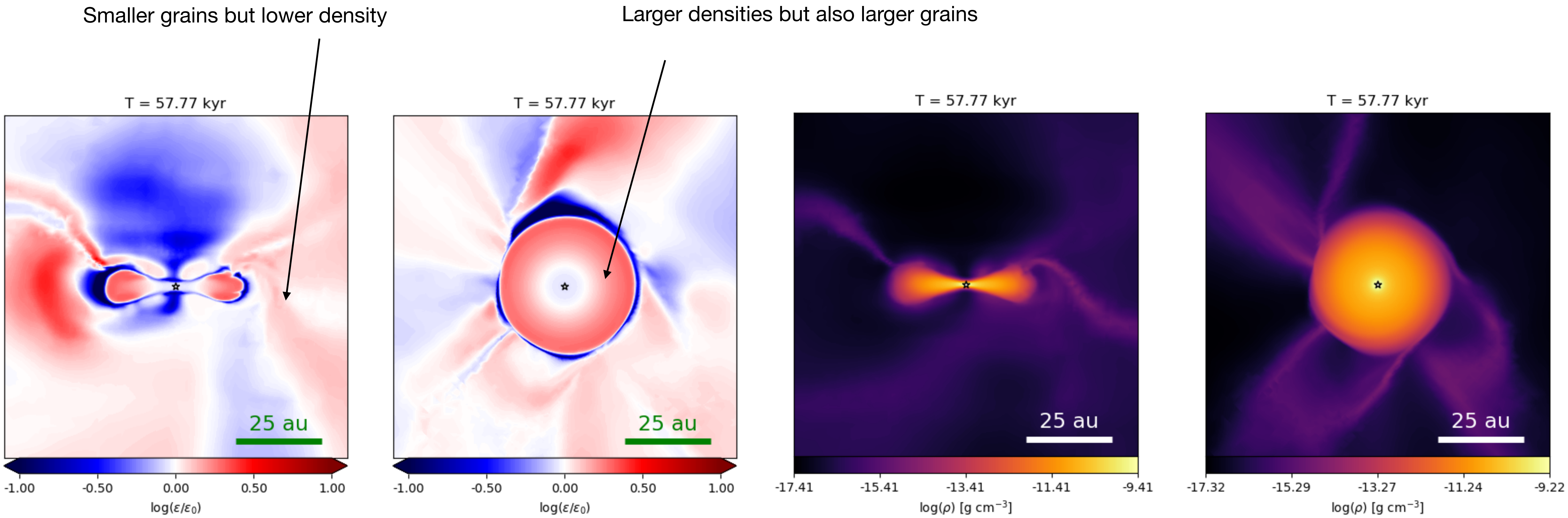




$$\frac{\partial \rho_d a_{\rm grain}}{\partial t} + \nabla \cdot \left(\rho_d a_{\rm grain} \vec{v}_{\rm grain}\right) = \frac{\rho_d a_{\rm grain}}{t_{\rm coag}}$$

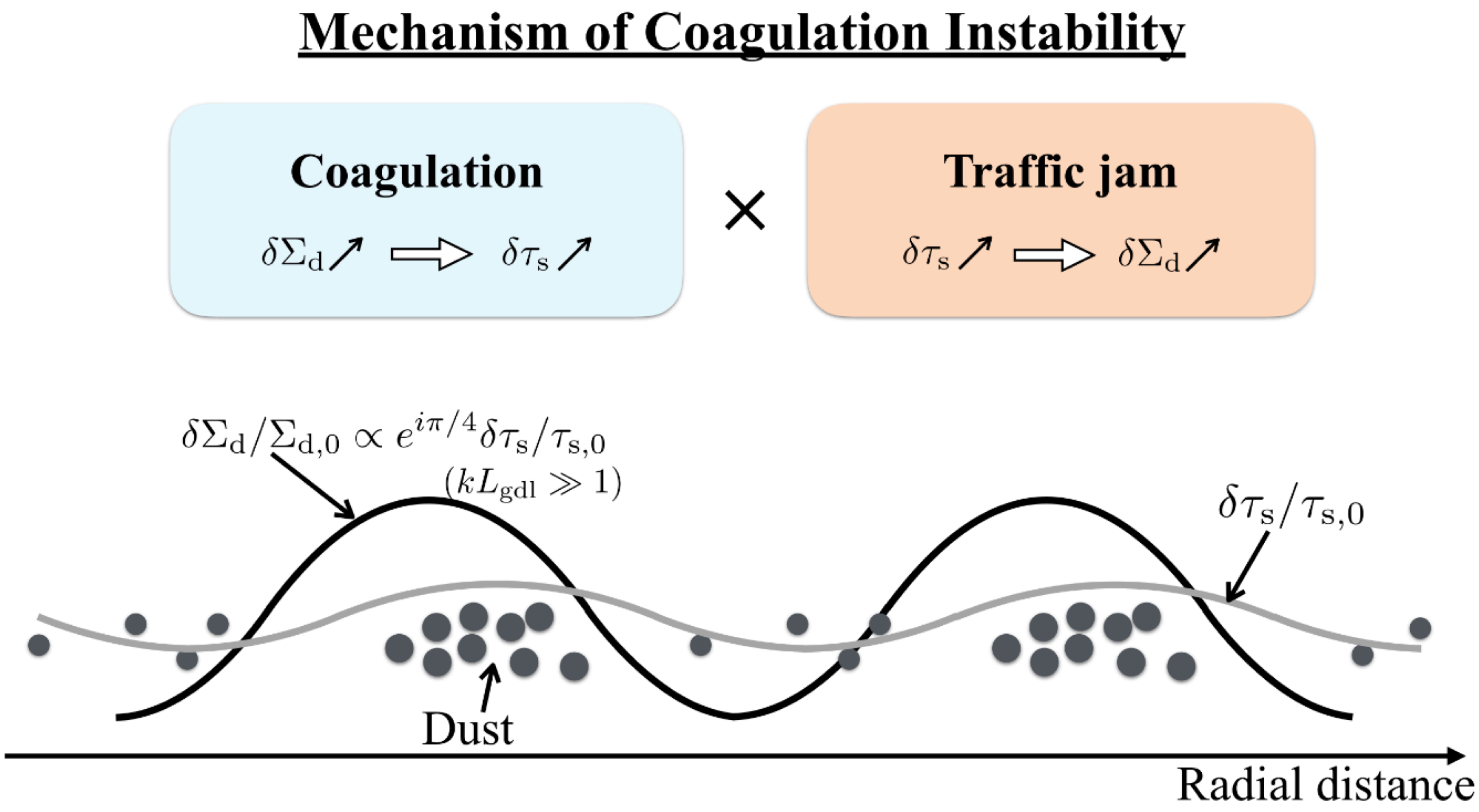
Tsukamoto et al. (2021)

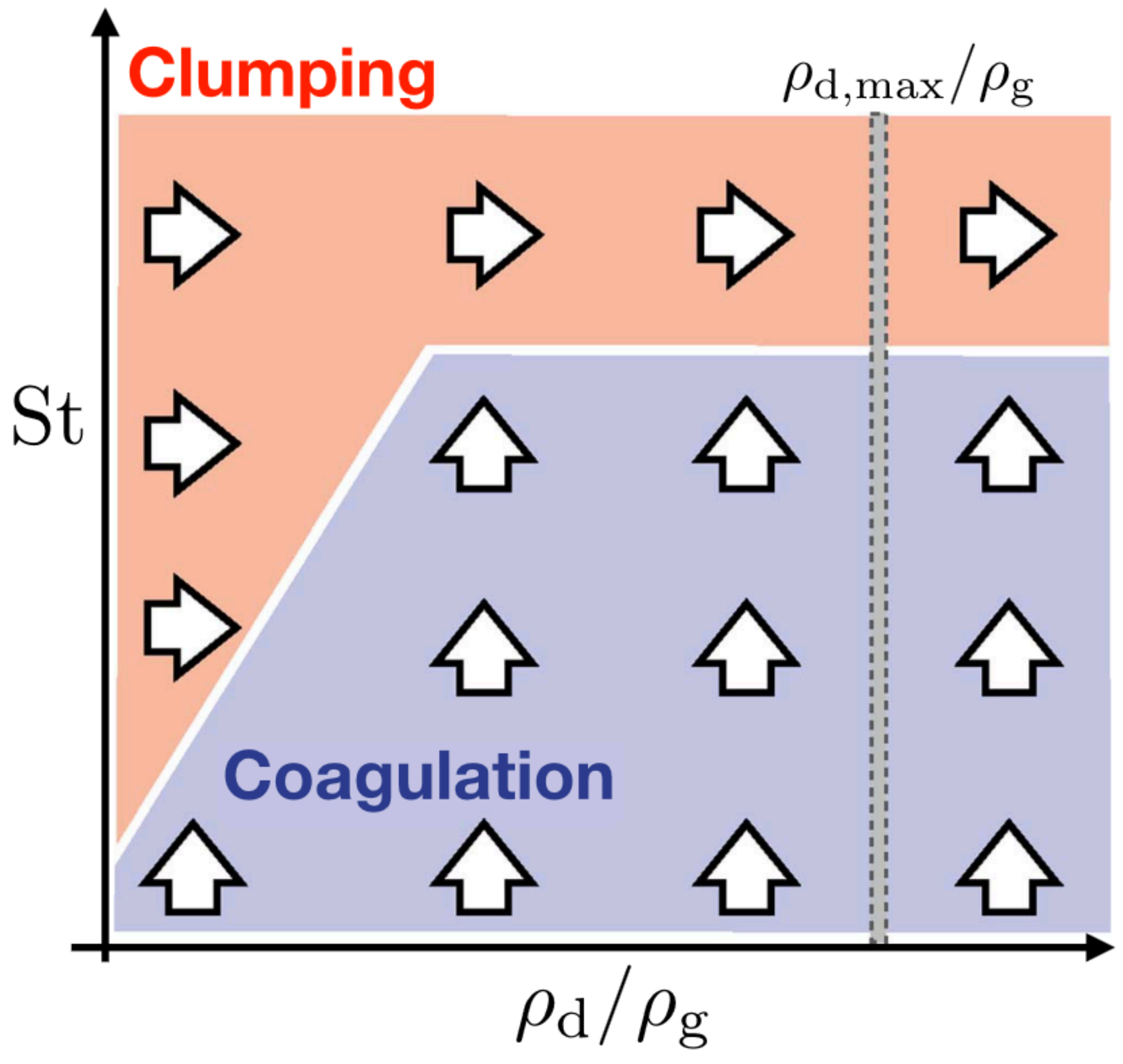
Dust replenishment from the outer edge of the disk



Lebreuilly et al. (in prep)

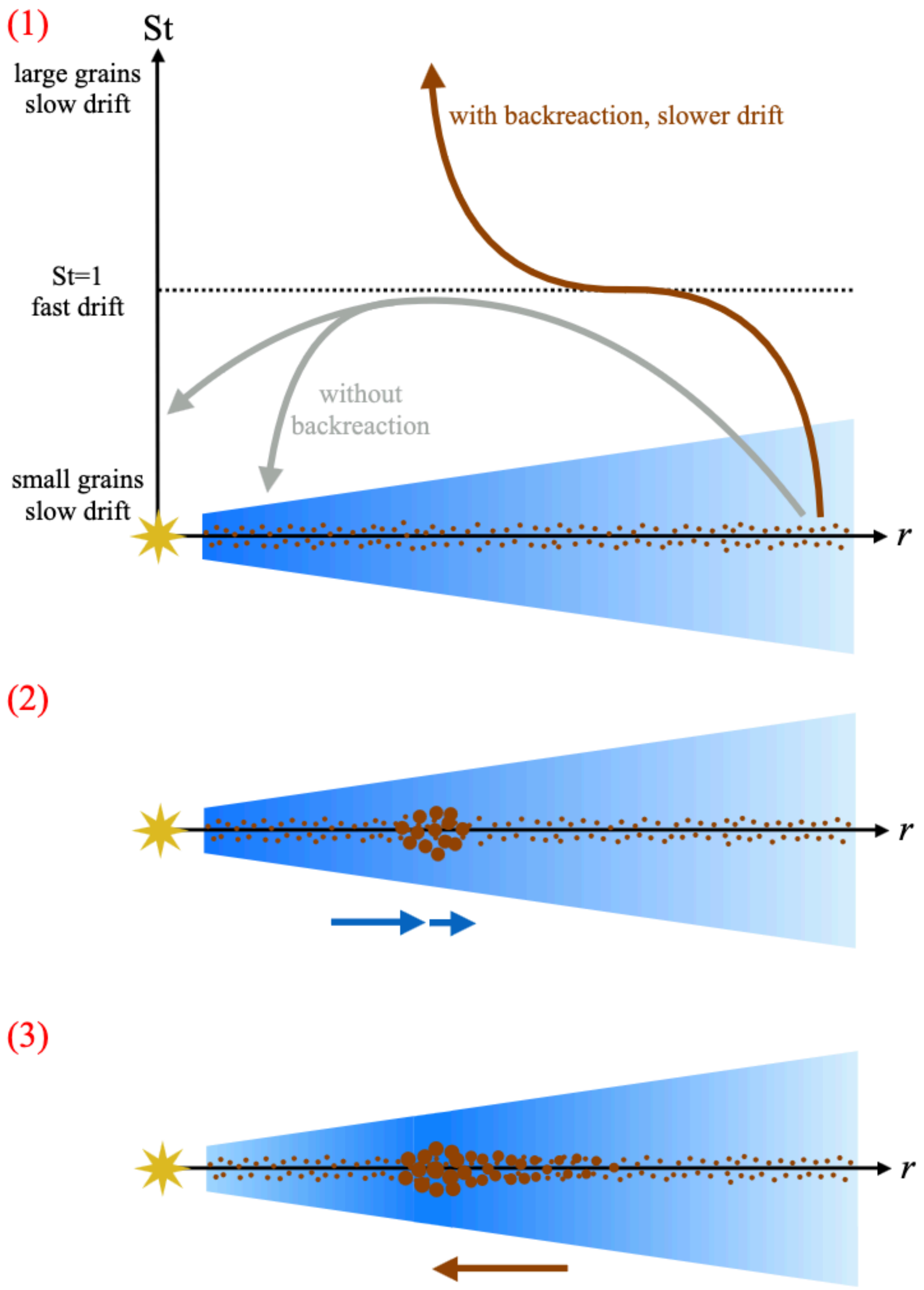
Coagulation instability : in disks (but not only?)

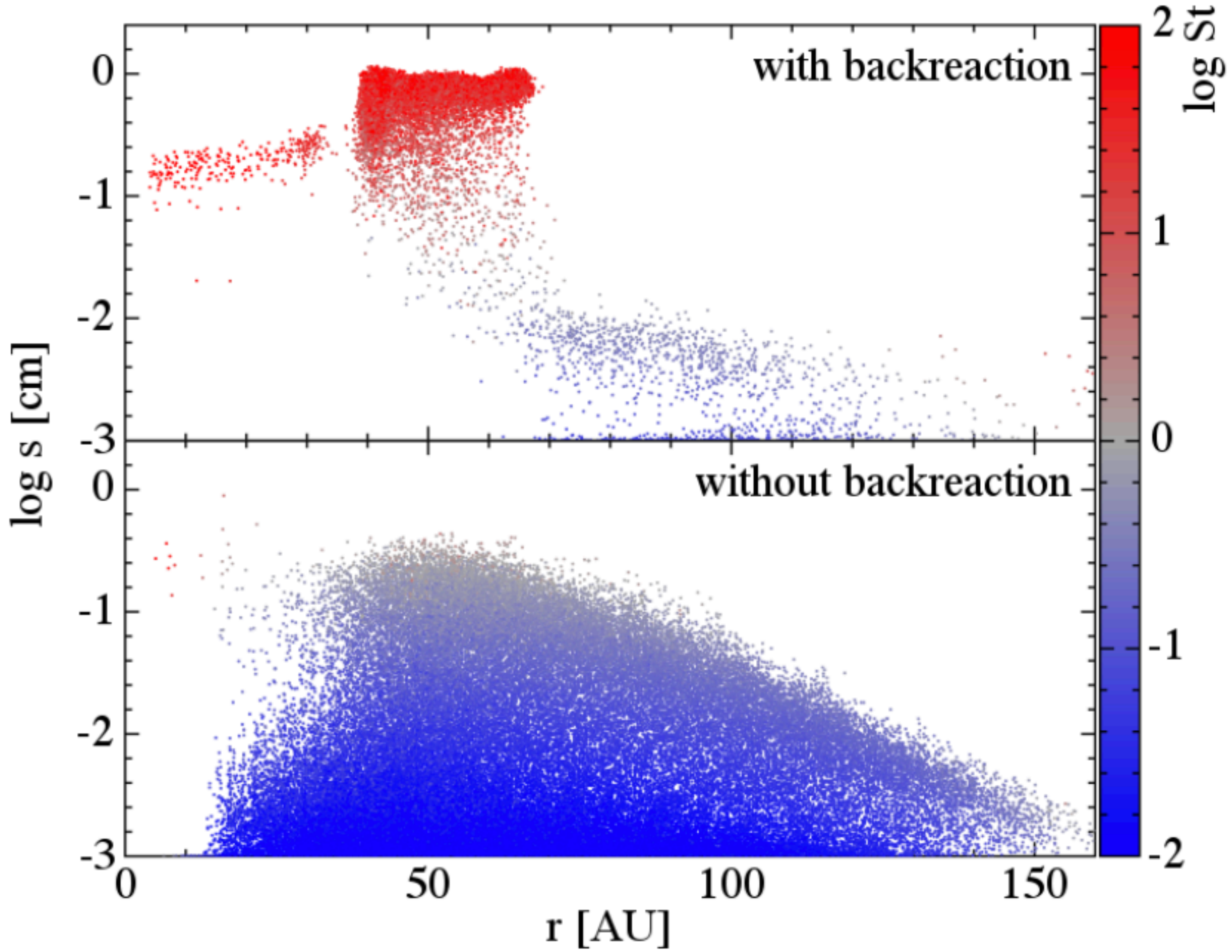




Tominaga et al. (2021); Tominaga & Tanaka (2023)

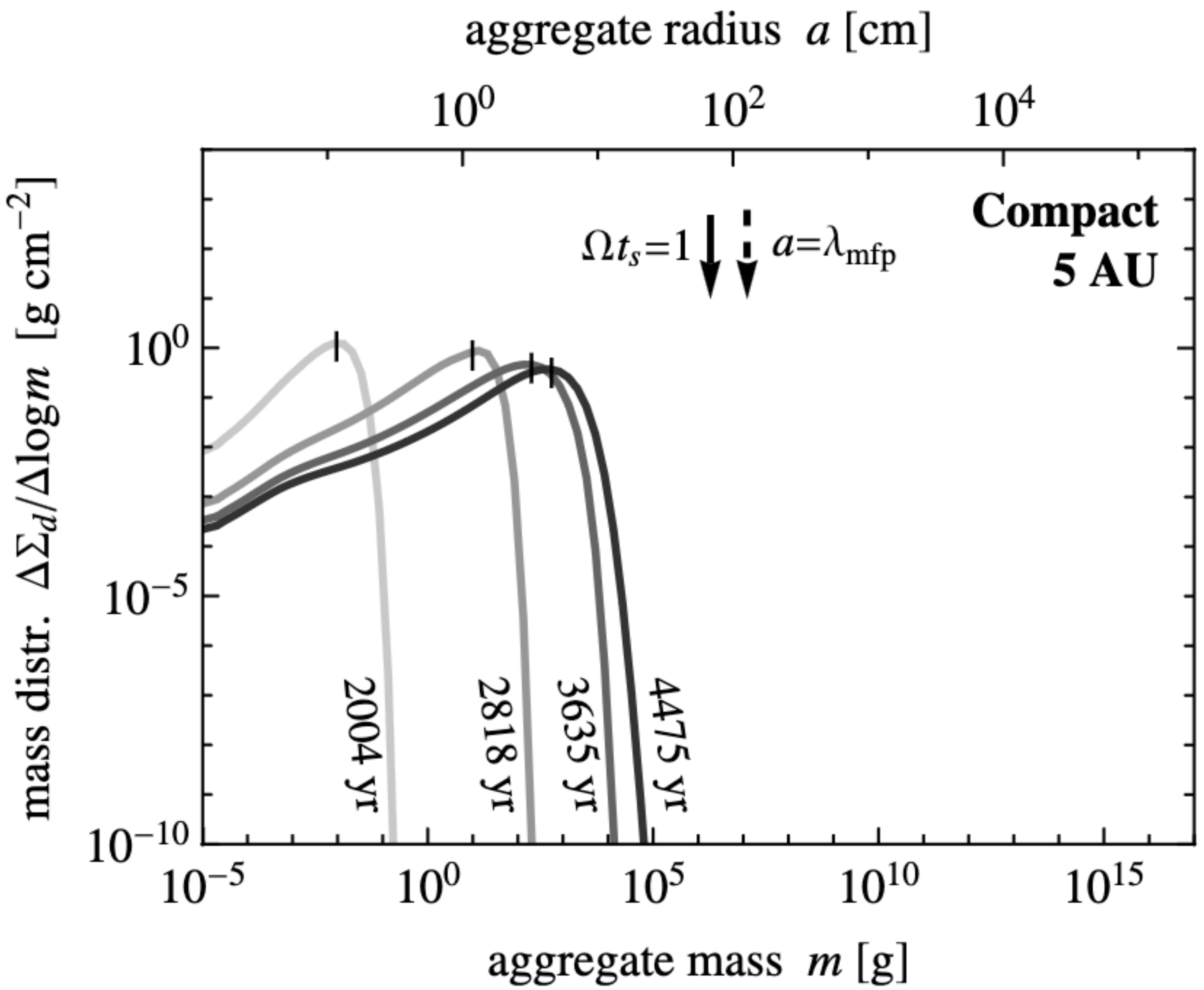
Self-induced dust traps

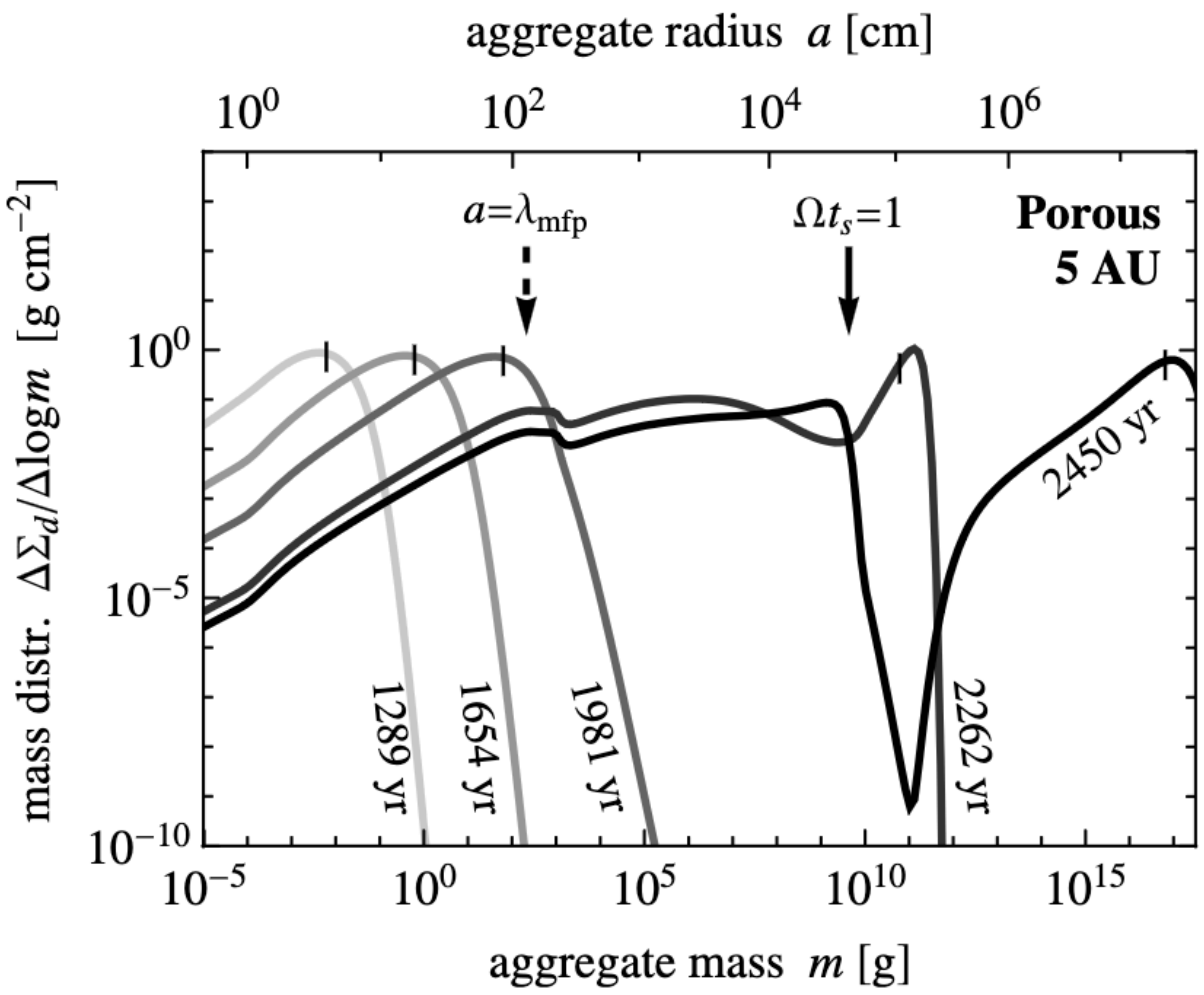




Gonzalez et al. (2017) ; Vericel & Gonzalez (2020) ; Vericel et al. (2021)

Porous grains could overcome the radial drift barrier





Porous grain might grow fast enough to break the drift barrier (but they must not break) : $t_{\rm coag} \propto \sqrt{\rho_{\rm grain}}\sqrt{a_{\rm grain}}$ in the turbulent case

e.g. Okuzumi et al. 2012

More barriers…

Dammit…

Btw we don’t what is the correct solution

Fragmentation

If the grain-grain collision becomes too energetic, then the bonds between the 'monomers' constituting the gains can break.

→ **This is fragmentation**

Experimentally (Dominik & Tielens 1997, *see also* Ormel et al. 2009) a collision between a grain i and j will lead to completed destruction of both grains when :

$$E_{\mathrm{kin},i,j} > E_{\mathrm{frag},i,j} = 5N_{\mathrm{mono}}\,E_{\mathrm{br}}$$

Or

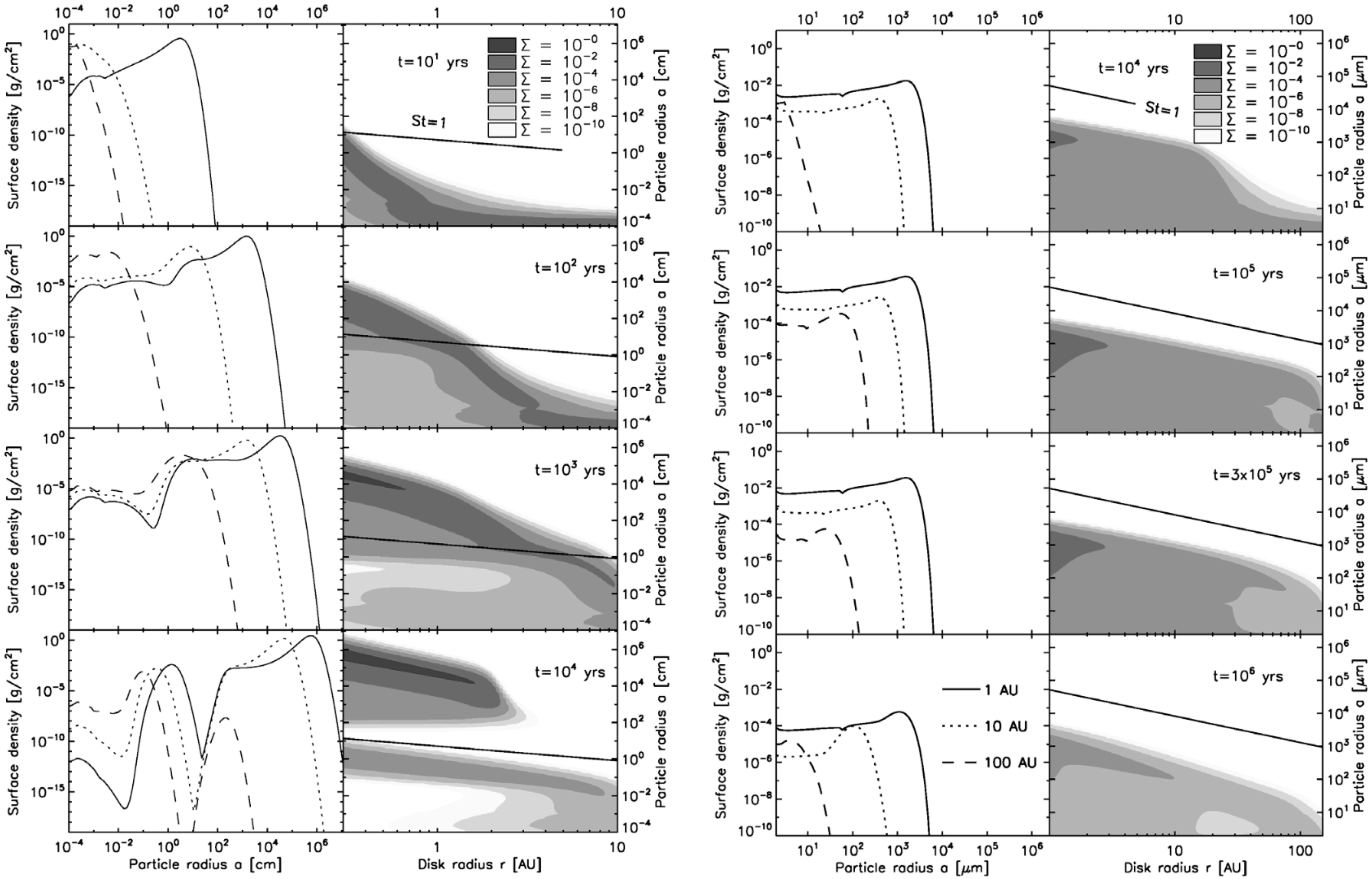
$$\frac{1}{2}\frac{m_i m_j}{m_i + m_j}\Delta v^2_{\mathrm{frag},i,j} = 5\left(\frac{m_i + m_j}{m_{\mathrm{mono}}}\right)E_{\mathrm{br}}$$

If we assume that the dust grains is composed of monomers that all have the same size (or a distribution see Kawasaki et al. 2022) we have

$$\Delta v_{i,j,\mathrm{frag}} = \left(1 + m_j/m_i\right)\sqrt{\frac{10 m_i E_{br}}{m_j\, m_{\mathrm{mono}}}}$$

→ the fragmentation velocity is not constant but depend on the mass ratio between the grains

Fragmentation: yet another barrier



If grains grow fast (e.g. high dust-to-gas ratio) —> they can overcome the radial drift barrier.

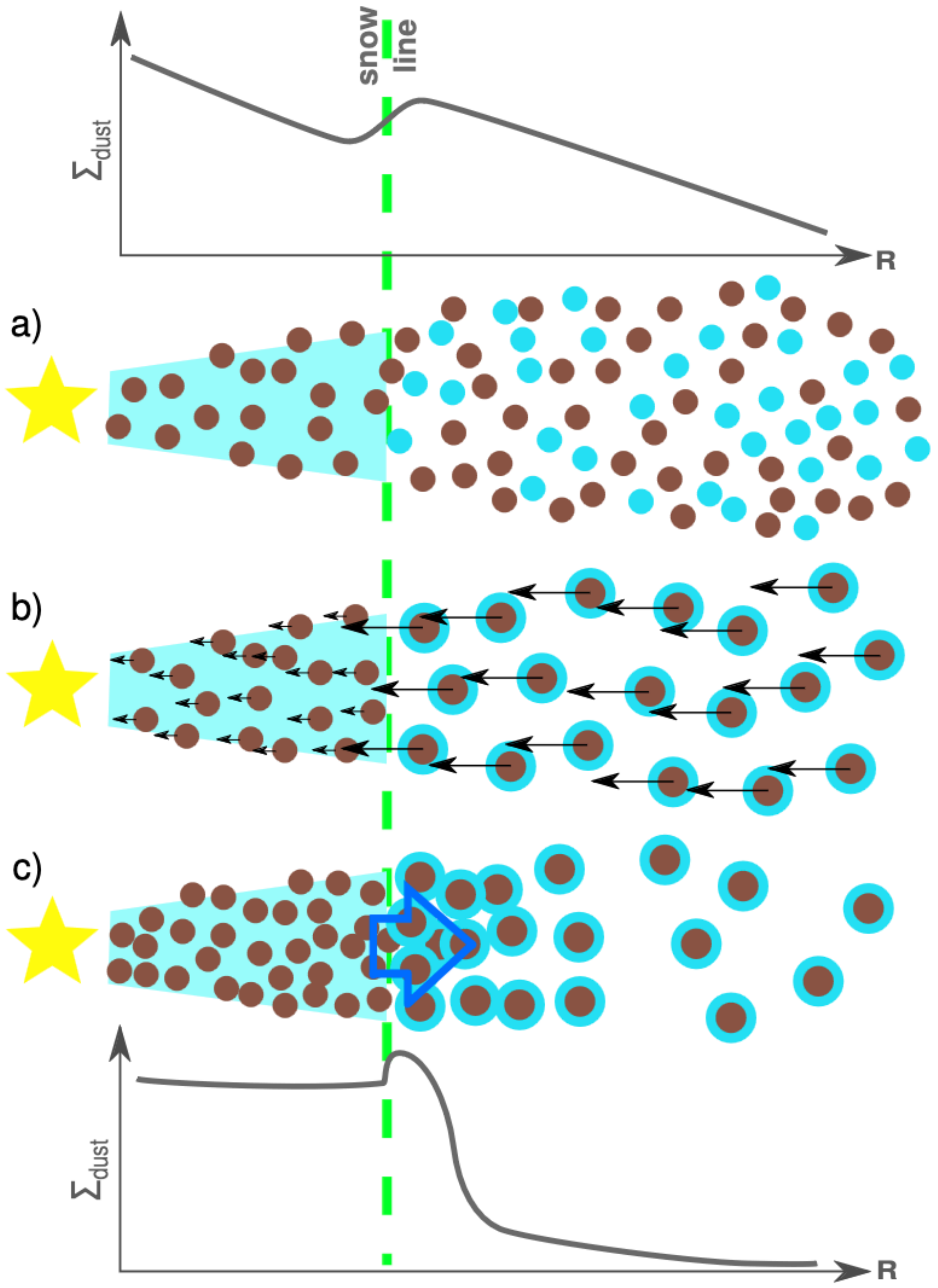
— —> Unfortunately grains could break below the barrier.

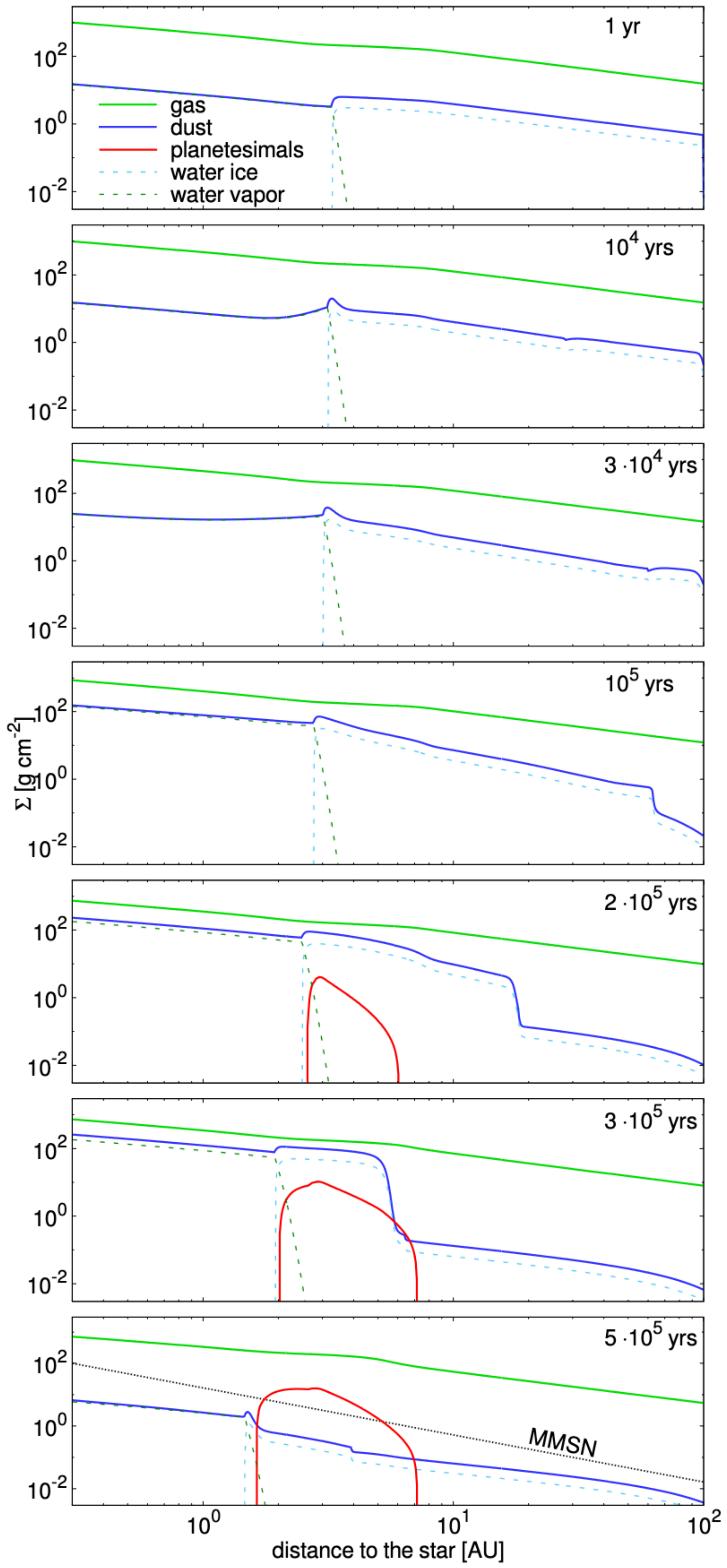
In that case, not only do we not have big grains but we also loose the small ones to radial drift… (fortunately we have seen some solutions)

Brauer et al. (2008)

Fragmentation: may it help ?

The change of fragmentation velocity at the snow line could trigger a traffic jam





→ increase in the dust to gas ratio

→ the streaming instability could be triggered at snow lines !

Draskowska et al. (2017)

Back to the protostellar collapse

Fragmentation : where is the threshold ?

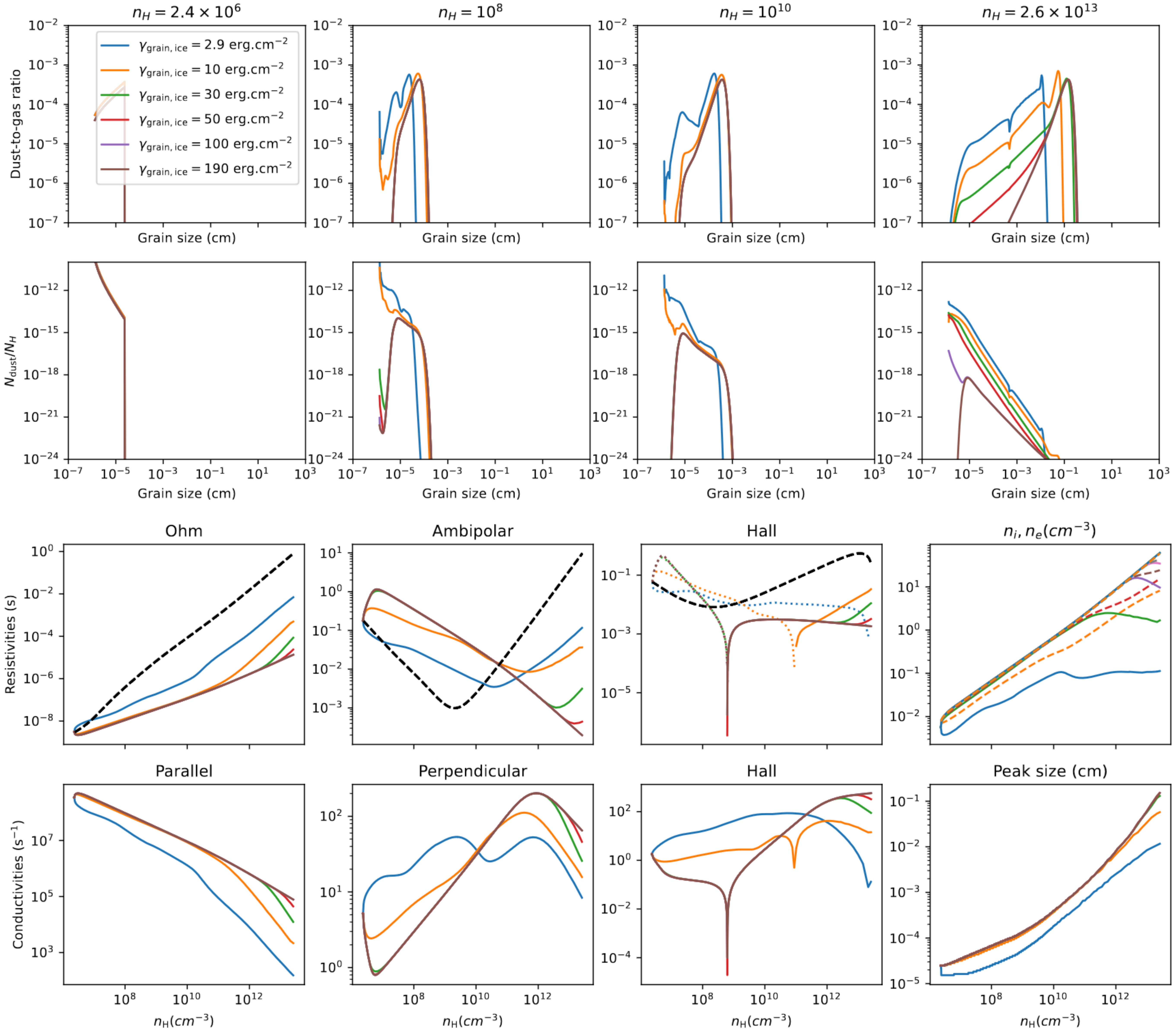
We do not know well the elastic properties of the grains !

The fragmentation velocity could range from as low as : 0.1m/s to 100 m/s (plus it's not supposed to be a velocity threshold)

Big problem for planet formation because we don't know what Stokes is reached.

Gundlach and Blum 2015; Poppe et al. 2000;Güttler et al. 2010; Musiolik et al. 2016a,b; Blum and Wurm 2008,*Musiolik & Wurm 2019, Yamamoto et al. (2014) etc..*

Impact of coagulation/fragmentation on the magnetic resistivities



Vallucci-Goy et al. sub see also :

Guillet et al. 2020; Silsee et al. 2020, Lebreuilly et al. 2023

Conclusions II

- Coagulations happens even during the protostellar collapse.
- It leads to the growth from ISM grains to grains up to mm/cm/dm? sizes
- Coagulation can either be modelled by the solving Smoluchowski equation or the Stekinski (monodisperse) equation
- Coagulation is hard to solve in 3D but new methods are on their way.
- Coagulation deeply affect the magnetic resistivities and therefore disk formation