MHD turbulence and star formation

Blakesley Burkhart

Rutgers University

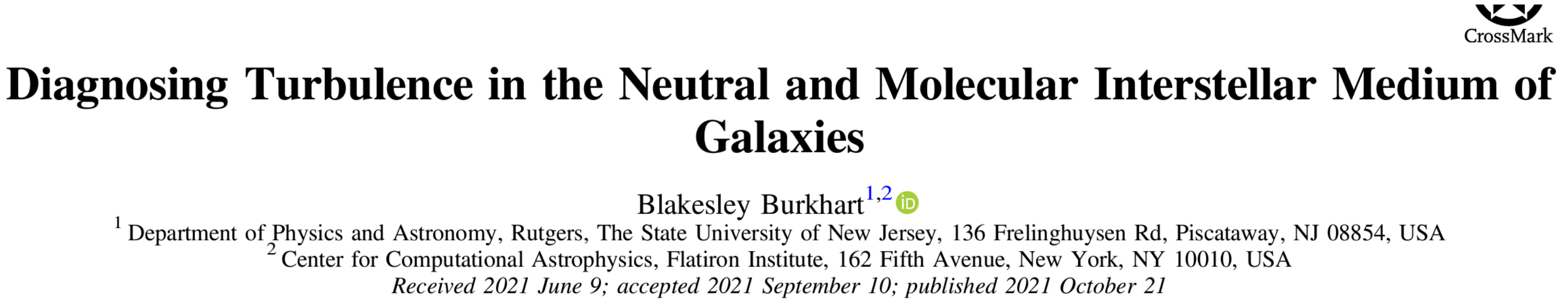
Flatiron Institute

- -what is MHD turbulence (cascade, intermittency, anisotropy, compressibility)
- Diagnostics
- Tomorrow: Consequences of turbulence for SF
- Tomorrow: Self regulation models

MHD Turbulence: A Biased Review

Alexander A. Schekochihin†
Rudolf Peierls Centre for Theoretical Physics, University of Oxford,
Clarendon Laboratory, Parks Road, Oxford OX1 3PU, UK
Merton College, Oxford OX1 4JD, UK

(compiled on 21 July 2022)



Diagnosing Turbulence in the Neutral and Molecular Interstellar Medium of Galaxies

Blakesley Burkhart[1,2]
[1] Department of Physics and Astronomy, Rutgers, The State University of New Jersey, 136 Frelinghuysen Rd, Piscataway, NJ 08854, USA
[2] Center for Computational Astrophysics, Flatiron Institute, 162 Fifth Avenue, New York, NY 10010, USA

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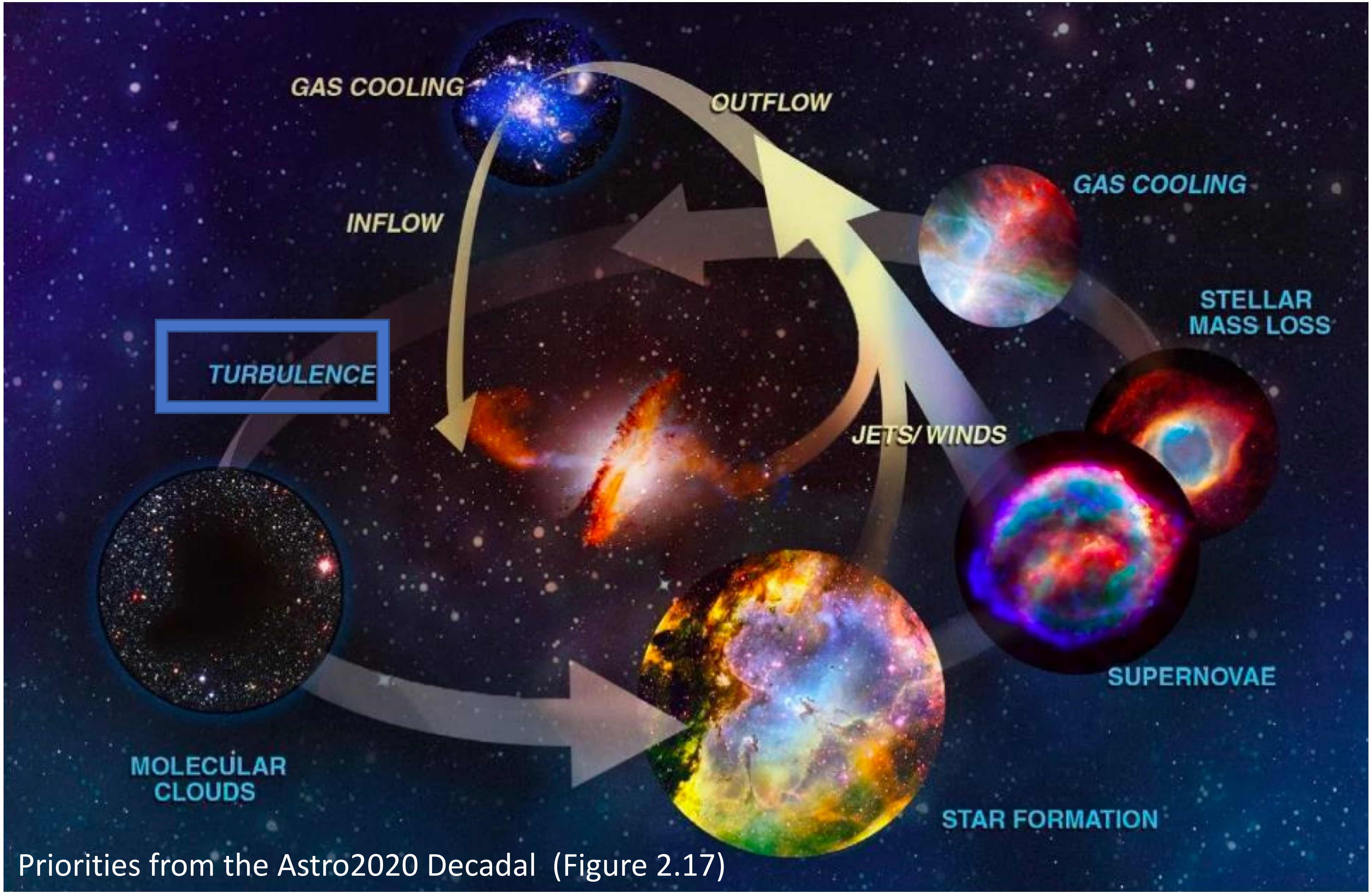


Priorities from the Astro2020 Decadal (Figure 2.17)

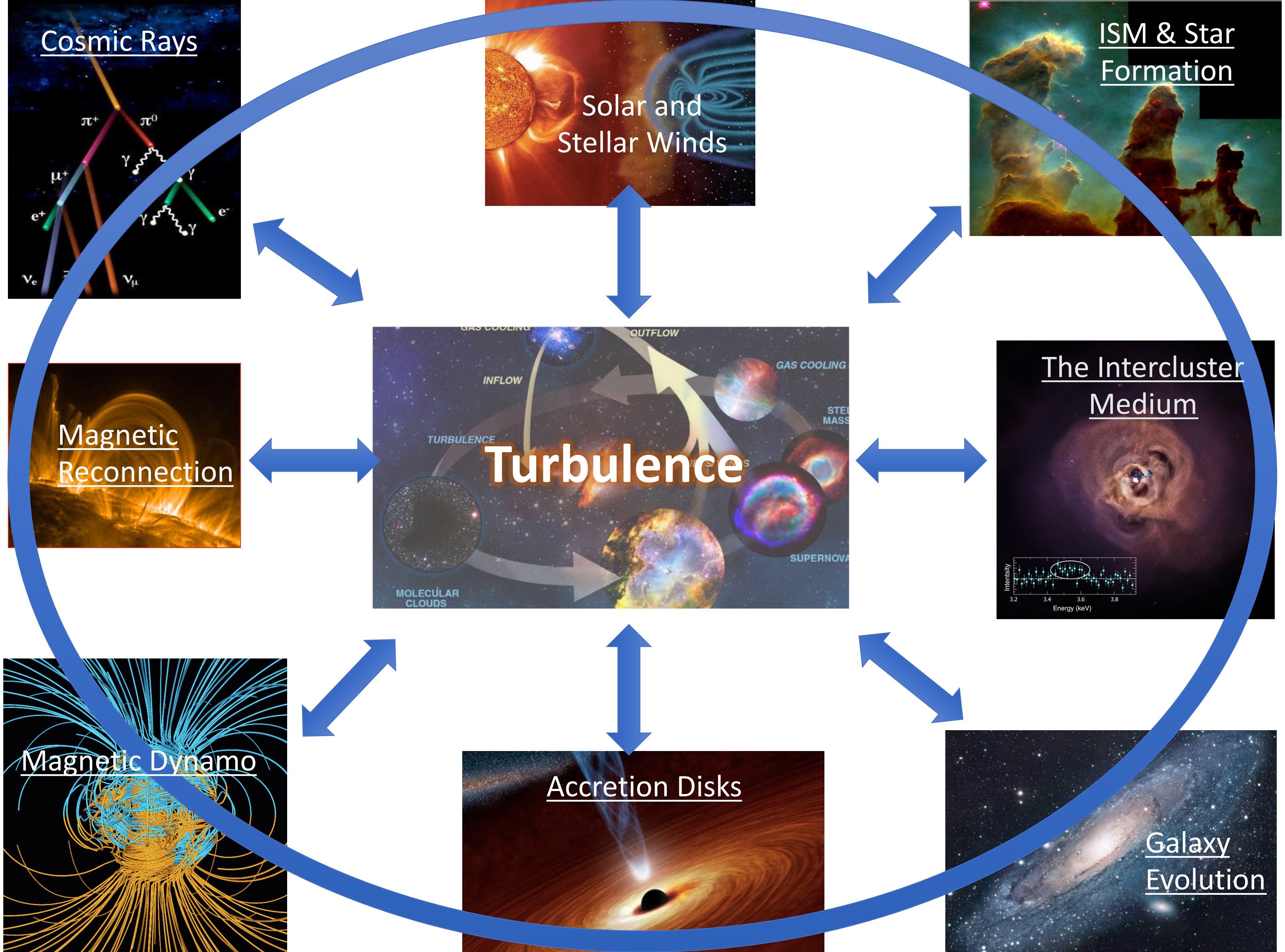
Cosmic Ecosystems

Astro 2020 Priority Area

Unveiling the Drivers of Galaxy Growth

Research in the coming decade will revolutionize our understanding of the origins and evolution of galaxies, from the cosmic webs of gas that feed them to the formation of stars.

Turbulence is the key ingredient for modeling all galactic gas flows.



Cosmic Rays

Solar and Stellar Winds

ISM & Star Formation

Magnetic Reconnection

Turbulence

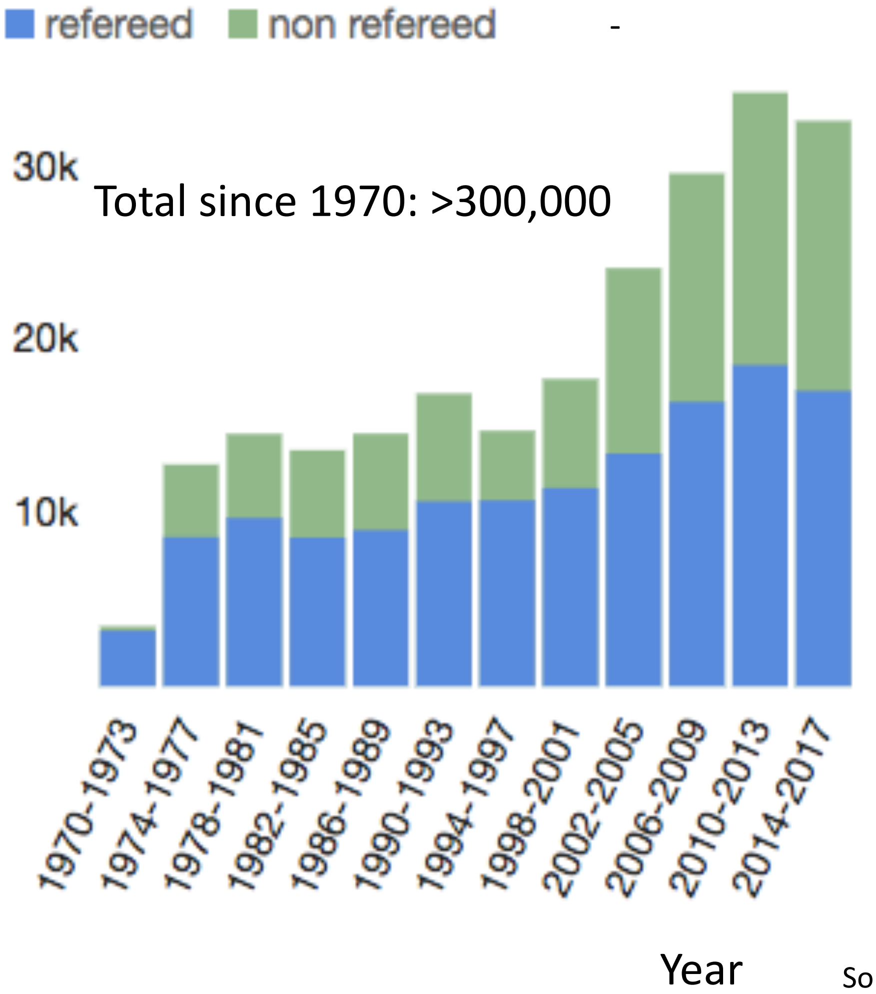
The Intercluster Medium

Magnetic Dynamo

Accretion Disks

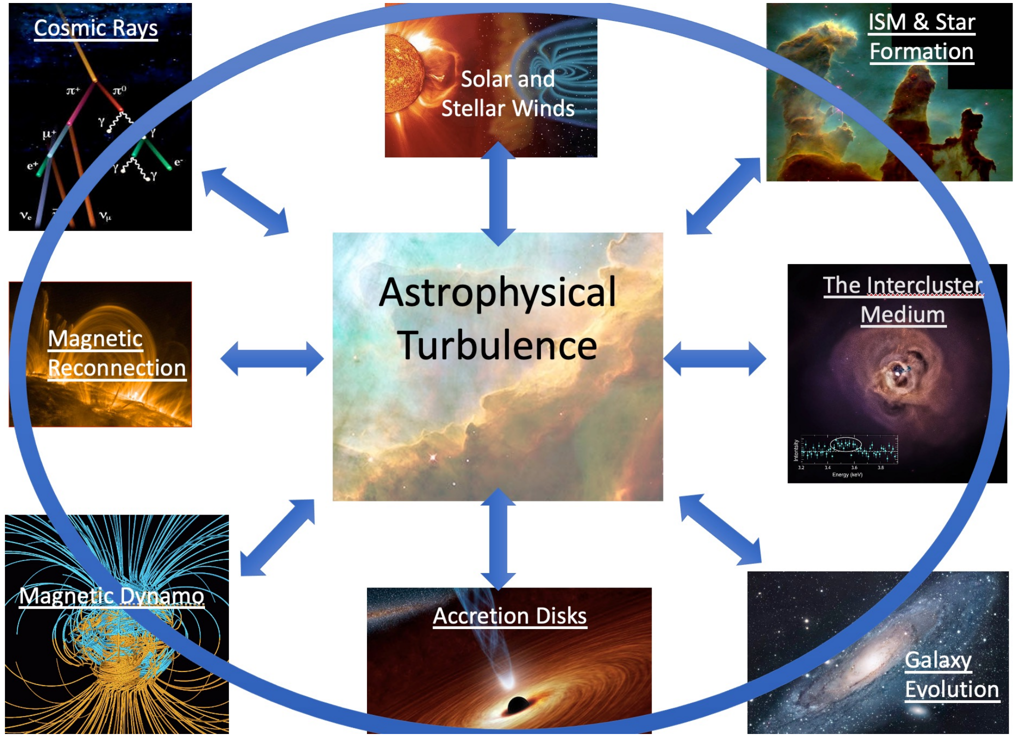
Galaxy Evolution

Astronomy Papers w/ “Turbulence” in Abstract



Total since 1970: >300,000

Source: ADS Bumblebee



Cosmic Rays

Solar and Stellar Winds

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Astrophysical Turbulence

The Intercluster Medium

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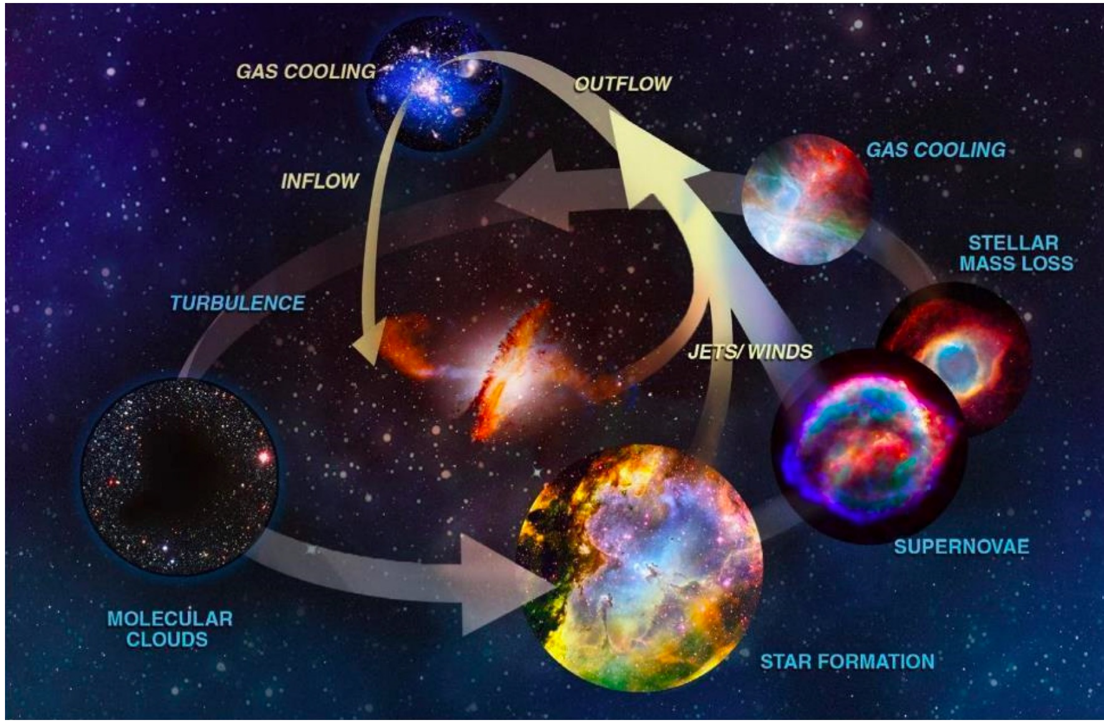
Galaxy Evolution

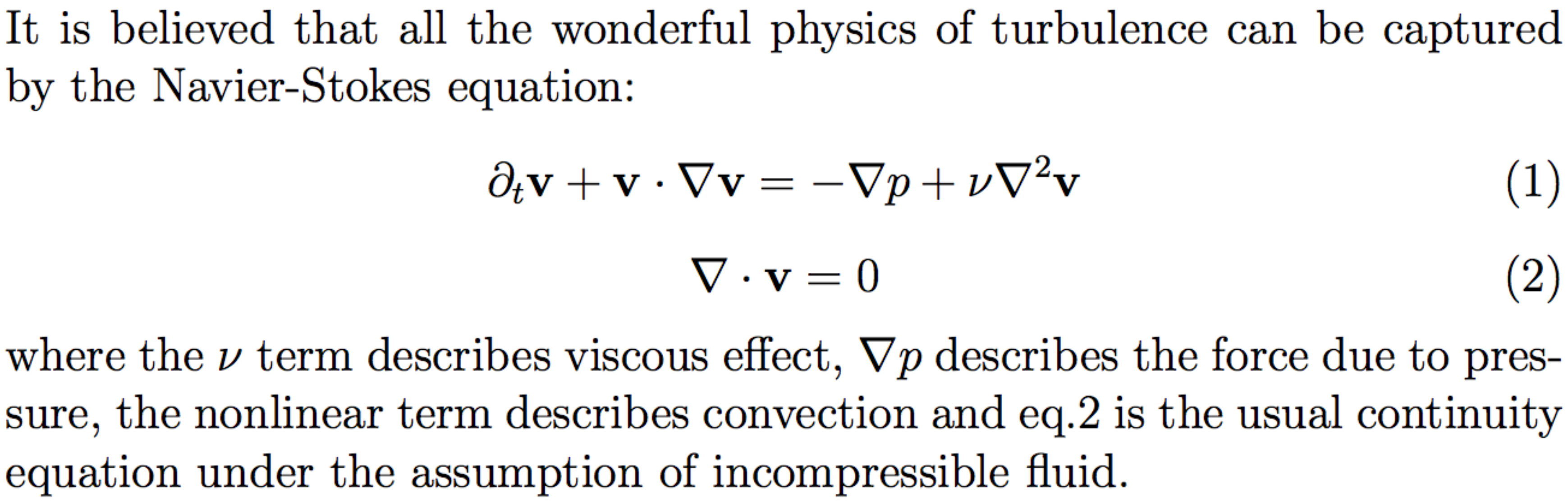
MHD Turbulence..



- What is turbulence?
- Hydro: Kolmogorov 41
- MHD (1995): GS95 and Critical Balance
- MHD (2006): Dynamic Alignment
- Intermittency
- Compressibility
- Diagnostics (tomorrow?)
- Star formation self-regulation via turbulence and feedback (tomorrow?)

What is hydro turbulence?

Navier-Stokes



It is believed that all the wonderful physics of turbulence can be captured by the Navier-Stokes equation:

$$\partial_t \mathbf{v} + \mathbf{v} \cdot \nabla \mathbf{v} = -\nabla p + \nu \nabla^2 \mathbf{v} \tag{1}$$

$$\nabla \cdot \mathbf{v} = 0 \tag{2}$$

where the ν term describes viscous effect, ∇p describes the force due to pressure, the nonlinear term describes convection and eq.2 is the usual continuity equation under the assumption of incompressible fluid.

Turbulence: The unsolved Millennium Prize Problem

Prove or give a counter-example of the following statement:

"In three space dimensions and time, given an initial velocity field, there exists a vector velocity and a scalar pressure field, which are both smooth and globally defined, that solve the Navier–Stokes equations."

What is hydro turbulence: Reynolds Number

- Reynolds number: $Re=VL/\nu \longleftarrow \boxed{(V^2/L) / (\nu V/L^2)}$

$$\frac{\partial \mathbf{v}}{\partial t} = -\underset{\underset{V^2/L}{\uparrow}}{(\mathbf{v}\cdot\nabla)\mathbf{v}} + \underset{\underset{\nu V/L^2}{\uparrow}}{\nu\nabla^2\mathbf{v}}$$

- When $Re \ll Re_{critical}$, flow = laminar

  When $Re \gg Re_{critical}$, flow = turbulent

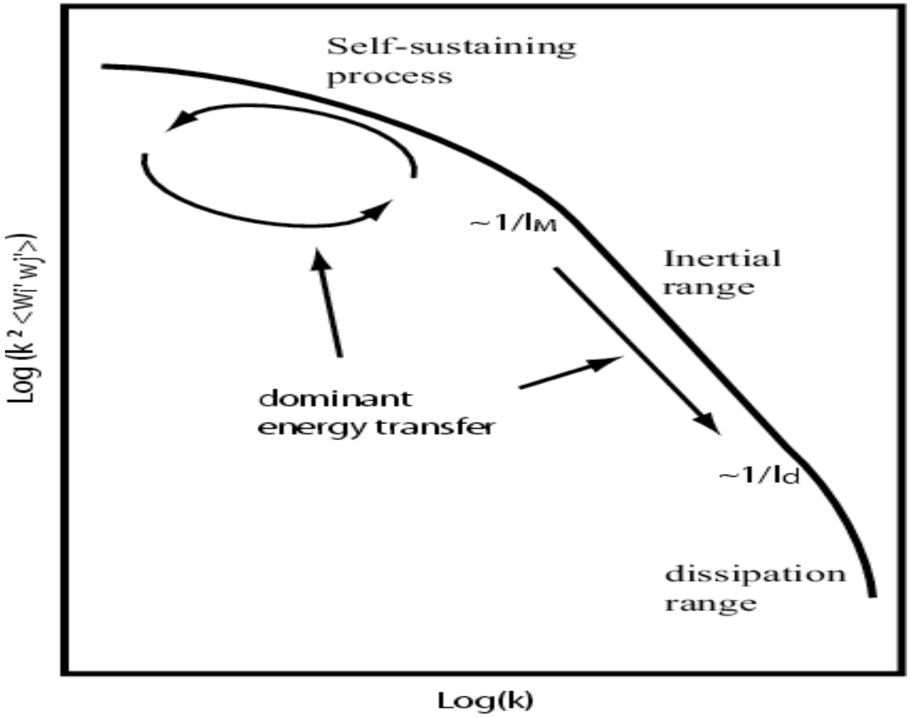
Image credit: Gary Settles

GMCs can have flows $Re>10^{10}$…. There is no question galaxies are turbulent!

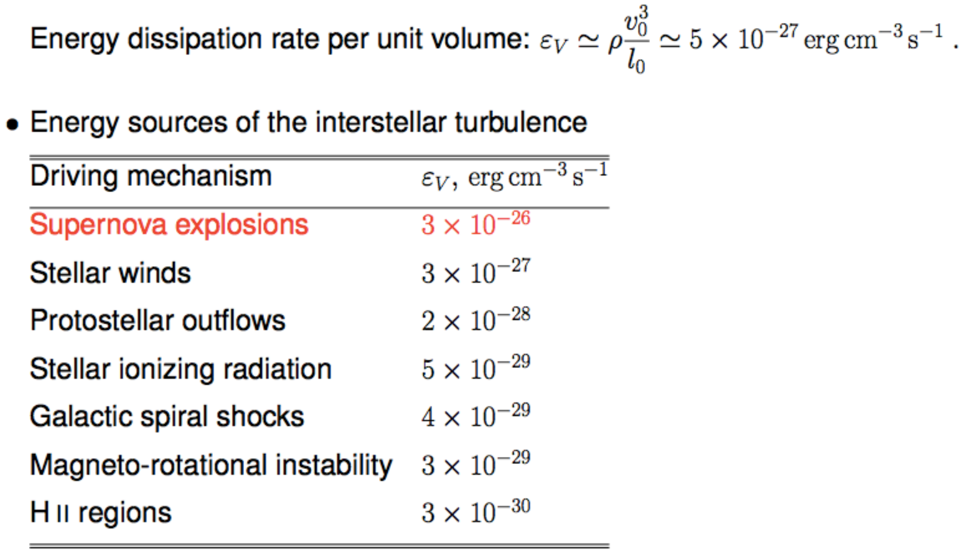
What is Turbulence:
Energy Cascade

Turbulence is not just ‘chaos’. Turbulence is an energy transfer in space/time. It has specific statistical properties which can be seen when averaged over space/time.

$$P(\overline{k}) = \sum_{\overline{k}=const.} \tilde{F}(\overline{k}) \cdot \tilde{F}^{*}(\overline{k})$$



Three ranges of scales of interest:
driving scale(s), inertial range, dissipation scale(s)



Energy dissipation rate per unit volume: $\varepsilon_V \simeq \rho \frac{v_0^3}{l_0} \simeq 5 \times 10^{-27}\,\mathrm{erg\,cm^{-3}\,s^{-1}}$.

- Energy sources of the interstellar turbulence

Driving mechanism	ε_V, $\mathrm{erg\,cm^{-3}\,s^{-1}}$
Supernova explosions	3×10^{-26}
Stellar winds	3×10^{-27}
Protostellar outflows	2×10^{-28}
Stellar ionizing radiation	5×10^{-29}
Galactic spiral shocks	4×10^{-29}
Magneto-rotational instability	3×10^{-29}
H II regions	3×10^{-30}

What is Turbulence: Sources of Energy

$$\left(\frac{dE}{dA}\right)_{\rm turb} \approx \frac{3}{2}\Sigma_{\rm g}\sigma_{\rm g}^2 = 3.1\times 10^9\,\Sigma_{\rm g,10}\sigma_{\rm g,10}^2\ \mathbf{erg\ cm}^{-2}$$

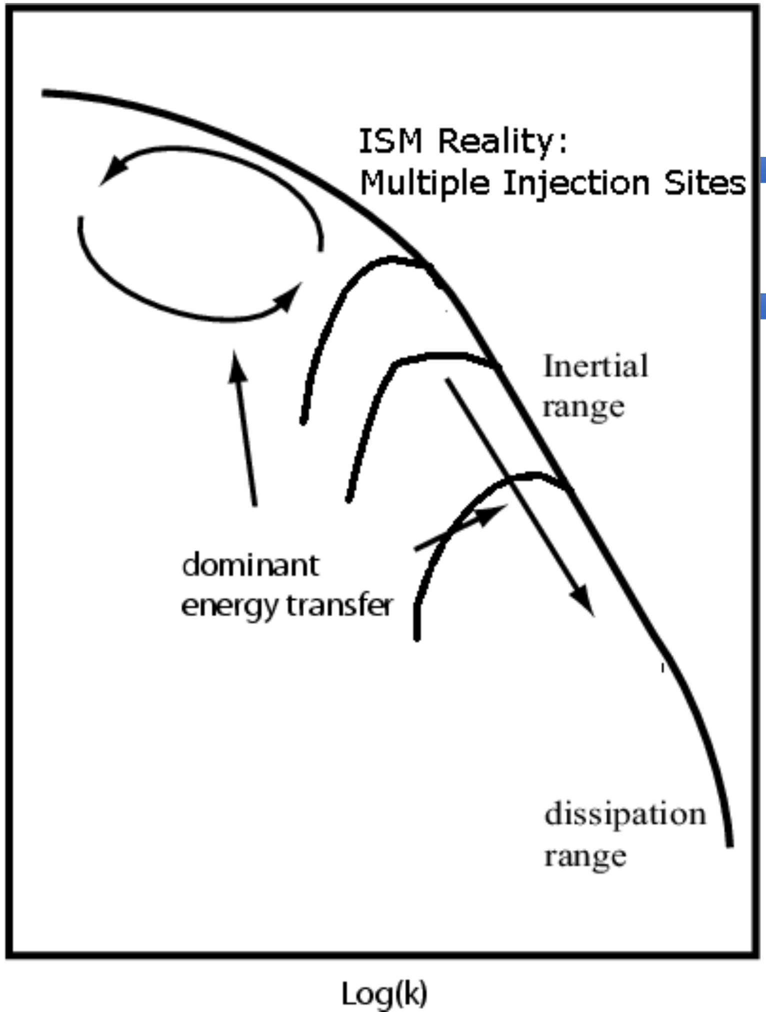
What sources this?

$$\begin{aligned}\left(\frac{dE}{dA}\right)_{\rm sf} &\approx \dot{\Sigma}_* \left\langle \frac{p_*}{m_*} \right\rangle \sigma_{\rm g}\frac{r}{v_\phi}\\ &= 3.1\times 10^9\,\dot{\Sigma}_{*,-3}\sigma_{\rm g,10}r_{10}v_{\phi,200}^{-1}\ \mathbf{erg\ cm}^{-2}\end{aligned}$$

$$\left(\frac{dE}{dA}\right)_{\rm inflow} \approx \frac{\dot{M}_{\rm in}v_\phi}{2\pi r} = 6.5\times 10^9\,\dot{M}_{\rm in,1}v_{\phi,200}r_{10}^{-1}\ \mathbf{erg\ cm}^{-2},$$

Mass inflow can be an important energy source in galaxies…as important as feedback… more on this tomorrow!

Origins of Turbulence: Multiple Drivers

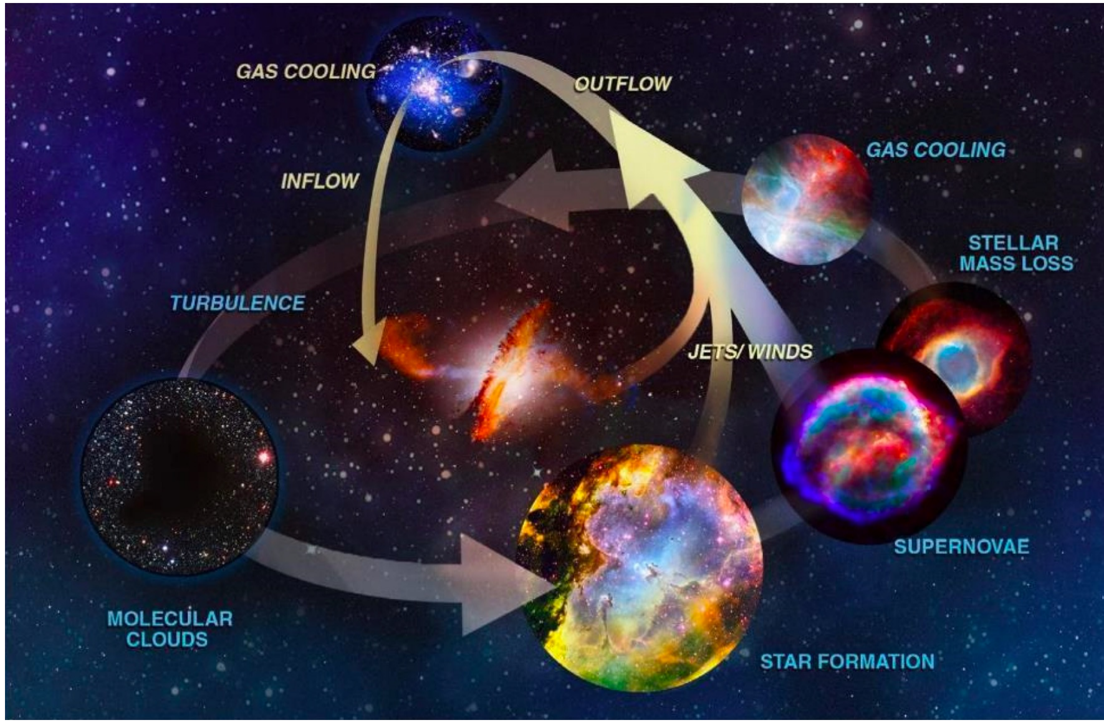


<u>1000 Pc scales:</u>
Galaxy mergers (major/minor), Expanding SNe shells, disk instability

<u>100 Pc scales:</u>
supernova, expanding shells, MRI, cloud collisions…

<u>10 pc-sub-pc scales:</u>
Winds, outflows, stellar feedback, stellar wakes

MHD Turbulence..



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In the beginning (in 1941) was Kolmogorov...

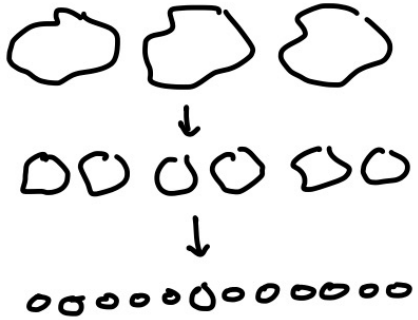
$$\frac{\delta u_\lambda^2}{\tau_\lambda} \sim \epsilon$$ energy flux through scale λ (assumed local in λ)

τ_λ: cascade time

$$\tau_\lambda \sim \frac{\lambda}{\delta u_\lambda}$$ (what else?)

$$\Downarrow$$

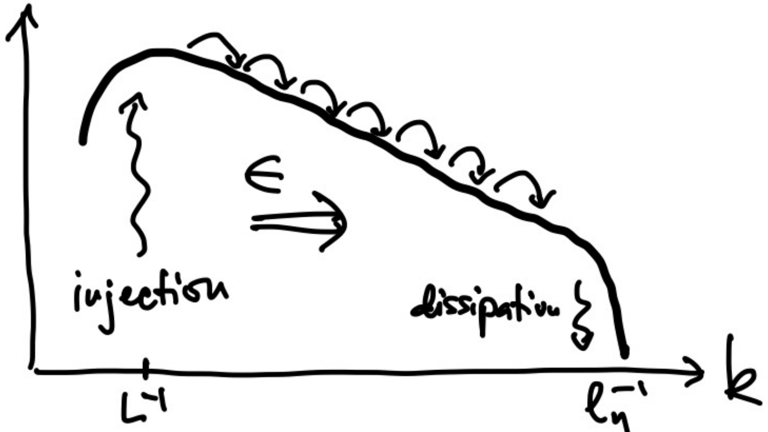
$$\delta u_\lambda \sim (\epsilon\lambda)^{1/3} \Leftrightarrow \boxed{E(k) \sim \epsilon^{2/3} k^{-5/3}}$$



"K41"

$$k = \frac{1}{\text{length}} \qquad \varepsilon = \frac{\text{length}^2}{\text{time}^3} \qquad E(k) = \frac{\text{length}^3}{\text{time}^2}$$

Only possible combination with the right dimensions:



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[2] "All changes in nature occur in such a way that if anything is added anywhere, the same amount is subtracted from somewhere else. [...] As this is a universal law of nature, it extends to the laws of motion..."—Lomonosov (1748).

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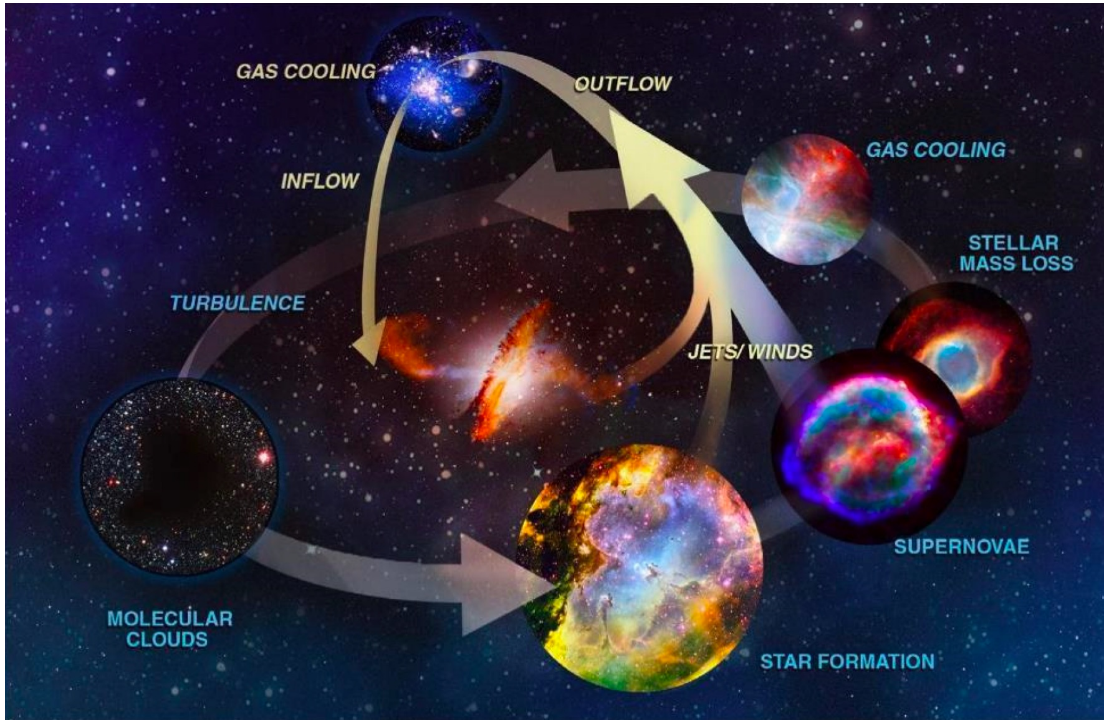
MHD Turbulence..



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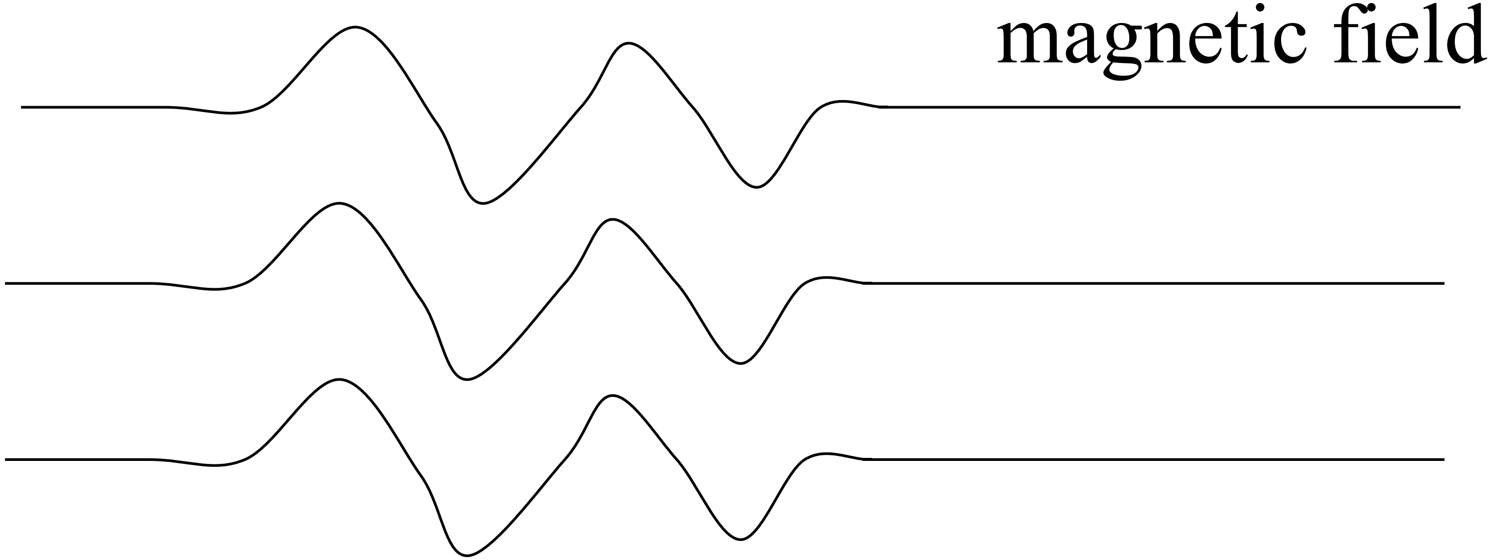
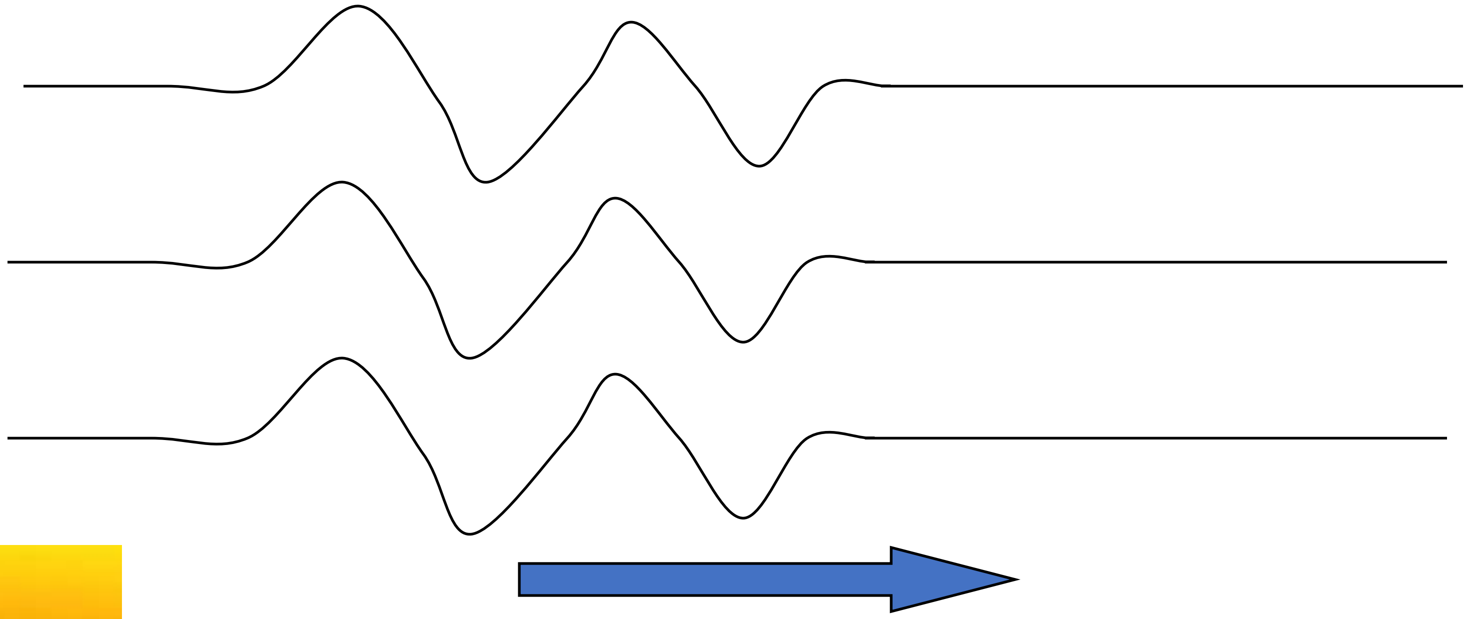
Incompressible MHD turbulence: 1) weak (wave like)



Suppose that we perturb magnetic field lines.
We will only consider Alfvenic perturbations.
(restoring force=tension)
We can make the wave packet move in one direction.

Dynamics of one wave packet

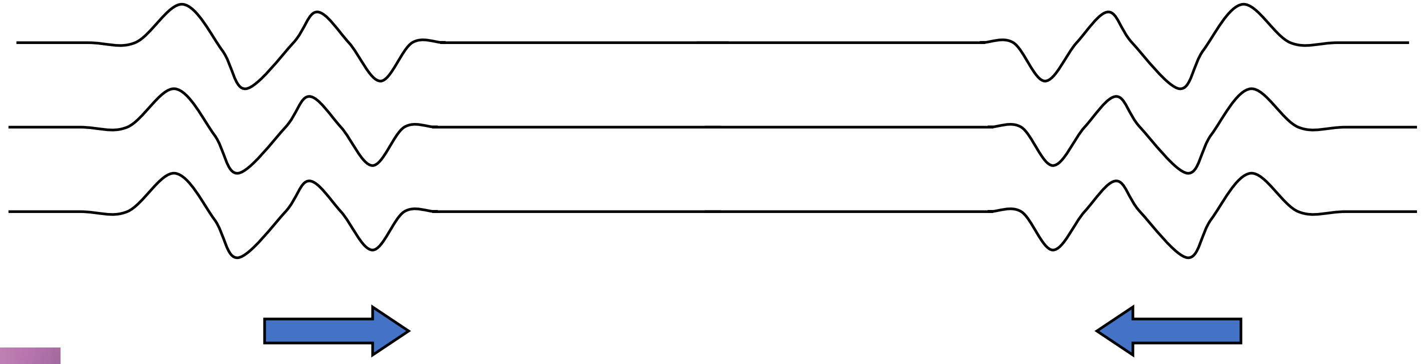
Suppose that this packet is moving to the right.
What will happen?



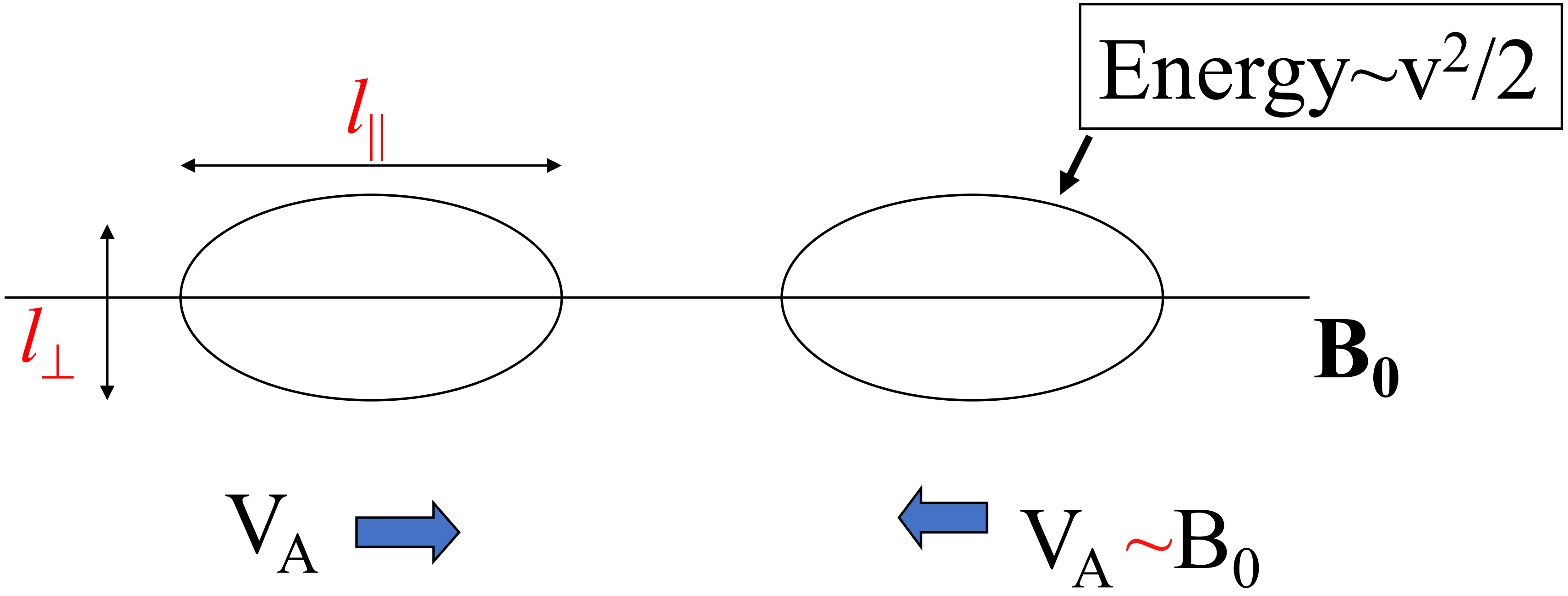
V_A: Alfven speed

Dynamics of two opposite-traveling wave packets

Now we have two colliding wave packets.
What will happen?



What happens when two Alfvenic wave packets collide?



Energy~$v^2/2$

$l_{||}$

$l_\perp$

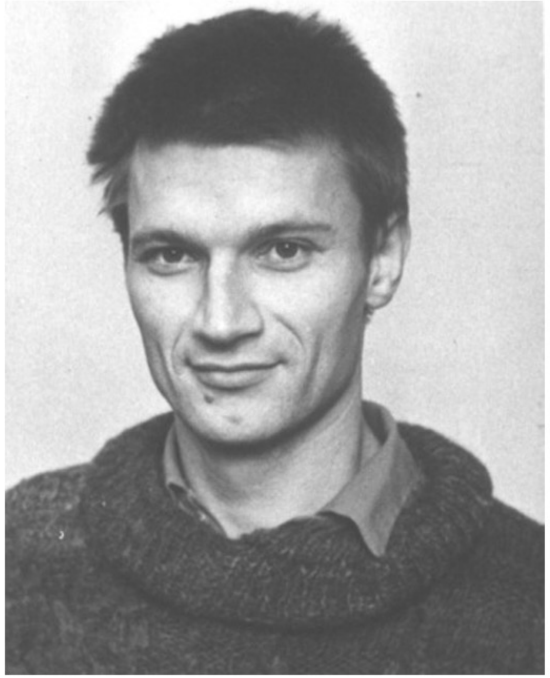
$\mathbf{B_0}$

V_A

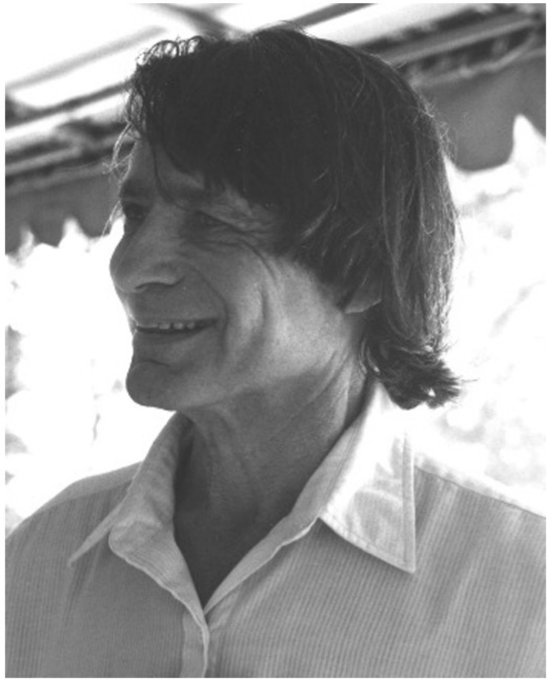
$V_A \sim B_0$

First inclusion of the magnetic field in turbulence was Iroshnikov 1963 and Kraichnan 1965

Isotropic wave-like Alfvenic cascade scales (dimensionally works) like:



R. S. Iroshnikov (1937-1991)



R. H. Kraichnan (1928-2008)

Known as: IK Turbulence

$$E(k) \sim (\varepsilon v_{\mathrm{A}})^{1/2} k^{-3/2} \quad \Leftrightarrow \quad \delta u_{\lambda} \sim (\varepsilon v_{\mathrm{A}} \lambda)^{1/4}.$$

Intuitively there is a problem: mean B field is hard to bend and there can be separate motions parallel and perpendicular to B

In the beginning (in 1941) was Kolmogorov...

In MHD/plasma with straight $\vec{B}_0$,

$\tau_\lambda \sim \tau_{nl}$

not inevitable because there is Alfvén time:

$\tau_A \sim l_\parallel / v_A$

where $l_\parallel \gg \lambda$

$$\frac{\delta u_\lambda^2}{\tau_\lambda} \sim \epsilon$$

energy flux through scale λ (assumed local in λ)

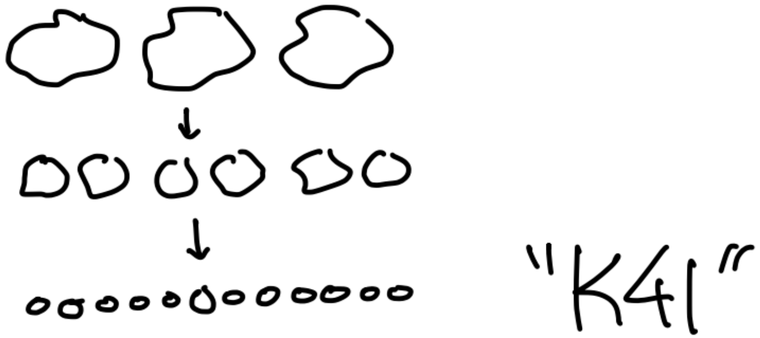
cascade time

$$\tau_\lambda \sim \frac{\lambda}{\delta u_\lambda} \equiv \tau_{nl}$$

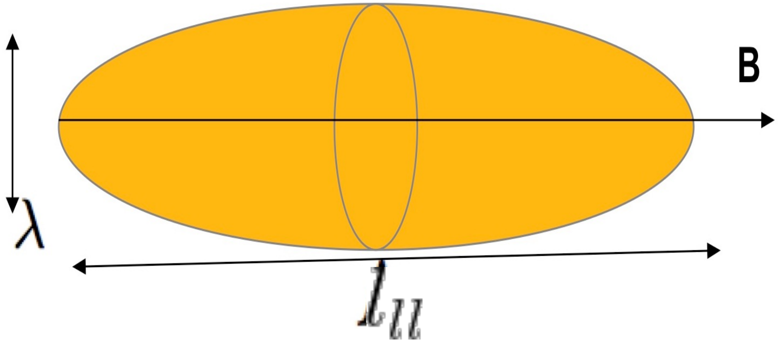
(what else?)

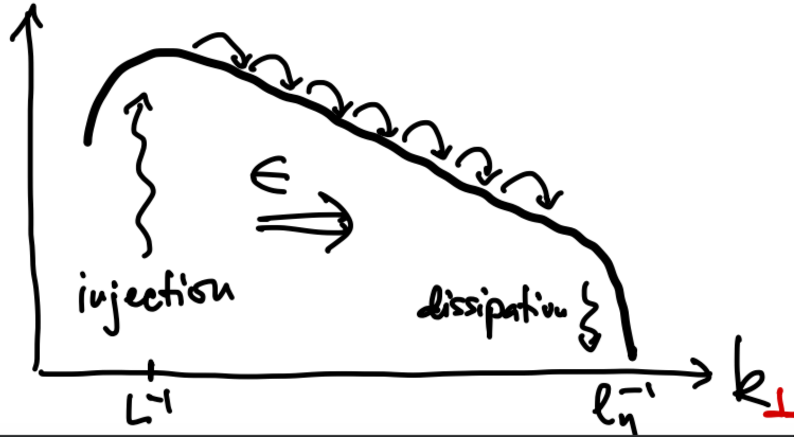
$$\Downarrow$$

$$\delta u_\lambda \sim (\epsilon\lambda)^{1/3} \Leftrightarrow \boxed{E(k) \sim \epsilon^{2/3} k^{-5/3}}$$



Anisotropic eddy





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where $l_\parallel \gg \lambda$

SOLUTION:

"critical balance"

$\tau_A \sim \tau_{nl}$

(Goldreich & Sridhar 1995) ≡ GS95

...but Higdon (1984) deserves a nod...

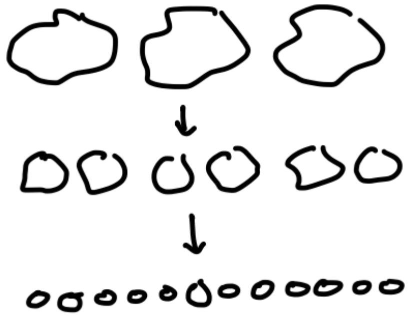
$$\frac{\delta u_\lambda^2}{\tau_\lambda} \sim \epsilon$$

energy flux through scale λ (assumed local in λ)

cascade time

$$\tau_\lambda \sim \frac{\lambda}{\delta u_\lambda} \equiv \tau_{nl}$$

(what else?)



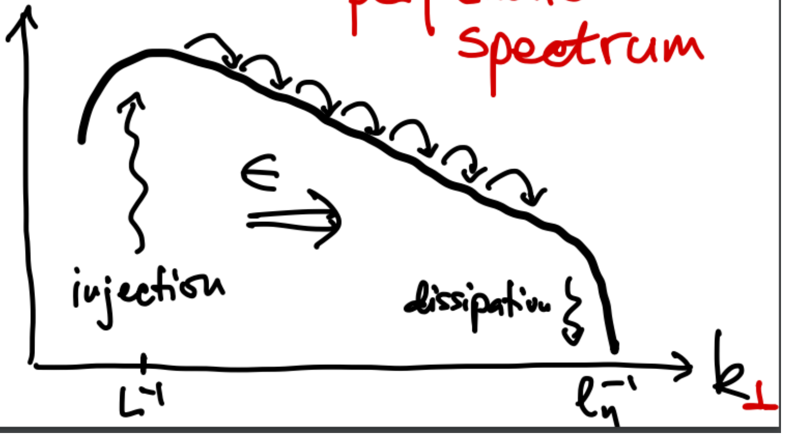
"K41"

$$\Downarrow$$

$$\delta u_\lambda \sim (\epsilon\lambda)^{1/3} \Leftrightarrow \boxed{E(k_\perp) \sim \epsilon^{2/3} k_\perp^{-5/3}}$$

back to K41

perpendicular spectrum



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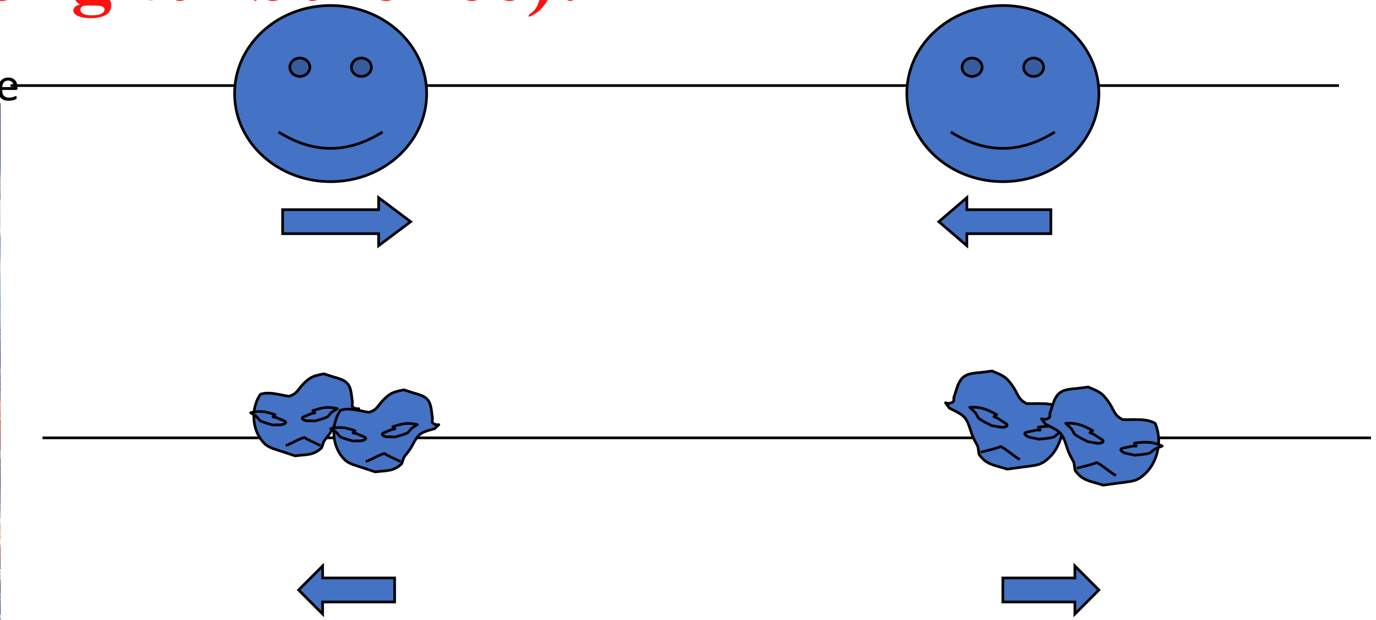
$$\chi \sim t_w/t_{eddy} \sim \ (v\, l_{||} / \, l_{\perp} V_A)$$

- Suppose that $\chi \sim 1$.

e.g.) When $V_A \sim v_l$ and $l_{||} \sim l_{\perp}$, we have $\chi \sim 1$.

=>**1 collision is enough to complete cascade (strong turbulence)!**

Wave vs eddy
Weak vs strong turbulence

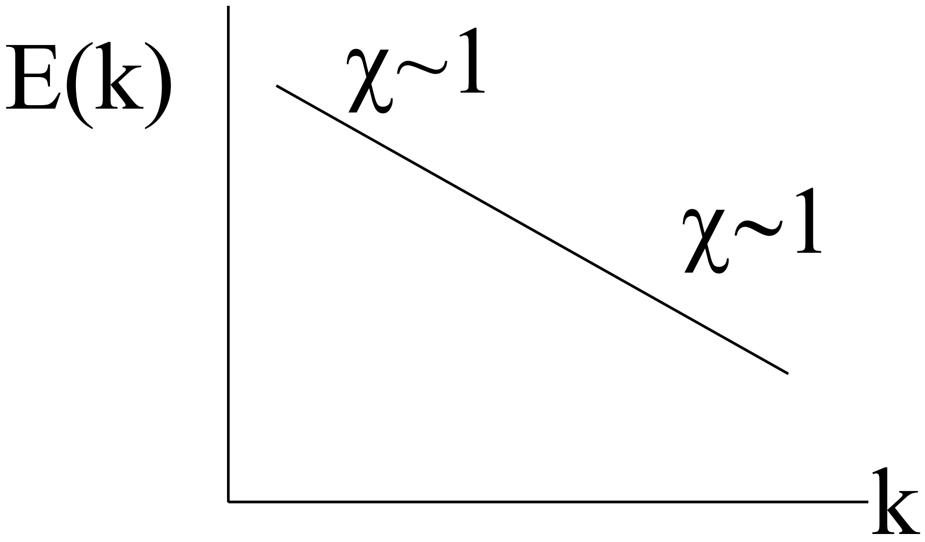


$$\chi \sim t_A / t_{nl} \sim (v\, l_{||} / l_{\perp} V_A)$$

- Goldreich & Sridhar (1995) found that, when $\chi \sim 1$ on a scale, $\chi \sim 1$ on all smaller scales.

* $\chi \sim 1$ is called critical balance

*This regime is called strong turbulence regime



Weak turbulence regime (lectures by Galtier): turbulence is purely wave-like with “weak” perturbations

Even if you are in a weak turbulence regime, eventually as cascade proceeds you will reach a scale where you get into critical balance (i.e. turbulence is strong).

$$\frac{\delta u_\lambda^2}{\tau_\lambda} \sim \epsilon$$

cascade time

$$\tau_\lambda \sim \frac{\lambda}{\delta u_\lambda} \equiv \tau_{nl}$$

(what else?)

$$\Downarrow$$

$$\delta u_\lambda \sim (\epsilon\lambda)^{1/3} \Leftrightarrow \boxed{E(k_\perp) \sim \epsilon^{2/3} k_\perp^{-5/3}}$$

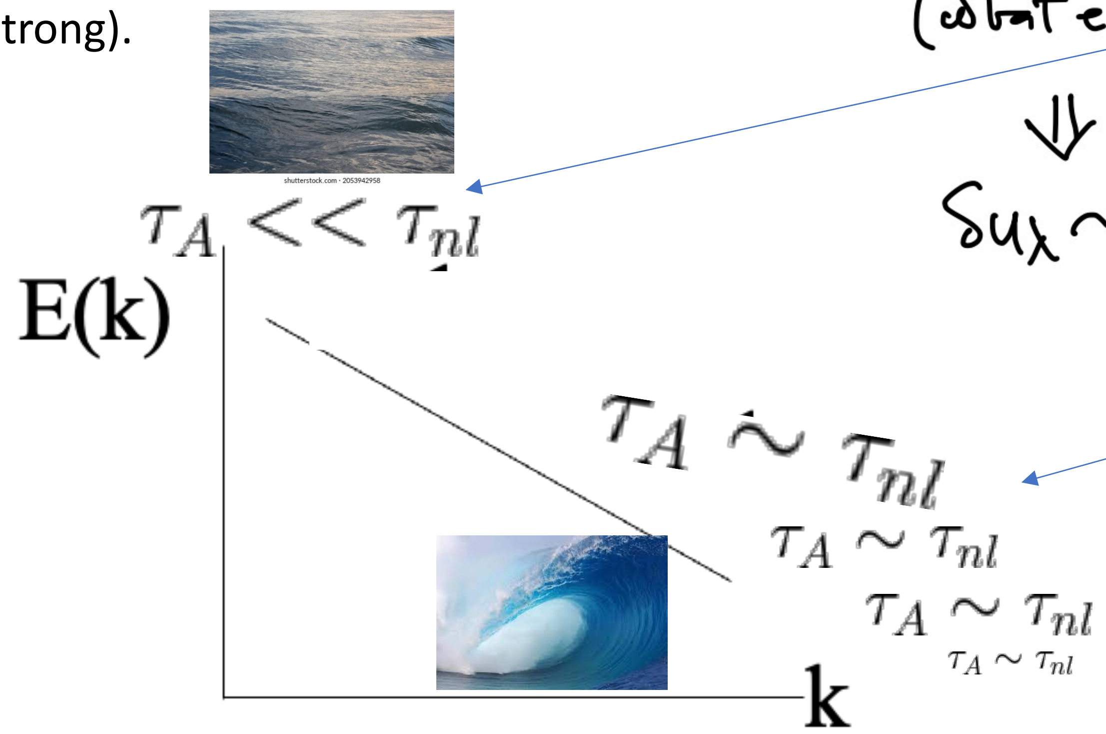
$$\frac{\delta u_{\ell_\parallel}^2}{\tau_A} \sim \epsilon$$ “parallel cascade”

$$\tau_A \wr \ell_\parallel / v_A$$

$$\Downarrow$$

$$\delta u_{\ell_\parallel} \sim \left(\frac{\epsilon \ell_\parallel}{v_A}\right)^{1/2}$$

$$\boxed{E(k_\parallel) \propto k_\parallel^{-2}}$$



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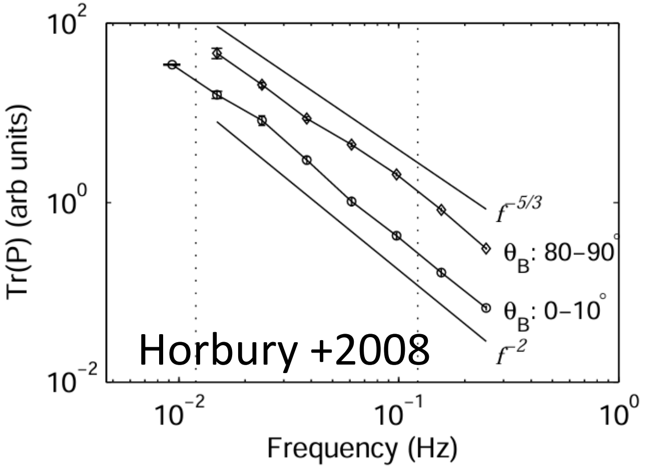
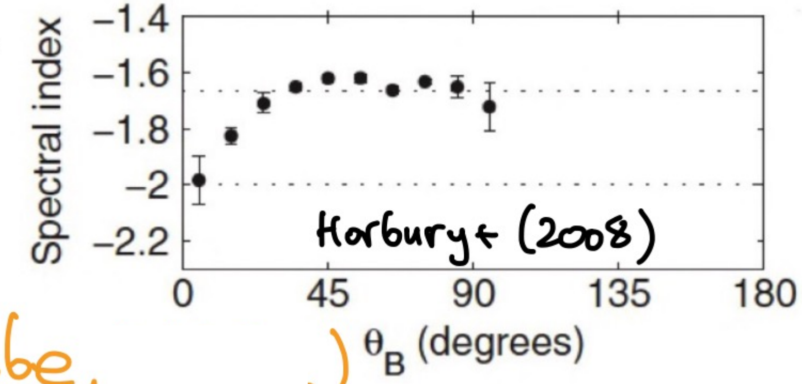


Horbury +2008

Solar wind observations: (Ulysses)

Thus started a GOLDEN AGE of SW turbulence (latest: Parker Solar Probe, .)



Horbury+ (2008)

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WHAT DOES CRITICAL BALANCE MEAN?

- $\tau_A \sim \tau_{nL}$ follows by causality (& WT break-down)
- MHD turbulence is anisotropic in a scale-dependent way

$$\frac{\delta u_\lambda^2}{\tau_\lambda} \sim \epsilon$$

cascade time

$$\tau_\lambda \sim \frac{\lambda}{\delta u_\lambda} \equiv \tau_{nl}$$

(what else?)

$$\Downarrow$$

$$\delta u_\lambda \sim (\epsilon\lambda)^{1/3} \Longleftrightarrow \boxed{E(k_\perp) \sim \epsilon^{2/3} k_\perp^{-5/3}}$$

$$\frac{\delta u_{l_\parallel}^2}{\tau_A} \sim \epsilon$$

"parallel cascade"

$$\tau_A \wr\, l_\parallel / v_A$$

$$\Downarrow$$

$$\delta u_{l_\parallel} \sim \left(\frac{\epsilon l_\parallel}{v_A}\right)^{1/2}$$

$$\boxed{E(k_\parallel) \propto k_\parallel^{-2}}$$

$$\boxed{l_\parallel \sim v_A \epsilon^{-1/3} \lambda^{2/3}}$$

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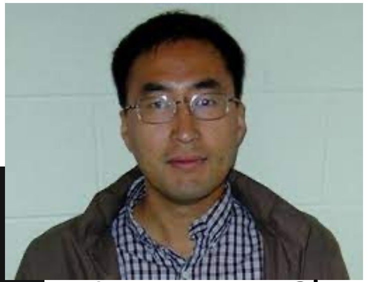
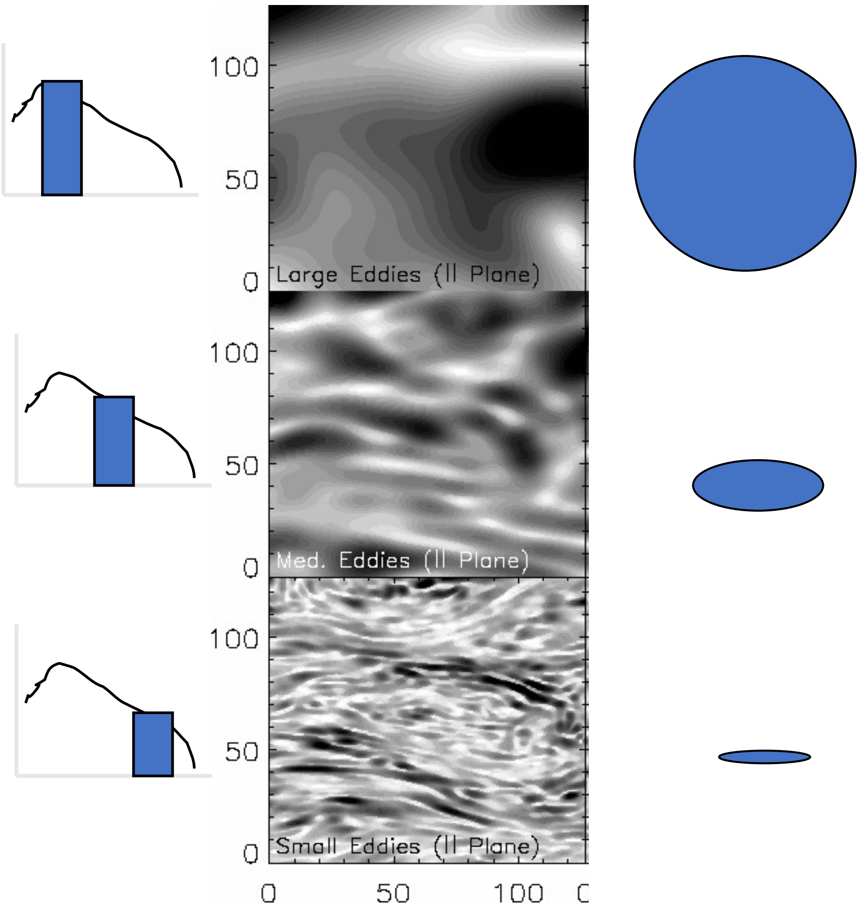
Magnetohydrodynamic Turbulence



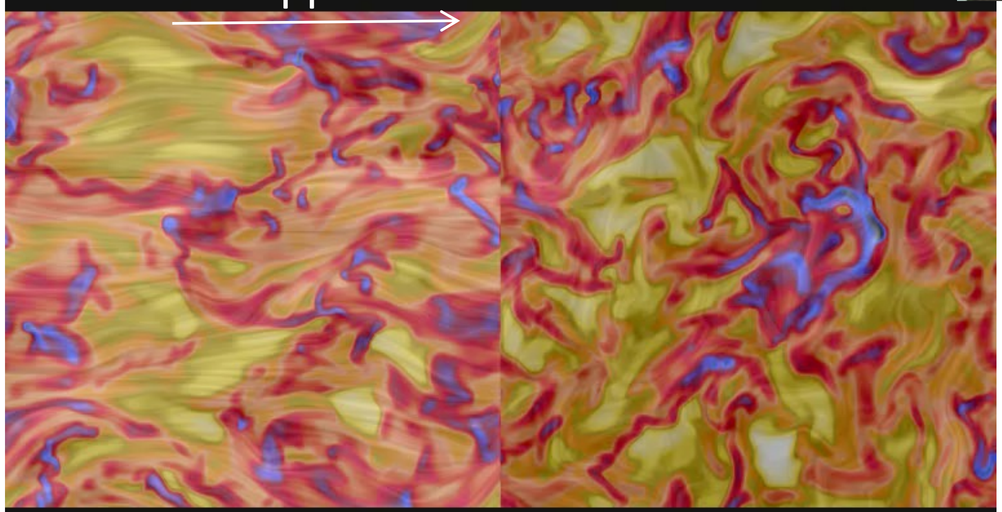
Jungyeon Cho



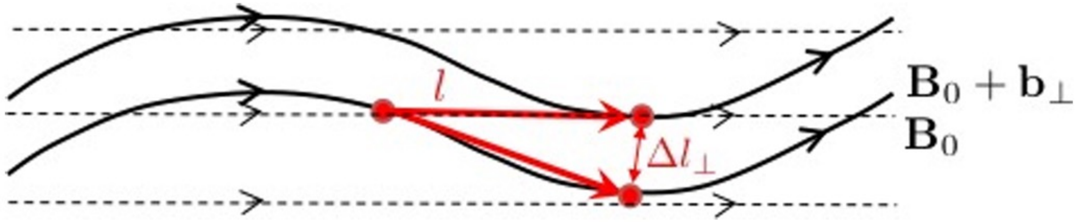
Magnetic field

B_0

Goldreich & Sridhar 1995, Cho et al. 2002

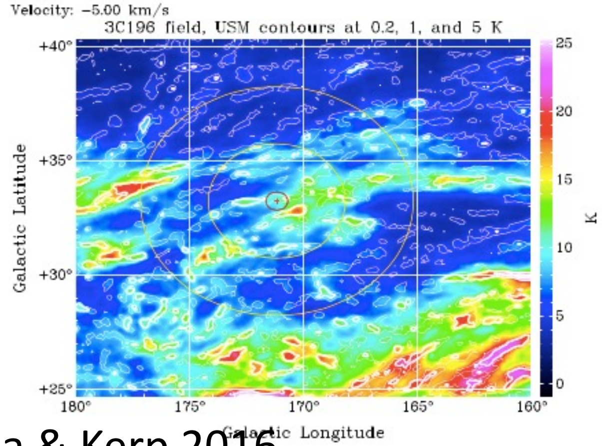


Density Field MHD Simulation, Burkhart et al. 2009



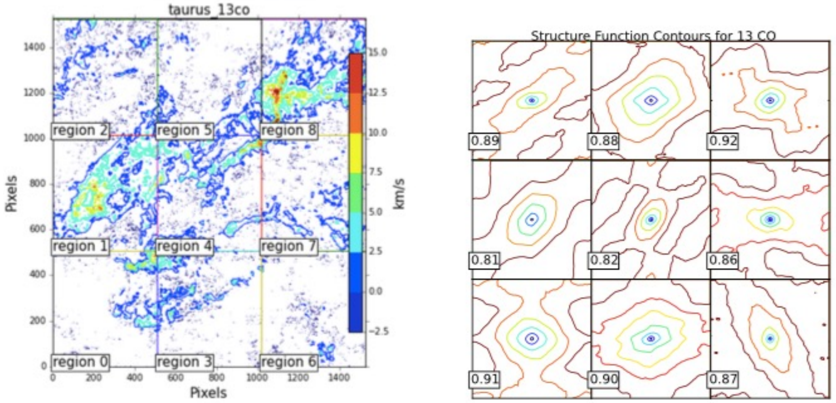
Magnetized ISM

Anisotropies in the HI gas distribution



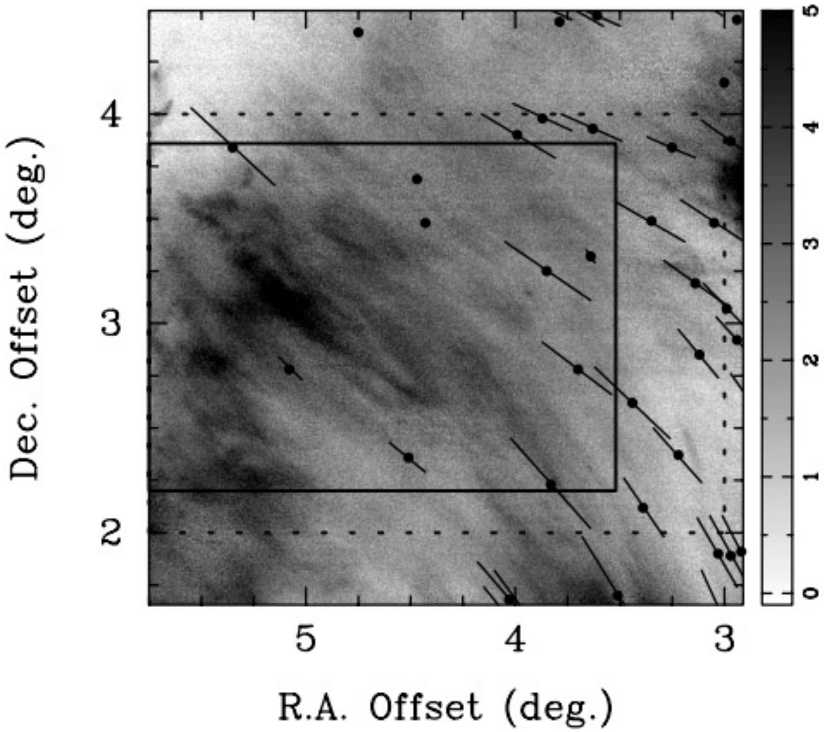
Kalberla & Kerp 2016

Velocity **anisotropy** in Taurus Cloud



Missy McIntosh, Thesis

Velocity **anisotropy** in Taurus Cloud



Heyer et al. 2008

Summary: Goldreich-Sridhar model (1995)

- Critical balance

$$\frac{l_\perp}{V_{L\perp}} = \frac{l_\parallel}{V_A}$$

- Constancy of energy cascade rate

$$\frac{v_\perp^2}{t_{\mathrm{cas}}} = \mathrm{const}$$

$$\frac{v_{\perp l}^2}{(l_\perp/v_{\perp l})} = \mathrm{const}$$

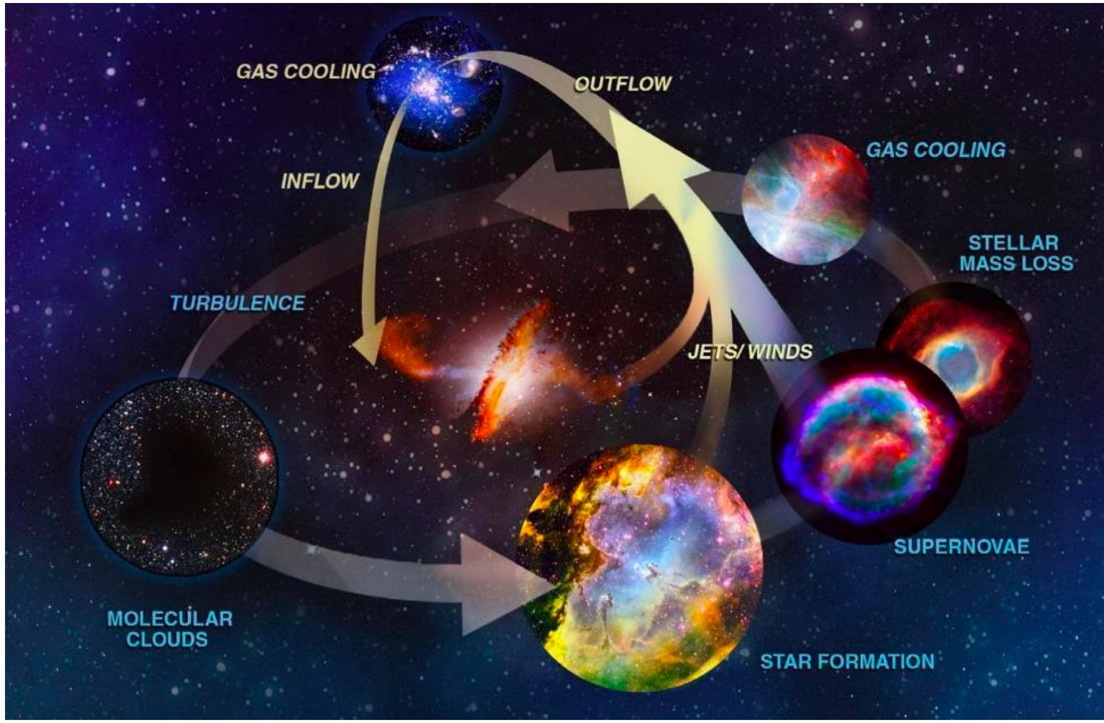
$v_\perp \sim l_\perp^{1/3}$

Or, $E(k) \sim k^{-5/3}$

$l_{||} \sim l_\perp^{2/3}$

MHD Turbulence...

- What is turbulence?
- Hydro: Kolmogorov 41
- MHD (1995): GS95 and Critical Balance
- MHD (2006): Dynamic Alignment
- Intermittency
- Reconnection
- Diagnostics
- Compressibility and star formation
- Star formation self-regulation via turbulence and feedback



- Critical balance is a robust feature of MHD. But do we have the right nonlinear time scale? Can be reduced by “dynamic alignment” Boldyrev2006

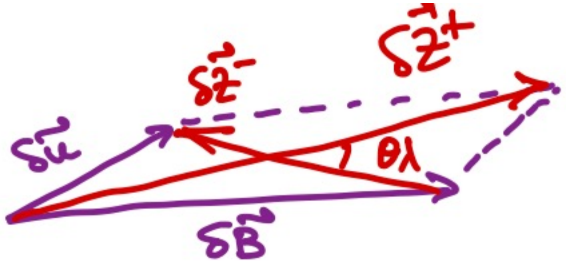
Ideal MHD dynamics tend to produce aligned fields:

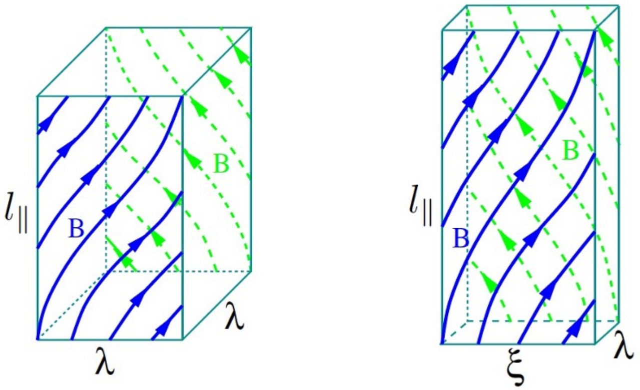
$$\frac{\partial \vec{Z}^{\pm}_{\perp}}{\partial t} \mp v_A \nabla_{\parallel} \vec{Z}^{\pm}_{\perp} \boxed{+ \vec{Z}^{\mp}_{\perp} \cdot \nabla_{\perp} \vec{Z}^{\pm}_{\perp}} = -\nabla_{\perp} p + \text{dissipation} + \text{forcing}$$

Elsasser fields $\vec{Z}^{\pm}_{\perp} = \vec{u}_{\perp} \pm \delta\vec{B}_{\perp}$

One E-field shears the other into alignment

Schekochihin 2022





Reduces the ‘turn over time’

$$\tau_{nl} \sim \frac{\lambda}{v_\lambda sin(\theta_\lambda)}$$

Cartoon of a GS95 eddy (left) vs. a Boldyrev (2006) aligned eddy (right).

- Critical balance is a robust feature of MHD. But do we have the right nonlinear time scale? Can be reduced by “dynamic alignment” Boldyrev2006

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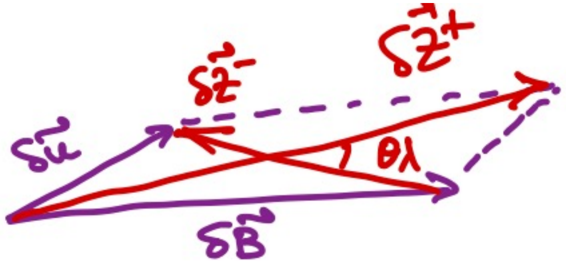
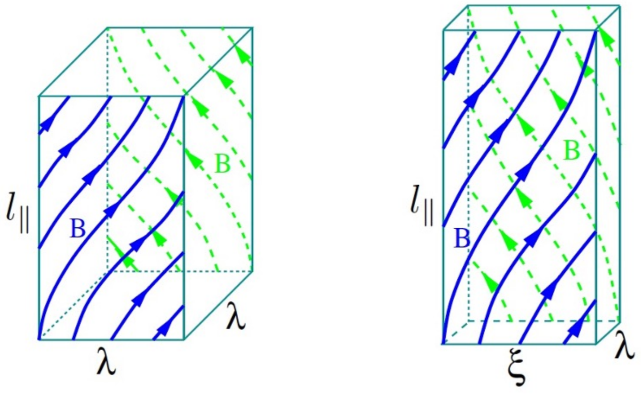
Elsasser fields
$\vec{Z}_\perp^\pm = \vec{u}_\perp \pm \delta\vec{B}_\perp$

one E-field shears the other into alignment

Schekochihin 2022





$$\tau_{nl} \sim \frac{\lambda}{v_\lambda sin(\theta_\lambda)}$$

But alignment cant be larger than small angle set by parallel cascade

$$sin(\theta) \sim \delta b/B \sim \delta v/V_A << 1$$

Cartoon of a GS95 eddy (left) vs. a Boldyrev (2006) aligned eddy (right).

- Critical balance is a robust feature of MHD. But do we have the right nonlinear time scale? Can be reduced by "dynamic alignment" Boldyrev2006

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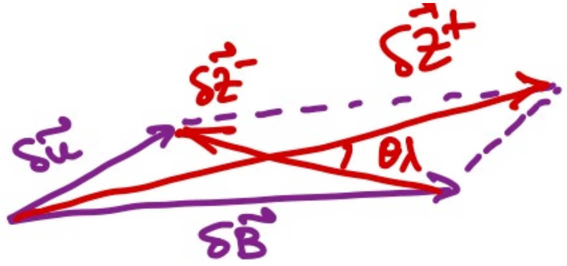
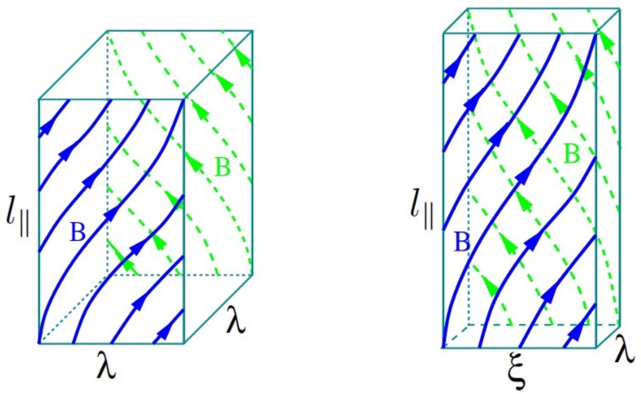
Elsasser fields
$\vec{Z}^{\pm}_{\perp} = \vec{u}_{\perp} \pm \delta\vec{B}_{\perp}$

One E field shears the other into alignment

Schekochihin 2022





Cartoon of a GS95 eddy (left) vs. a Boldyrev (2006) aligned eddy (right).

$$\tau_{nl} \sim \frac{\lambda}{v_\lambda sin(\theta_\lambda)}$$

$$sin(\theta) \sim \delta b/B \sim \delta v/V_A << 1$$

$$\theta_\lambda \sim \lambda^{1/4}$$

$$E(k_\perp) \propto k_\perp^{-3/2}.$$

(interesting we get back to IK Spectrum but for very different physical reasons)

- Critical balance is a robust feature of MHD. But do we have the right nonlinear time scale? Can be reduced by “dynamic alignment” Boldyrev2006

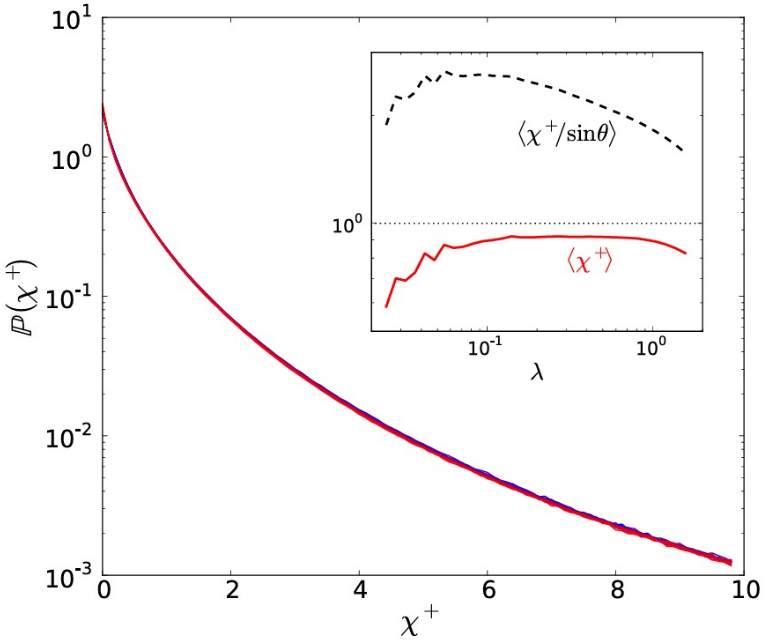
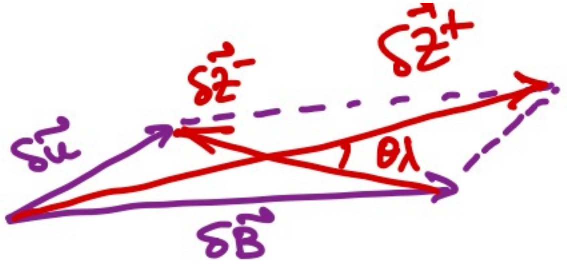


FIGURE 6. Refined critical balance: this figure, taken fr
probability density function (PDF) of the ratio $\chi^+ = \tau$
fact, 17 PDFs are plotted here, taken at different scales w
inertial range (this was a 1024^3 RMHD simulation)—the co
from blue (smaller scales) to red (larger scales), but this is
collapse on top of each other. The inset shows that the s
defined without the alignment angle (see § 6). [Reprinted fr
of the Royal Astronomical Society.]

Schekochihin 2022



$$\tau_{nl} \sim \frac{\lambda}{v_\lambda sin(\theta_\lambda)}$$

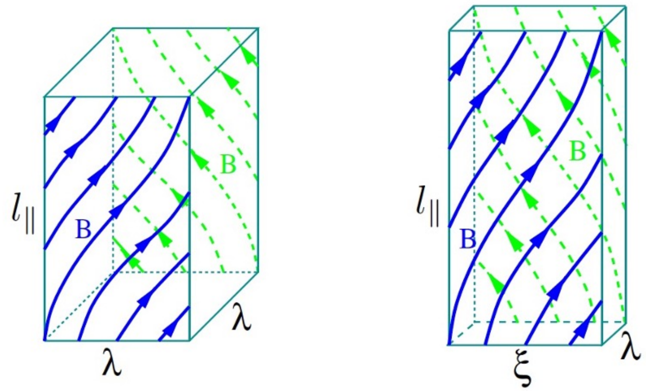
$$\chi^\pm \doteq \frac{\tau_A^\pm}{\tau_{nl}^\pm} = \frac{l_\parallel^\pm \delta z_\perp^\mp \sin\theta}{v_A \lambda},$$

Summary: Dynamic Alignment

As interactions occur, v and b will advect and shear each other, causing a 'dynamic alignment'. This introduces a degree of anisotropy in the 2D plane perpendicular to the magnetic field between b and v. Importantly this reduces the non linear time scale!

$$\sin\theta_\lambda \sim \theta_\lambda \sim \frac{\delta b_\lambda}{v_\mathrm{A}} \ll 1.$$

$$\theta_\lambda \sim \lambda^{1/4}$$



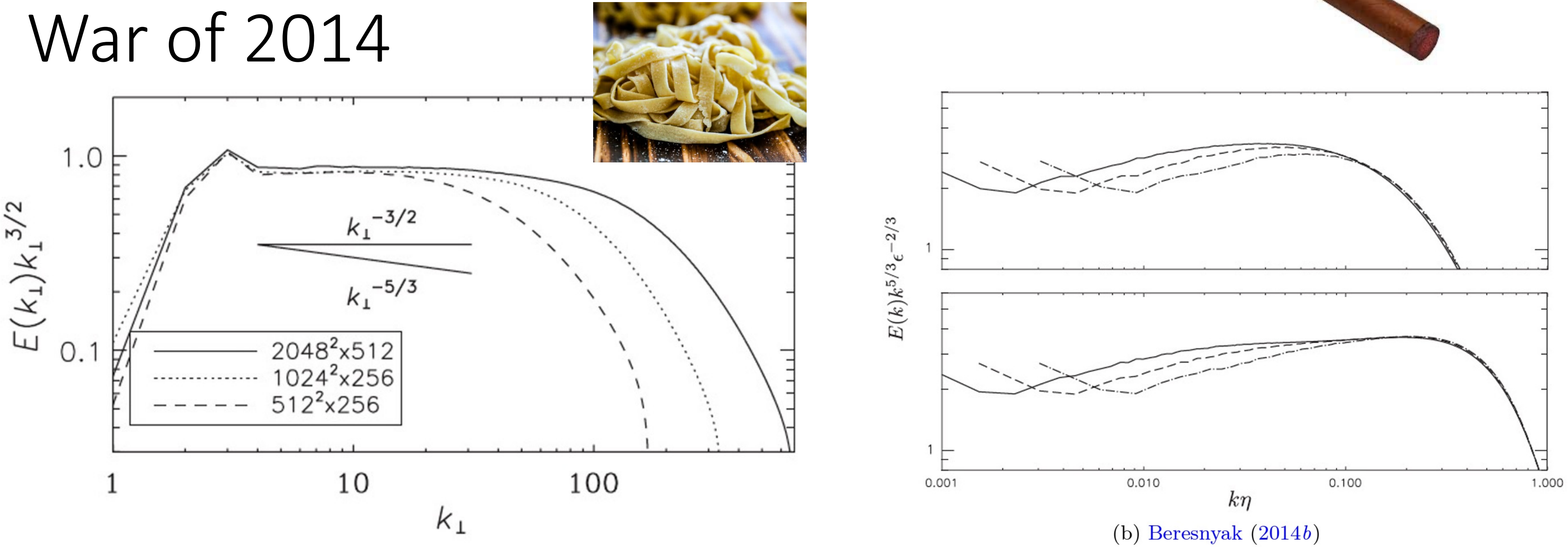
Cartoon of a GS95 eddy (left) vs. a Boldyrev (2006) aligned eddy (right).

The dynamic alignment in driven turbulence thus becomes scale-dependent. This leads to the field-perpendicular energy spectrum:

$$E(k_\perp) \propto k_\perp^{-3/2}.$$

More recent work shows scale dependency of alignment is likely controlled by intermittency. Strong fluctuations are more aligned…alignment is needed for scale invariant critical balance

The Great Spectral War of 2014



(a) Perez *et al.* (2012)

(b) Beresnyak (2014*b*)

FIGURE 10. The best-resolved currently available spectra of RMHD turbulence. (a) From simulations by Perez *et al.* (2012) (their figure 1), with Laplacian viscosity and resolution up to $2048^2 \times 512$. (b) From simulations by Beresnyak (2014*b*) (his figure 1, ©AAS, reproduced with permission), with Laplacian viscosity (top panel) and with 4th-order hyperviscosity (bottom panel); the resolution for the three spectra is 1024^3, 2048^3 and 4096^3. His spectra are rescaled

Solar Wind: Parker Solar Probe

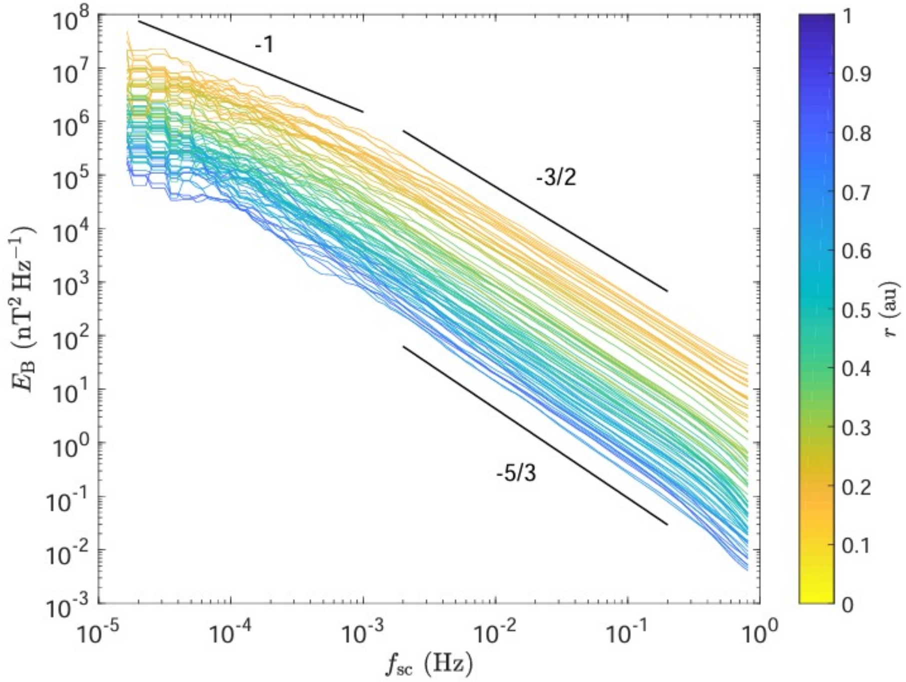


Figure 1. Magnetic field power spectrum, E_{B}, at different heliocentric distances, r, over the first two *PSP* orbits. Several power law slopes are marked for comparison. A turbulent inertial range is present at all distances, with a flattening at low frequencies. Deviations at high frequencies ($f_{\mathrm{sc}} \gtrsim 0.3\,\mathrm{Hz}$) are partly due to digital filter effects.

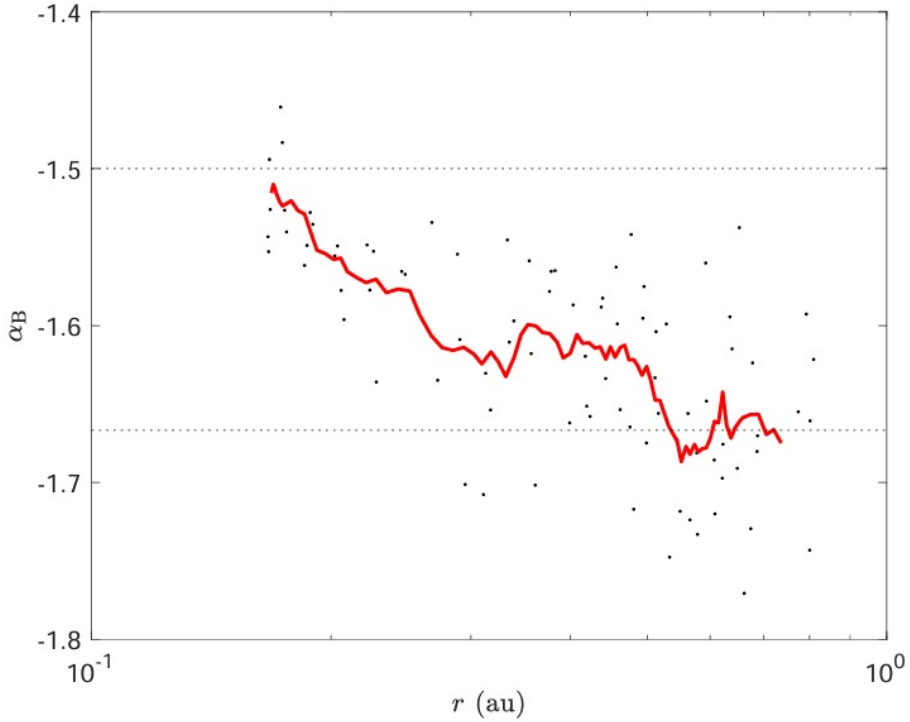
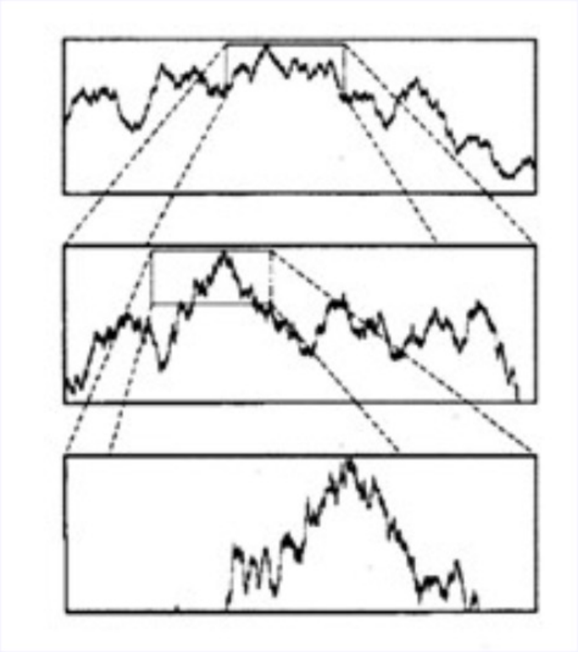


Figure 2. Variation of magnetic field spectral index, α_{B}, with heliocentric distance, r, in the MHD inertial range (10^{-2} Hz $< f_{\mathrm{sc}} <$ 10^{-1} Hz). The black dots show the spectral index measurements and the red line is a 10-point running mean. The horizontal dotted lines mark the theoretical predictions $-3/2$ and $-5/3$.

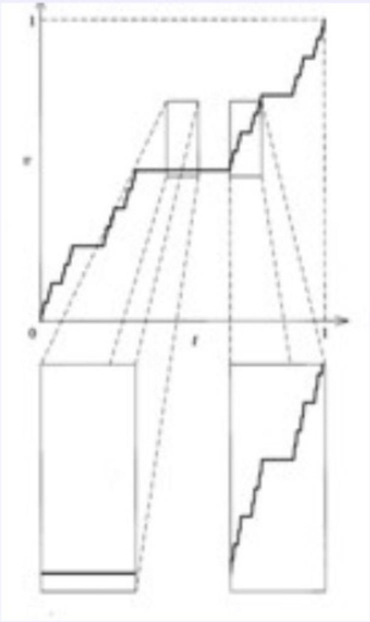
Chen et al. 2020

Self-similarity/Intermittency

- Self-similar fluctuations : if we magnify an arbitrary part, the statistical properties will be identical
- Intermittent fluctuations : alternance of intervals with high activity with quiet intervals



Brownian motion is self-similar



The Devil's staircase is intermittent

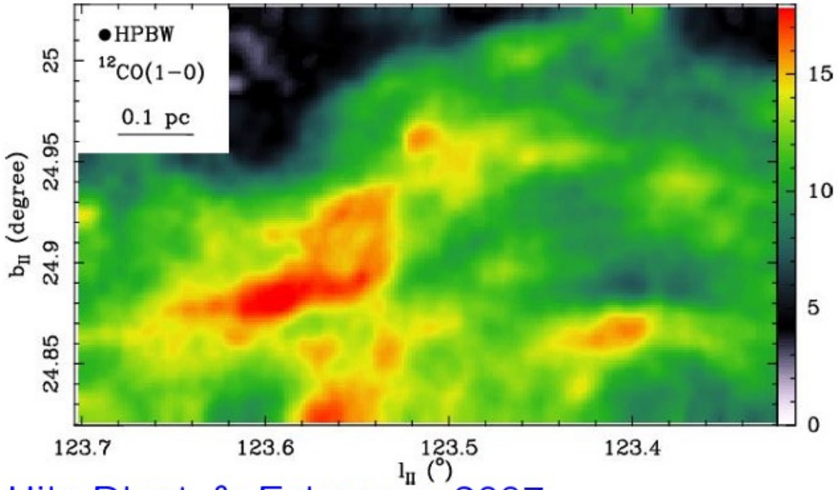
Intermittency is associated with increasing departure of PDFs from gaussianity when the scale δ decreases.

The tool: statistics of increments of line centroid velocities

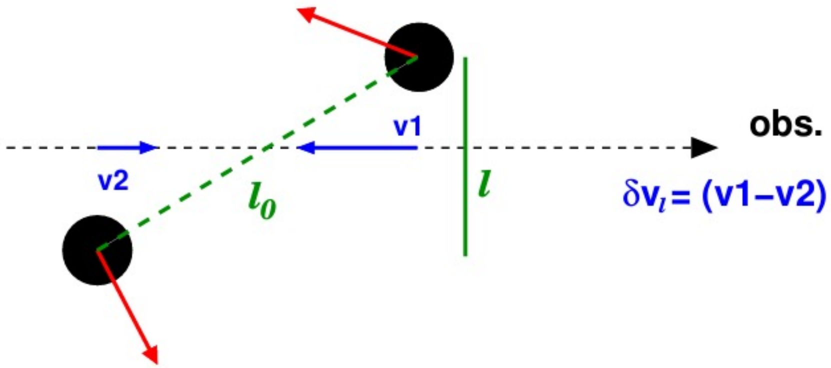
IRAM-30m,

8000 spectra (now 35000, resol 11")

Fully sampled, resolution 20"



Hily-Blant & Falgarone 2007



Line centroid velocity:

$$C(\mathbf{r}) = \int T(\mathbf{r}, v_x) v_x dv_x / \int T(\mathbf{r}, v_x) dv_x$$

Miesch & Scalo 1999, Pety & Falgarone 2003, Brunt et al. 2003, ...

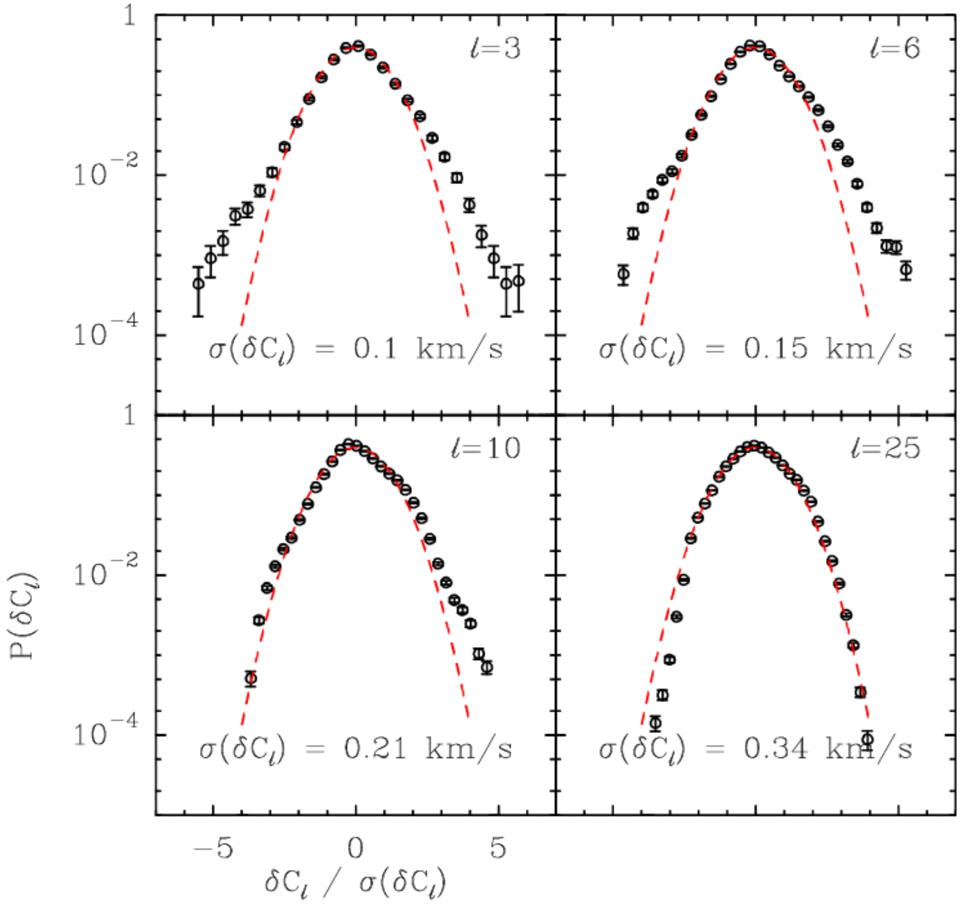
Extrema of line centroid increments trace extrema of $(\langle\omega_y\rangle^2 + \langle\omega_z\rangle^2)^{1/2}$

Lis et al. 1996

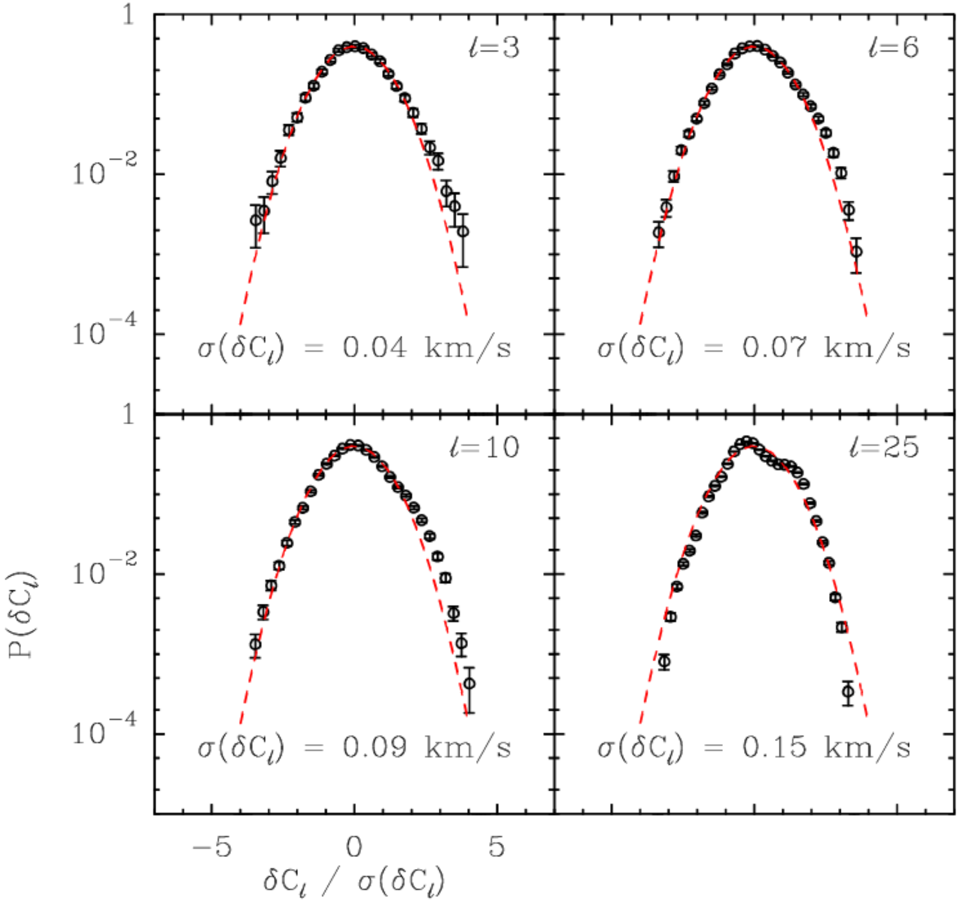
PDFs of Centroid Velocity Increments with variable lags

$M \sim 5$ in Polaris,
$M \sim 2$ in Taurus

Polaris



Taurus



Hily-Blant, Falgarone, Pety 2008

Summary for incompressible MHD

- Spectrum Goldreich-Sridhar (1995), $E(k) \sim k^{-5/3}$
- Spectrum Boldyrev (2006), $E(k) \sim k^{-3/2}$

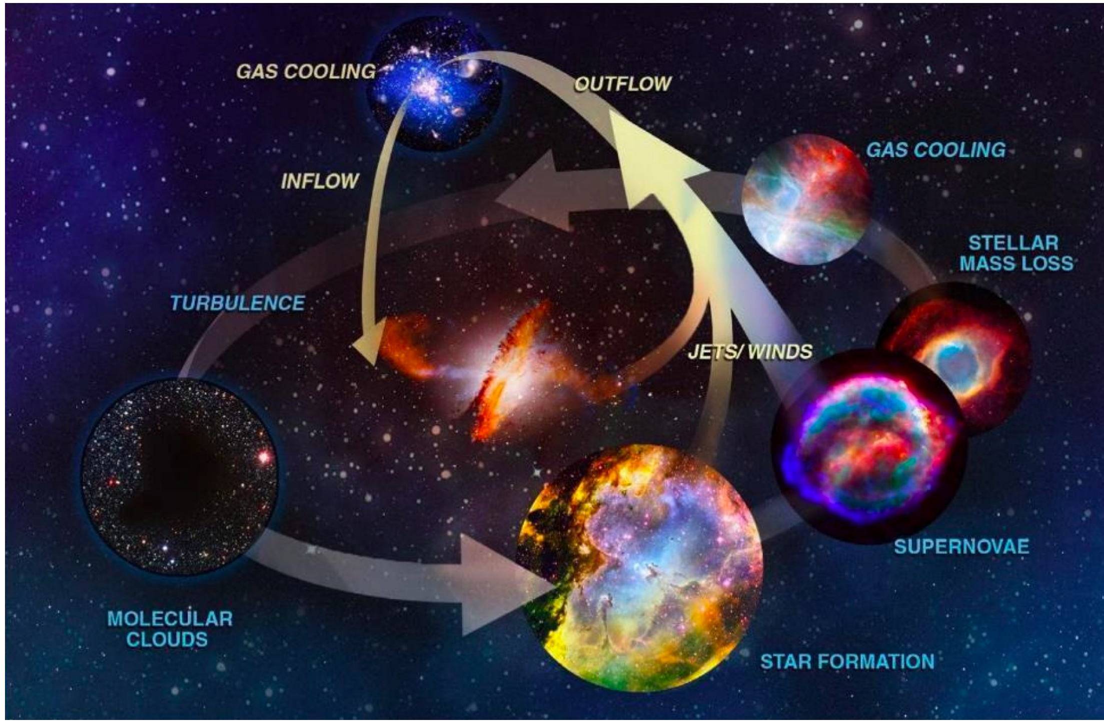
Numerical/theory debate on which is correct. My opinion: for astrophysics/star formation the difference likely isn't critical.

- Anisotropy: $l_{||} \sim l_{\perp}^{2/3}$

Critical Balance is on solid ground regardless. Anisotropy is agreed on and important for astrophysics!

Intermittency of turbulence is important and observable. Increases with Mach number/strength of turbulence

MHD Turbulence..



- What is turbulence?
- Hydro: Kolmogorov 41
- MHD (1995): GS95 and Critical Balance
- MHD (2006): Dynamic Alignment
- Intermittency
- Compressibility
- Star formation self-regulation via turbulence and feedback

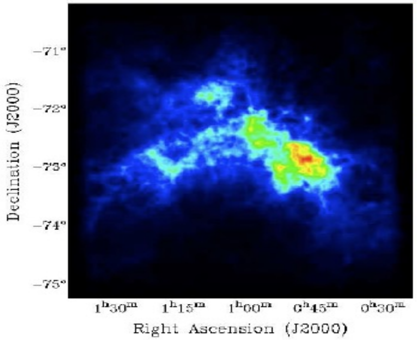
Cold Gas is supersonic: Sonic Mach Number in CNM

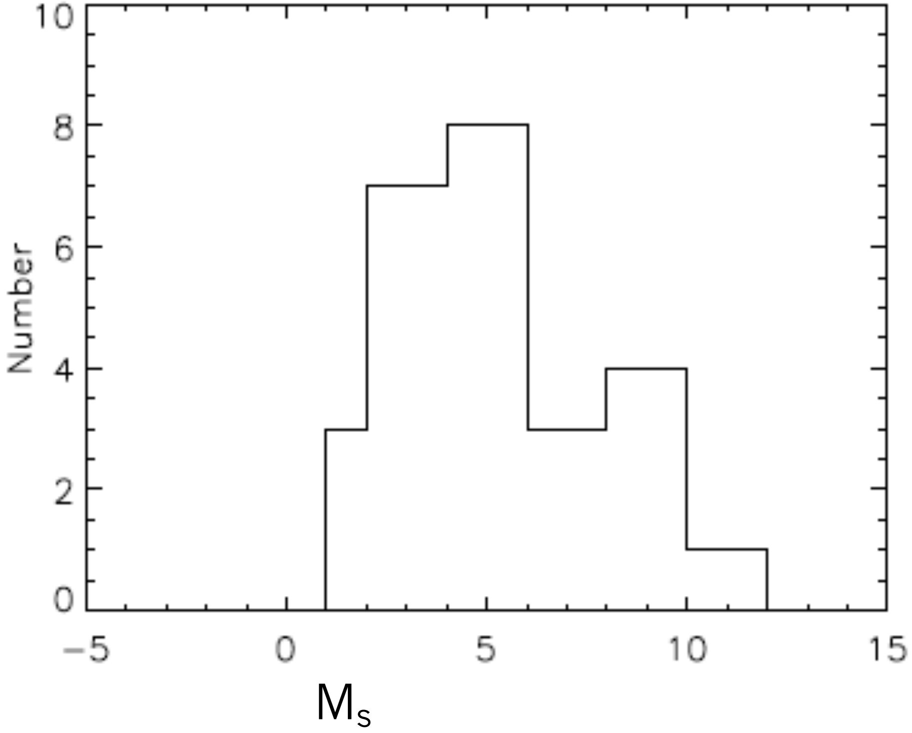
Observational Method for Cold Neutral 21cm Mach Numbers (need spin temperature).

$$\mathcal{M}_s{}^2 = \frac{V_{t,3\mathrm{D}}{}^2}{C_s{}^2} = 3.7\left(\frac{T_{k,\max}}{T_s} - 1\right) \qquad \frac{N_1}{N_0} \equiv \frac{g_1}{g_0}\exp\left(-\frac{h\nu_{10}}{kT_{\mathrm{s}}}\right)$$

Small Magellanic Cloud CNM Mach number

(spin temperatures from Dickey et al. 2001)



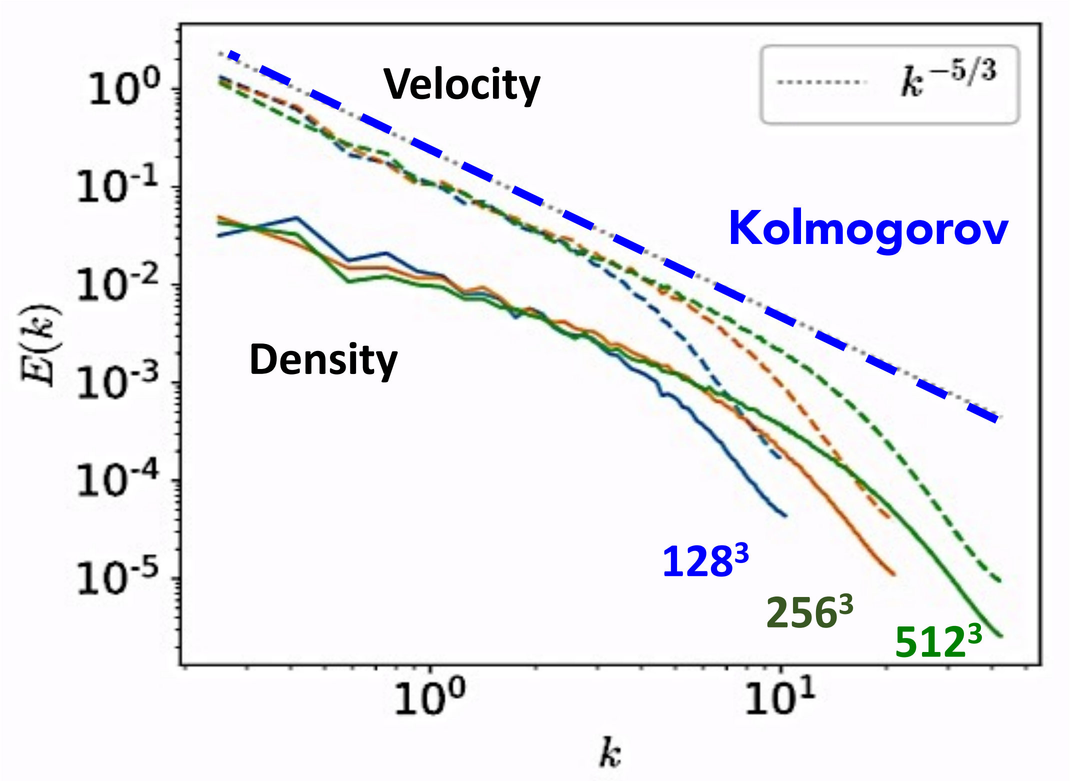


CNM PDF prediction:
M_s=3-20

Burkhart et al. 2010

Turbulent velocities induce density fluctuations

Density fluctuations passively follow the same cascade as turbulent velocities.



SX, SJ, & AL, submitted

MHD Eqs. and fluid simulations

-Solve the ideal MHD equations in a periodic box. Set equation of state/add energy equation.

-Include gravity, chemistry, heating/cooling, feedback etc.

-Include galactic initial conditions.

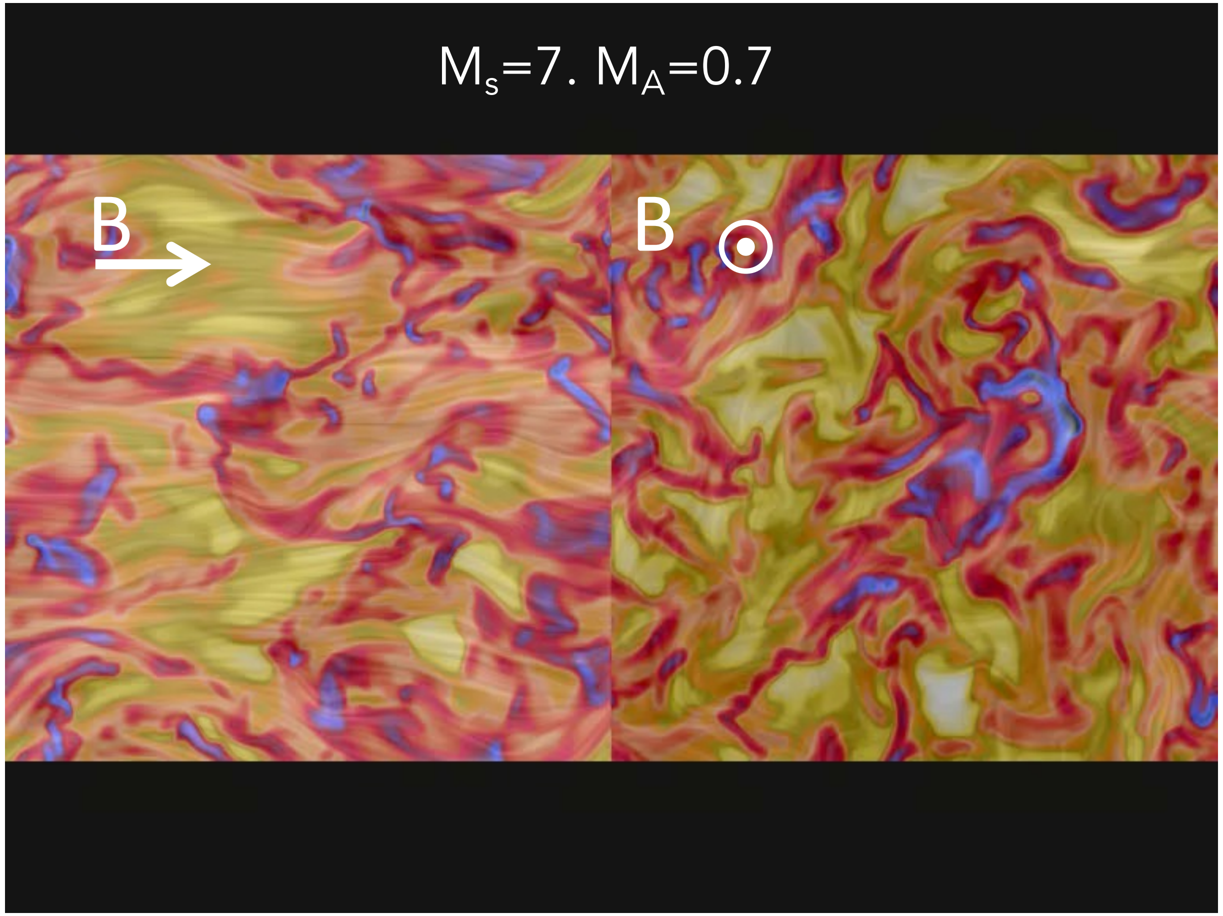
$$\frac{\partial \rho}{\partial t} + \nabla(\rho \mathbf{V}) = 0, \qquad \text{Mass Continuity Eq.,}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{P}{\rho^{\gamma}}\right) = 0, \qquad \text{Energy Eq.,}$$

$$\rho \frac{\mathrm{d}\mathbf{V}}{\mathrm{d}t} = -\nabla P - \frac{1}{\mu_0}\mathbf{B} \times (\nabla \times \mathbf{B}), \qquad \text{Euler's Eq.,}$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B}), \qquad \text{Induction Eq..}$$

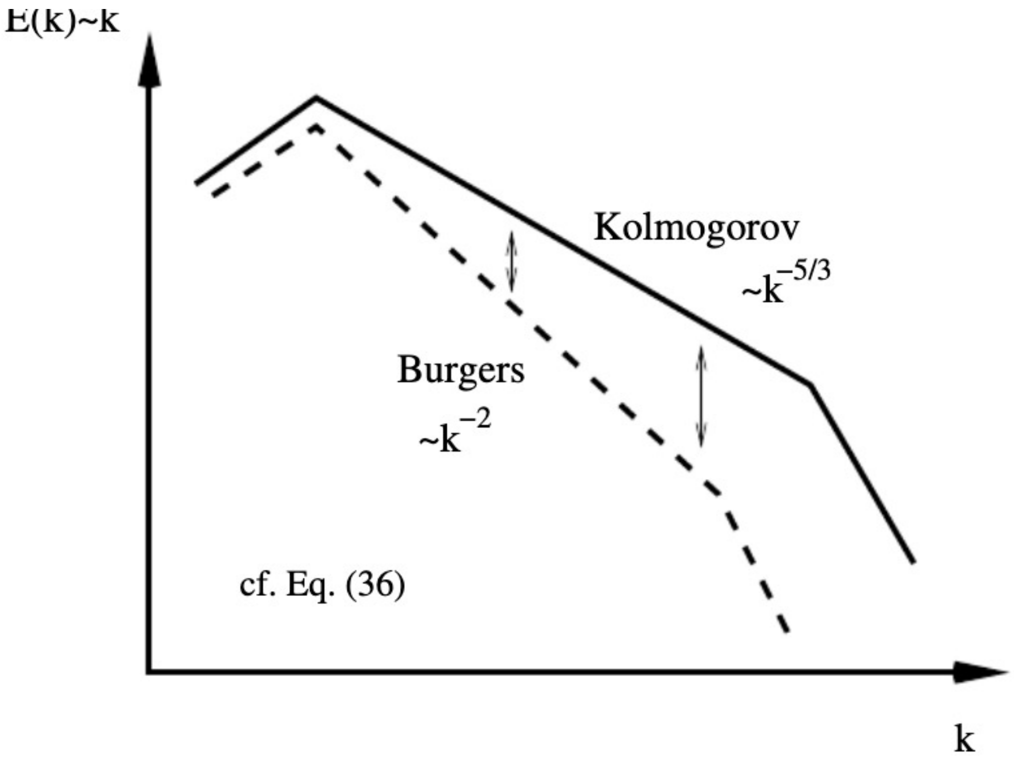
$$\nabla \cdot \mathbf{B} = 0.$$

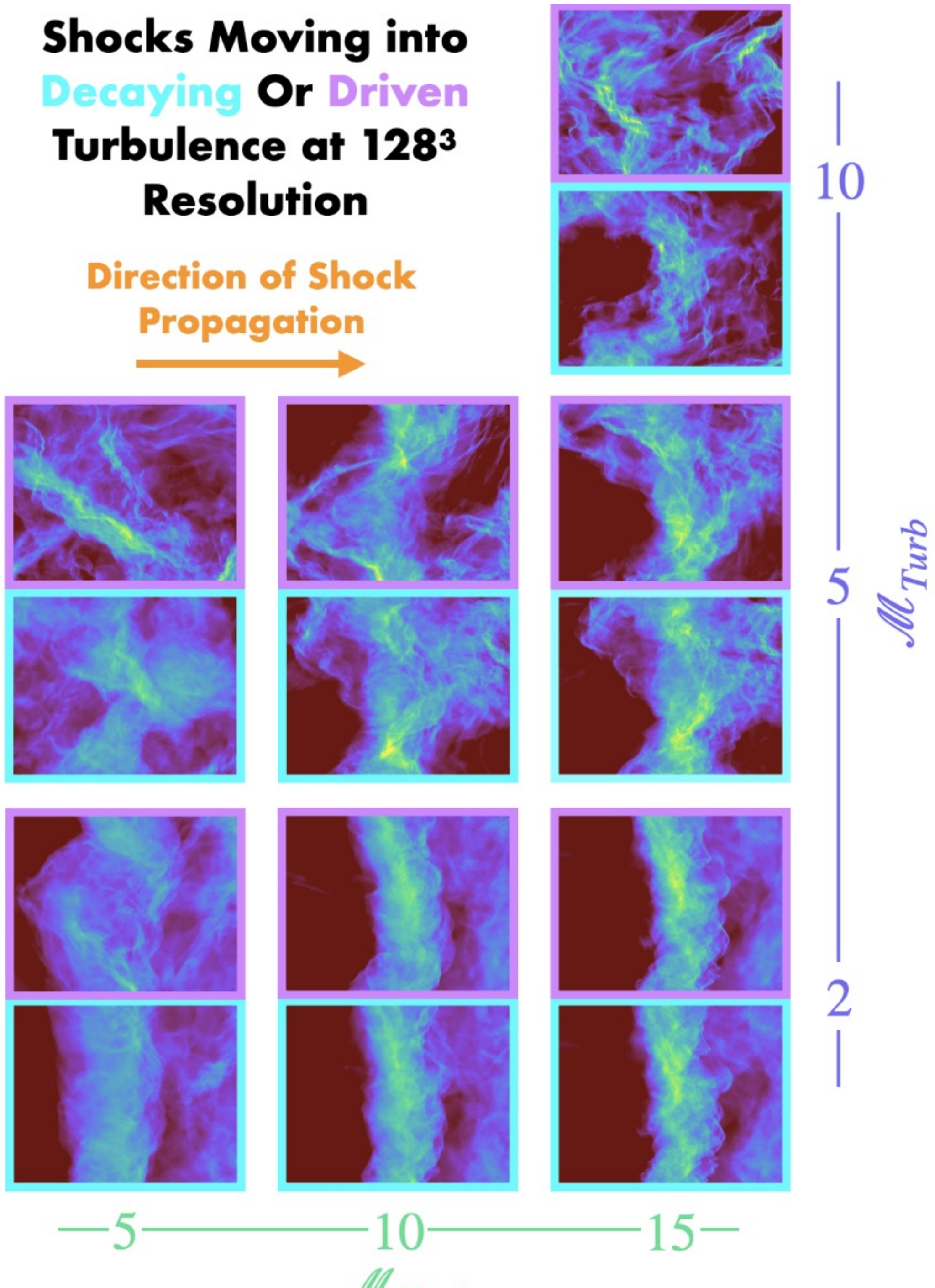


A large and exciting effort by many groups!

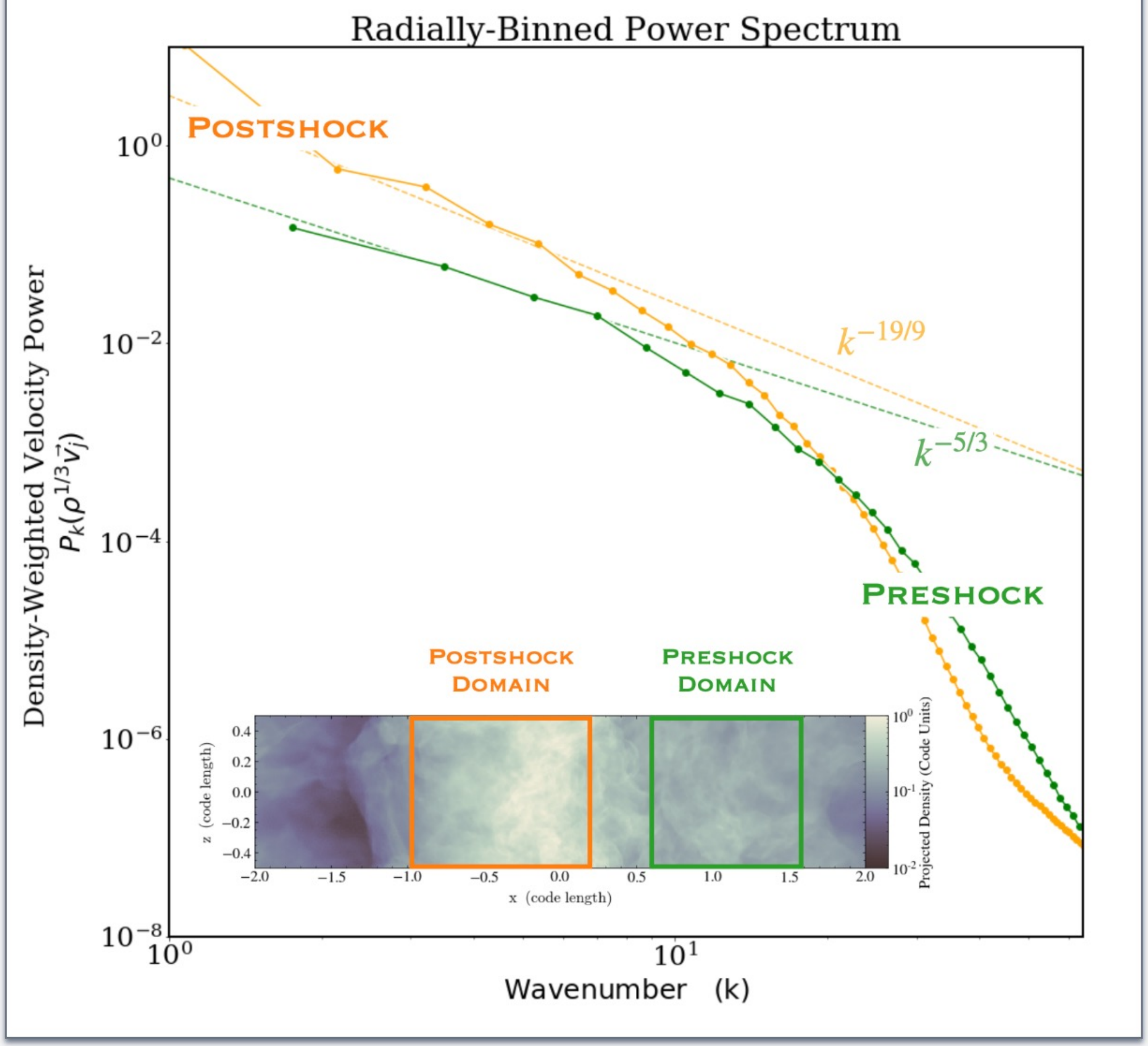
Vazquez-Semadeni, , Padoan, Passon, Stone, Mac Low, Klessen, Ostriker, Heitsch, Cho, Boldyrev, Li, Haugen, Jappsen, Ballestros, Mee, Brandenburg, Kritsuk, Dib, Offner, Kowal, Schmidt, Lemaster, Glover, Federrath, Price, DelSordo, Collins, Hopkins, Walch, Chevance, Semenov, Kruijssen, Robertson…++

Burgers Turbulence



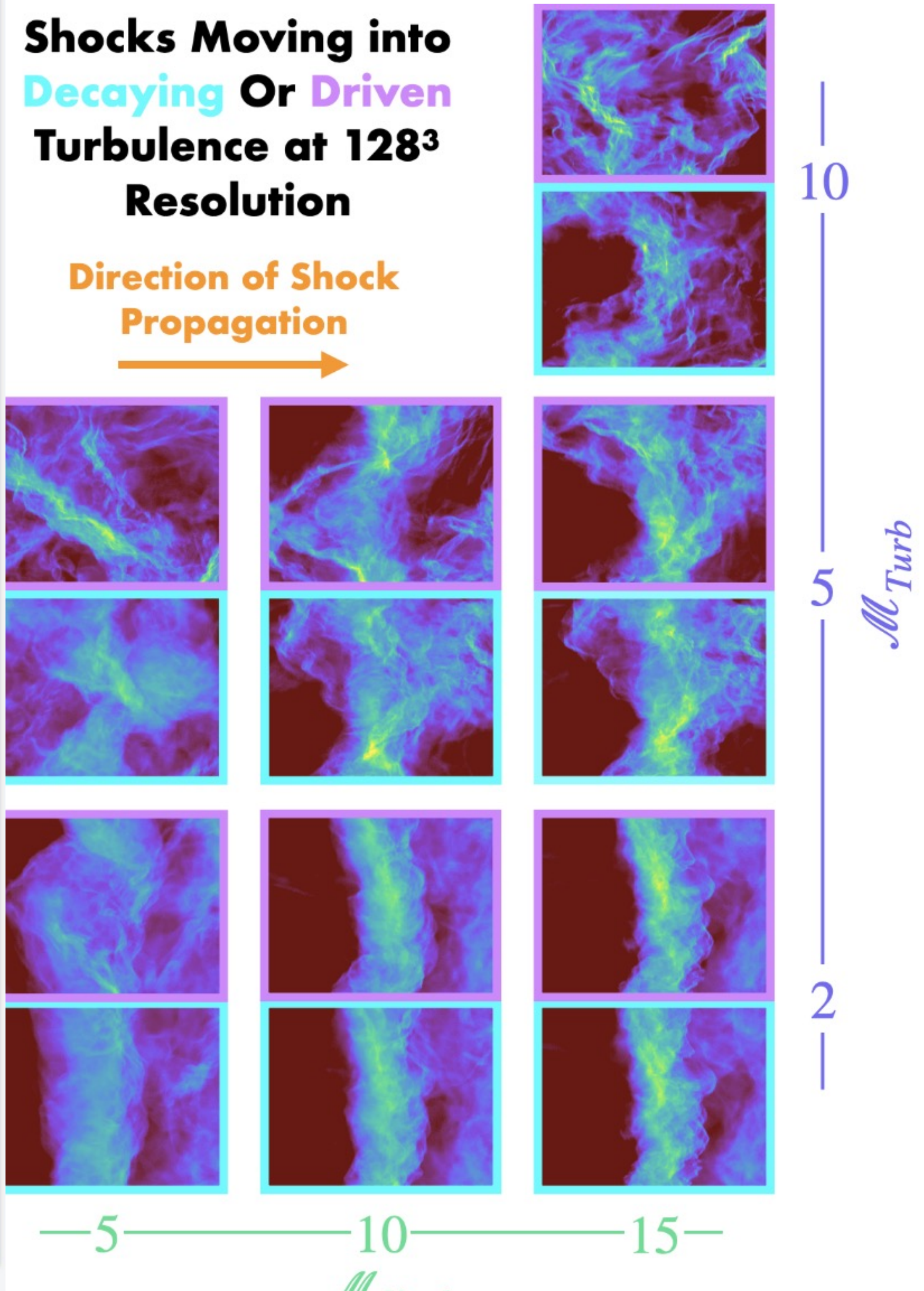


Foley et al. 2024



Shocks Moving into Decaying Or Driven Turbulence at 128^3 Resolution

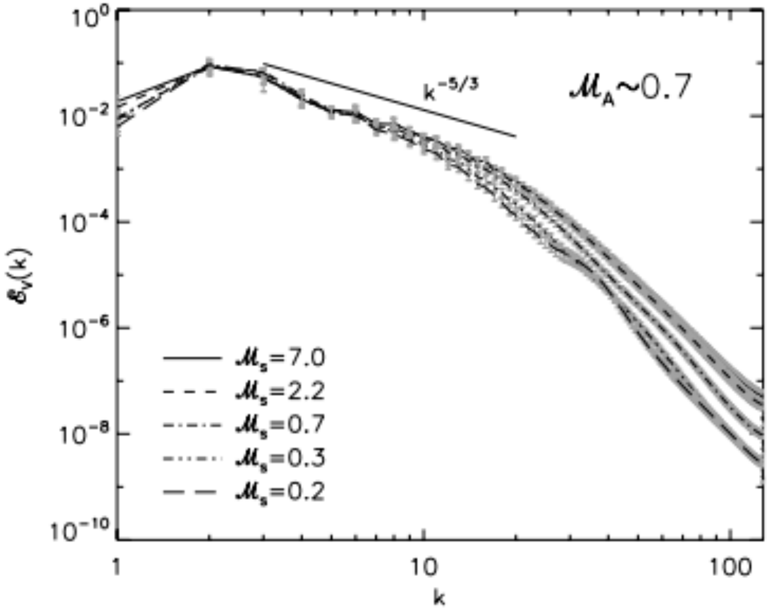
Direction of Shock Propagation →



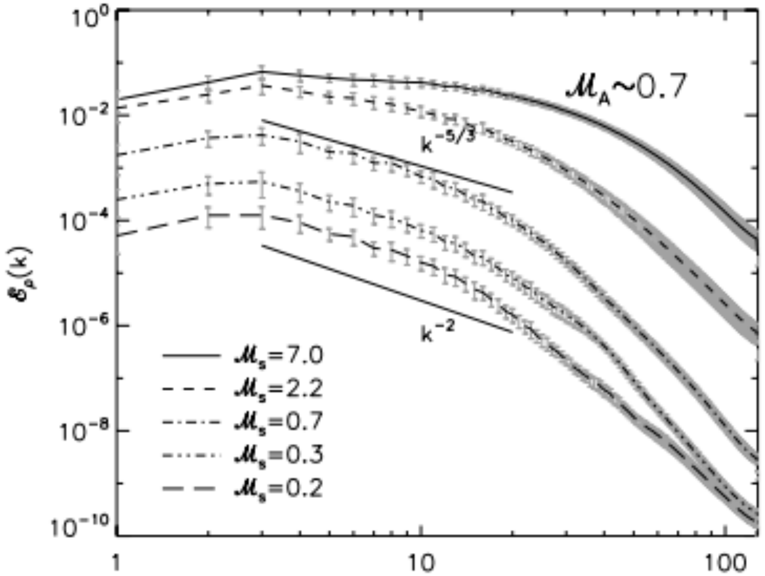
Supersonic Power Spectra

$$M_{\rm s} \equiv \frac{V_{\rm L}}{V_{\rm s}}$$

Velocity Power Spectrum



Density Power Spectrum

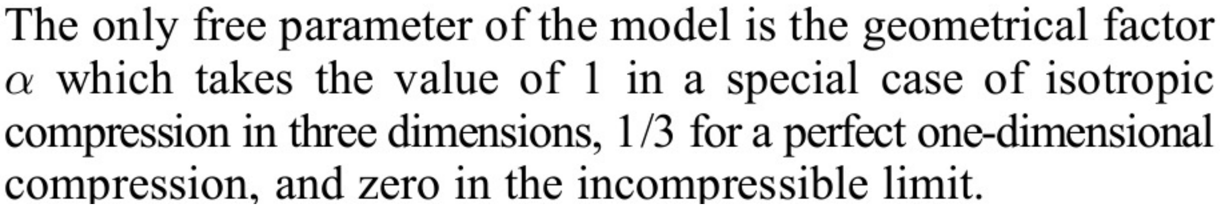


Kowal & Lazarian 2010
Burkhart et al. 2010

Fleck 1996 model of compressible turbulence: velocity steepens and density shallows relative to the incompressible kolmogorov slope

Fleck 1996

Fleck (1996) derived a set of scaling relations for the velocity, specific kinetic energy, density, and mass of a compressible flow assuming also mass conservation and that density is hierarchical, (i.e., vonWeizsäcker (1951)



The only free parameter of the model is the geometrical factor α which takes the value of 1 in a special case of isotropic compression in three dimensions, 1/3 for a perfect one-dimensional compression, and zero in the incompressible limit.

$$\frac{\rho_\nu}{\rho_{\nu-1}} = \left(\frac{l_\nu}{l_{\nu-1}}\right)^{-3\alpha}.$$

$$u \sim l^{1/3+\alpha},$$

$$\mathcal{E}(k) \sim k^{-\beta} \sim k^{-5/3-2\alpha},$$

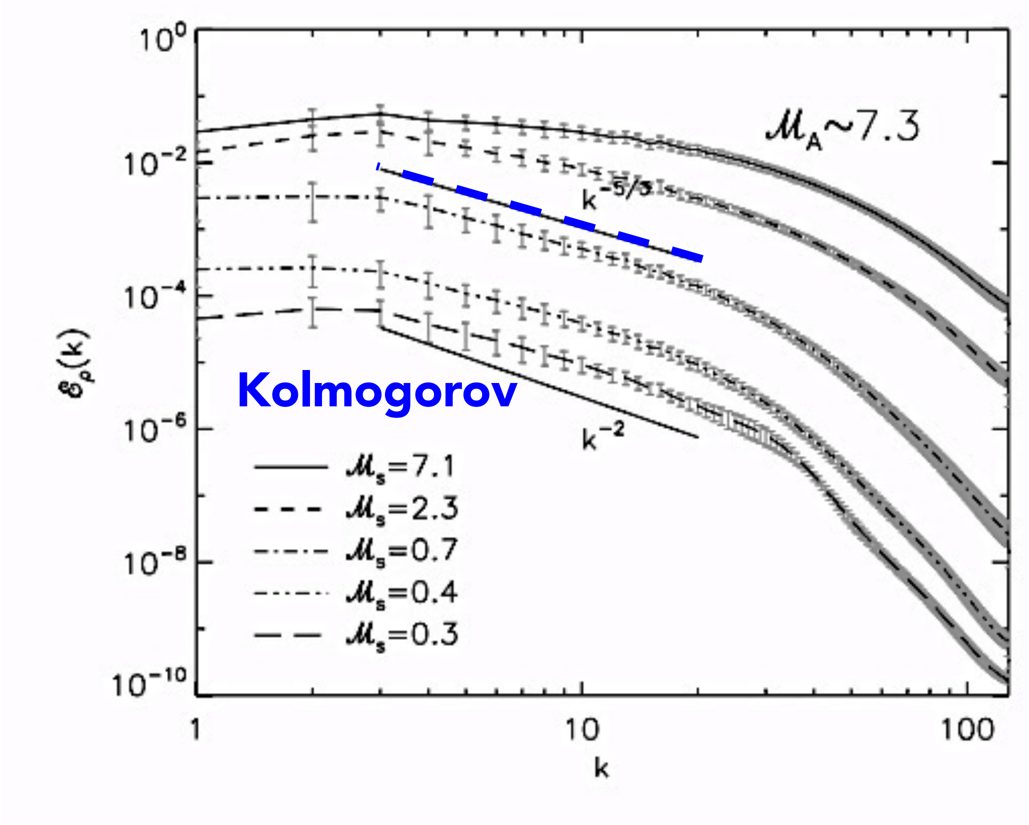
$$\rho \sim l^{-3\alpha},$$

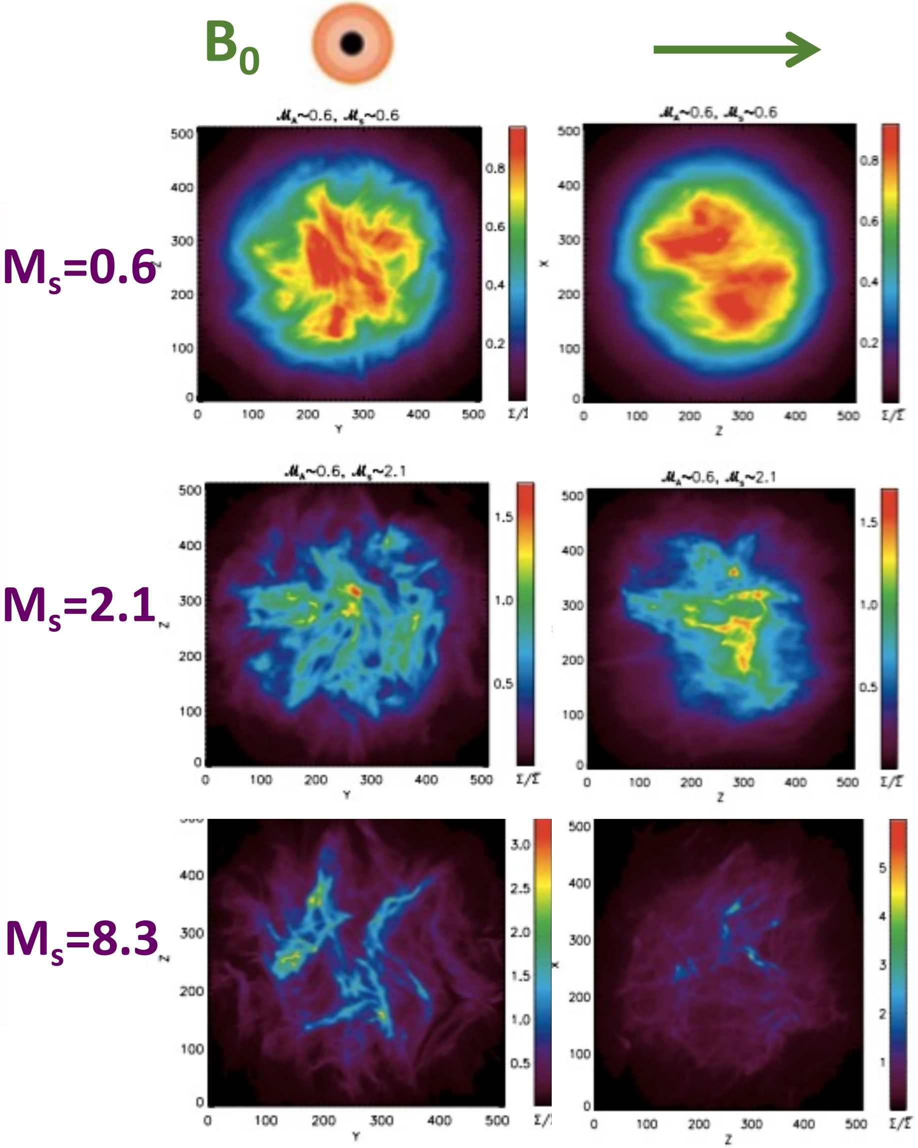
$$M(l) \sim l^{D_m} \sim l^{3-3\alpha},$$

$\underline{\alpha}$ ~0.2-.3 Kritsuk et al. 2007

Shallow density spectrum in supersonic turbulence

Kowal et al. 2007



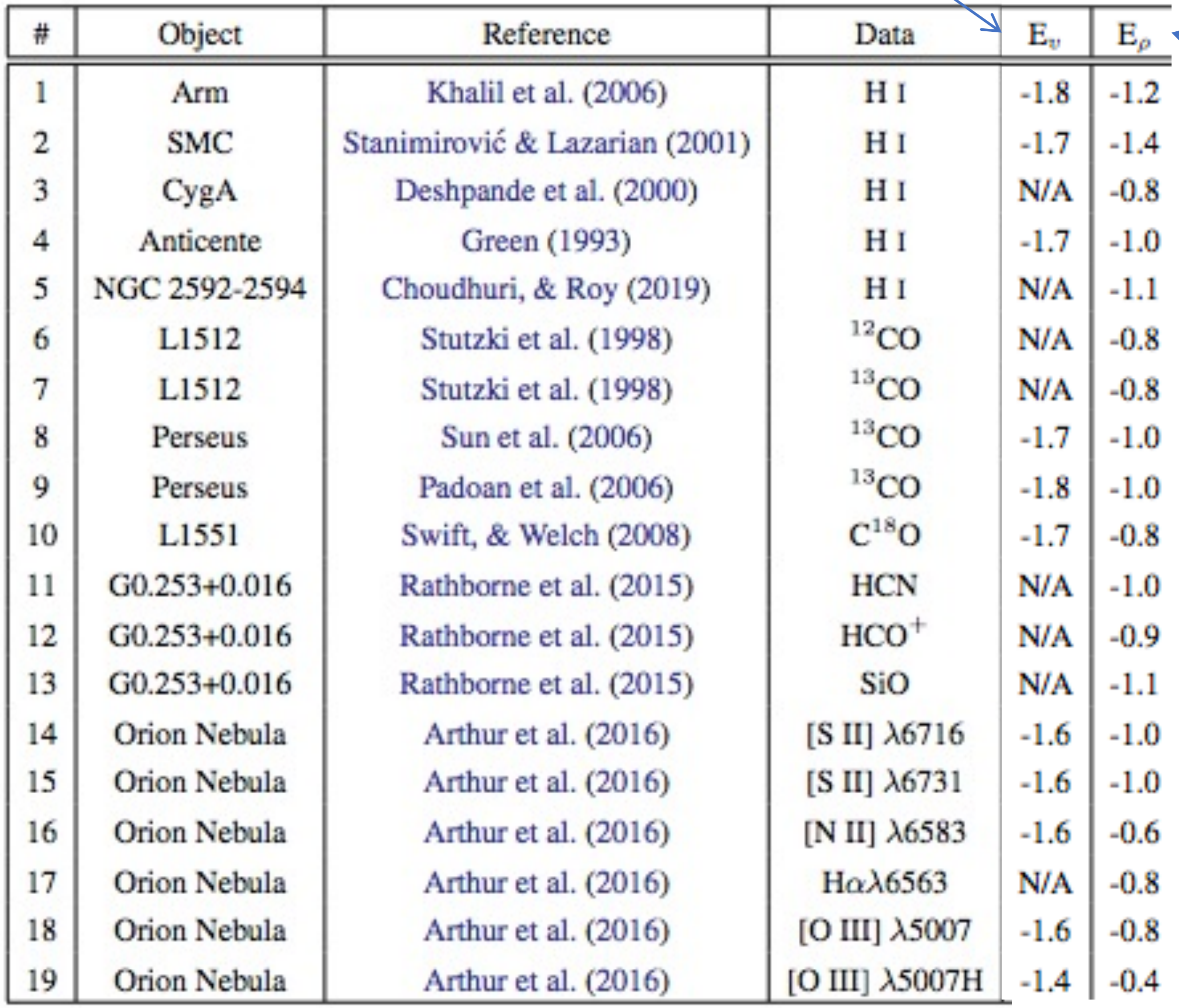


Velocity/density power spectrum reveals the galaxy is supersonically turbulent

For Supersonic Turbulence: density spectrum become shallower and velocity spectrum becomes steeper (relative to Kolmogorov spectrum)

Velocity power spectral index

Density power spectral index

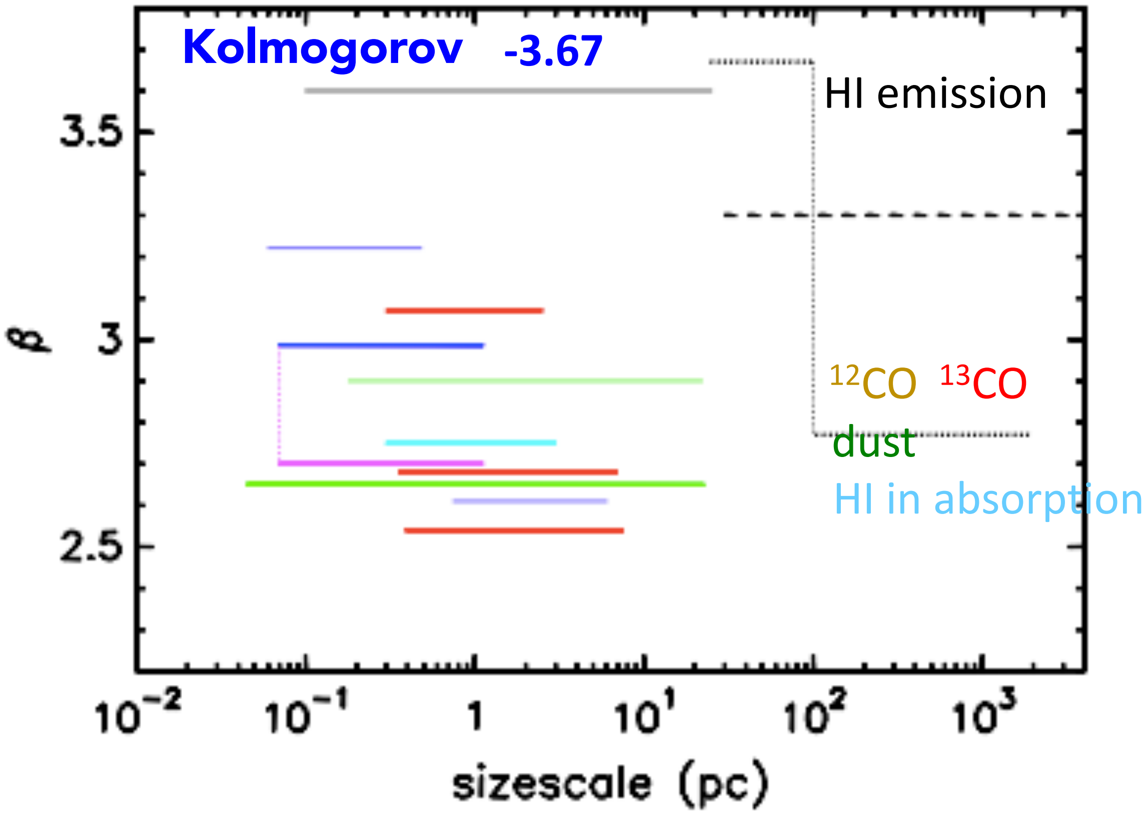


#	Object	Reference	Data	E_v	E_ρ
1	Arm	Khalil et al. (2006)	H I	-1.8	-1.2
2	SMC	Stanimirović & Lazarian (2001)	H I	-1.7	-1.4
3	CygA	Deshpande et al. (2000)	H I	N/A	-0.8
4	Anticente	Green (1993)	H I	-1.7	-1.0
5	NGC 2592-2594	Choudhuri, & Roy (2019)	H I	N/A	-1.1
6	L1512	Stutzki et al. (1998)	^{12}CO	N/A	-0.8
7	L1512	Stutzki et al. (1998)	^{13}CO	N/A	-0.8
8	Perseus	Sun et al. (2006)	^{13}CO	-1.7	-1.0
9	Perseus	Padoan et al. (2006)	^{13}CO	-1.8	-1.0
10	L1551	Swift, & Welch (2008)	C^{18}O	-1.7	-0.8
11	G0.253+0.016	Rathborne et al. (2015)	HCN	N/A	-1.0
12	G0.253+0.016	Rathborne et al. (2015)	HCO$^+$	N/A	-0.9
13	G0.253+0.016	Rathborne et al. (2015)	SiO	N/A	-1.1
14	Orion Nebula	Arthur et al. (2016)	[S II] λ6716	-1.6	-1.0
15	Orion Nebula	Arthur et al. (2016)	[S II] λ6731	-1.6	-1.0
16	Orion Nebula	Arthur et al. (2016)	[N II] λ6583	-1.6	-0.6
17	Orion Nebula	Arthur et al. (2016)	H$\alpha\lambda$6563	N/A	-0.8
18	Orion Nebula	Arthur et al. (2016)	[O III] λ5007	-1.6	-0.8
19	Orion Nebula	Arthur et al. (2016)	[O III] λ5007H	-1.4	-0.4

Burkhart et al. 2013

Various density spectra in the ISM

Spectral slopes of density spectra (3D)



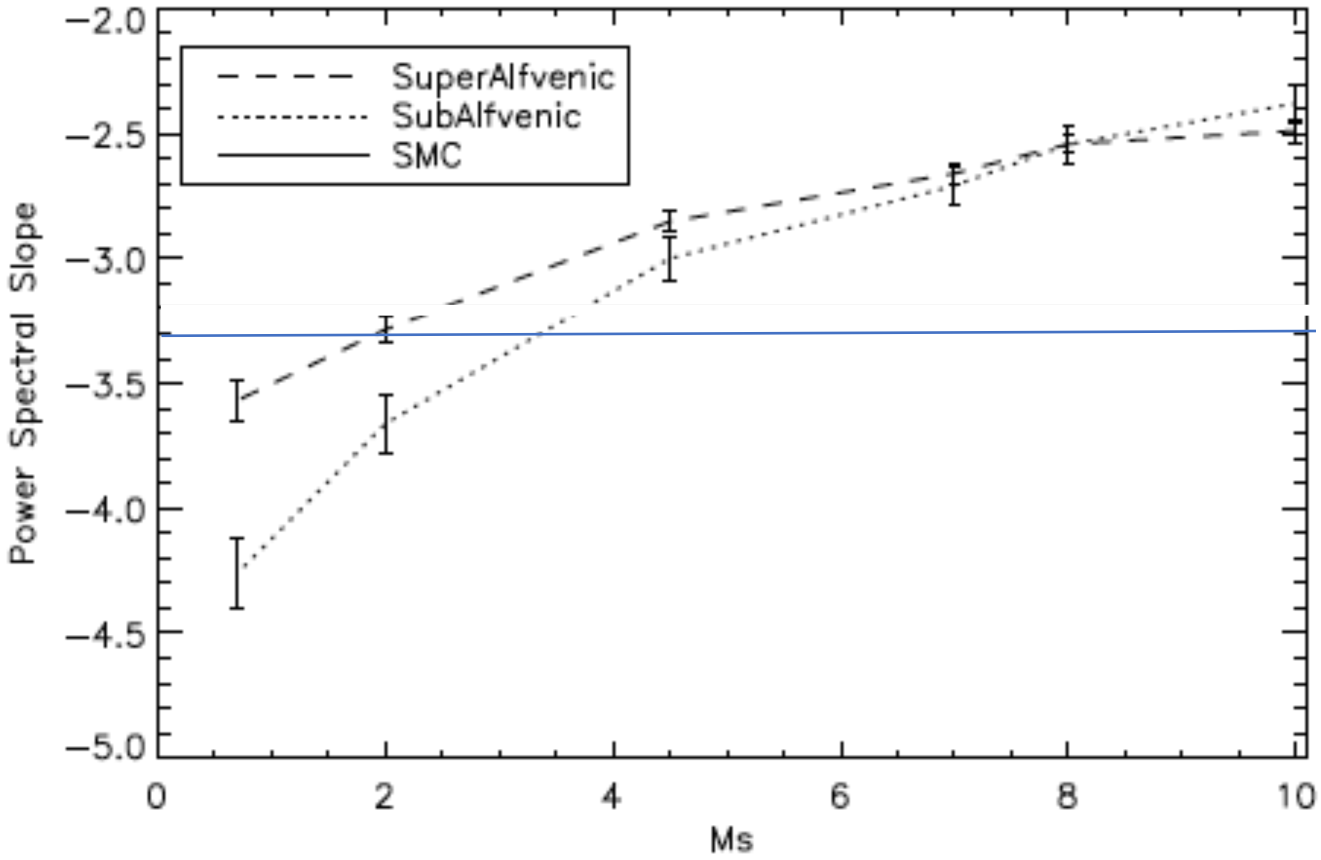
Hennebelle & Falgarone 12

e.g. Stutzki et al. 1998;
Deshpande et al. 2000;
Padoan et al. 2004; Swift 2006;
Lazarian 2009

Lazarian & Pogosyan 2004, k^{-3}

Density Spectrum Compared with 3D MHD Simulations

Density spectral index=-3.3 for SMC (Lazarian & Stanimirovic 2001)



Burkhart et al. 2010

Kolmogorov ~ k $^{-11/3}$

Structure functions

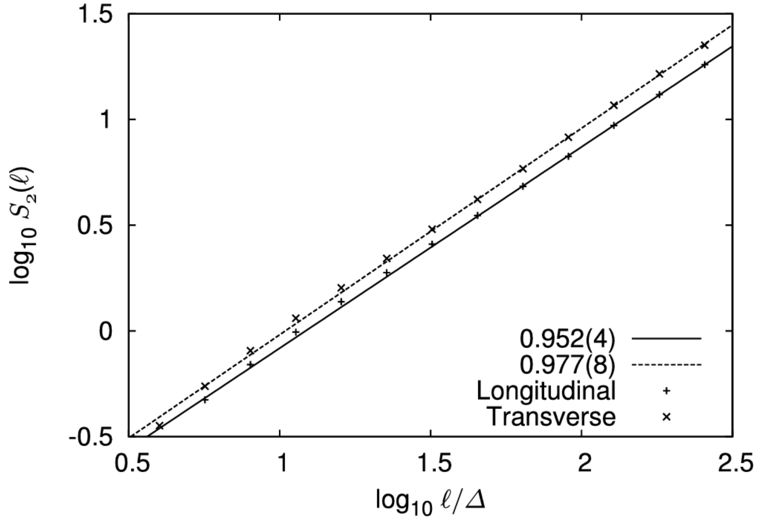
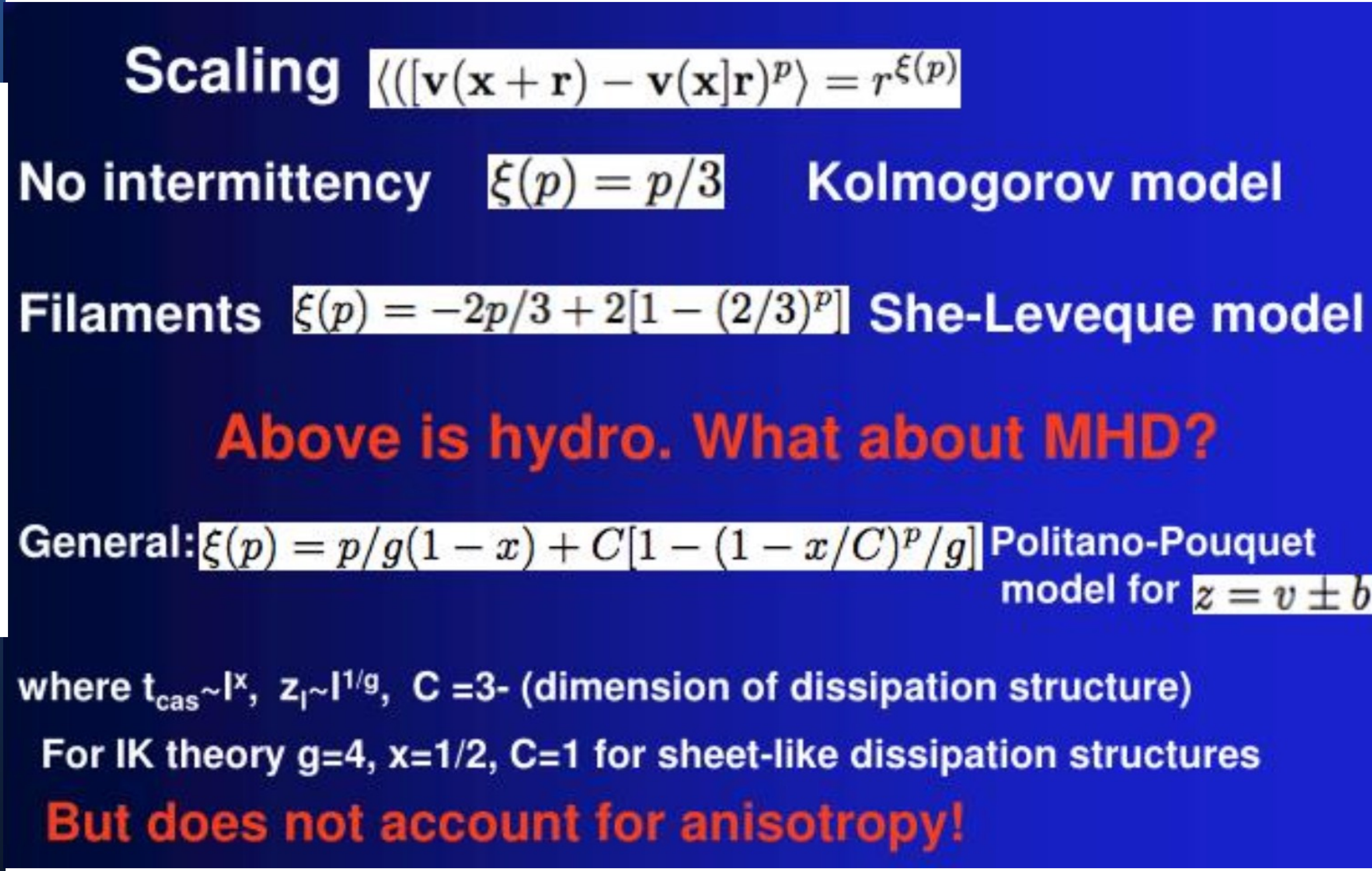


FIG. 11.—Same as Fig. 10, but for the second-order structure functions. [*See the electronic edition of the Journal for a color version of this figure.*]

Kritsuk et al. 2007 find agreement with Burgers-2 value for 2nd order SF

$$\zeta_2^{\parallel} = 0.952 \pm 0.004 \text{ and } \zeta_2^{\perp} = 0.977 \pm 0.008,$$



Scaling $\langle(([\mathbf{v}(\mathbf{x}+\mathbf{r})-\mathbf{v}(\mathbf{x}]\mathbf{r})^p\rangle = r^{\xi(p)}$

No intermittency $\xi(p) = p/3$ **Kolmogorov model**

Filaments $\xi(p) = -2p/3 + 2[1-(2/3)^p]$ **She-Leveque model**

Above is hydro. What about MHD?

General: $\xi(p) = p/g(1-x) + C[1-(1-x/C)^p/g]$ **Politano-Pouquet model for** $z = v \pm b$

where $t_{cas} \sim l^x$, $z_l \sim l^{1/g}$, **C =3- (dimension of dissipation structure)**

For IK theory g=4, x=1/2, C=1 for sheet-like dissipation structures

But does not account for anisotropy!

PDF moments (variance, skewness, and kurtosis) of the density PDF are related to the sonic Mach number

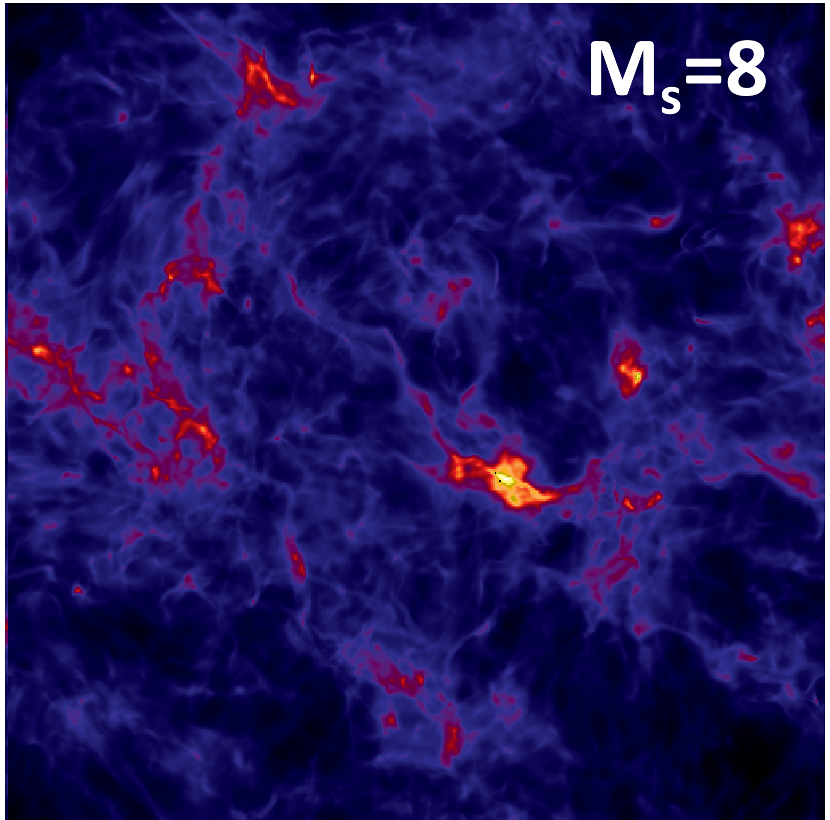
$$\sigma_s^2 = ln(1 + b^2 M_s^2)$$

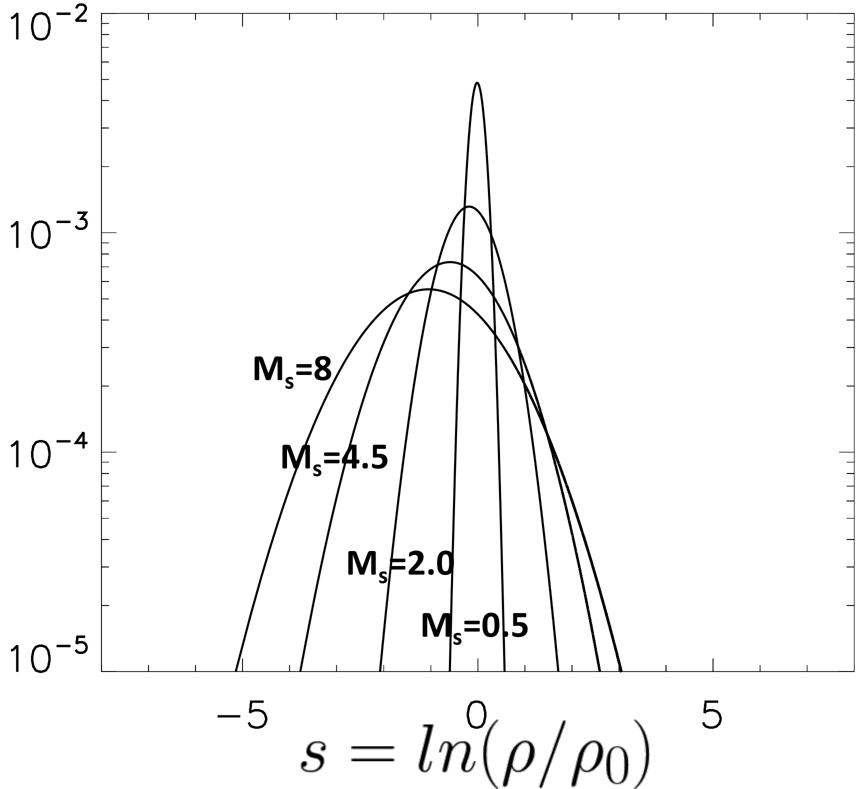
$$\boxed{\sigma^2_{\rho/\rho_0} = b^2 M^2}$$

3D normalised density variance

numerically determined parameter

3D rms Mach number





PDFs of Column Density-M_S

2nd moment: Variance (σ^2 linear and log PDF) vs. M_s
3rd moment: Skewness(linear PDF) vs. M_s
4th moment: Kurtosis(linear PDF) vs. M_s

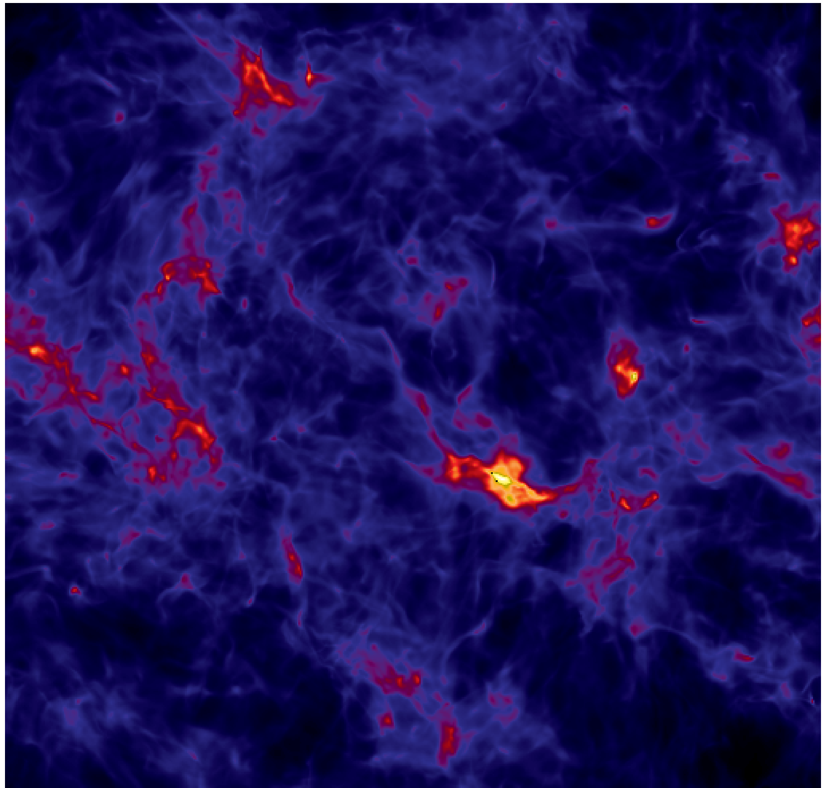
Column density PDFs:
Kowal et al. 07; Burkhart et al. 09,10; Burkhart & Lazarian 12; Kainulainen & Tan 13

$$\sigma^2_{\rho/\rho_0} = b^2\mathcal{M}_s^2$$

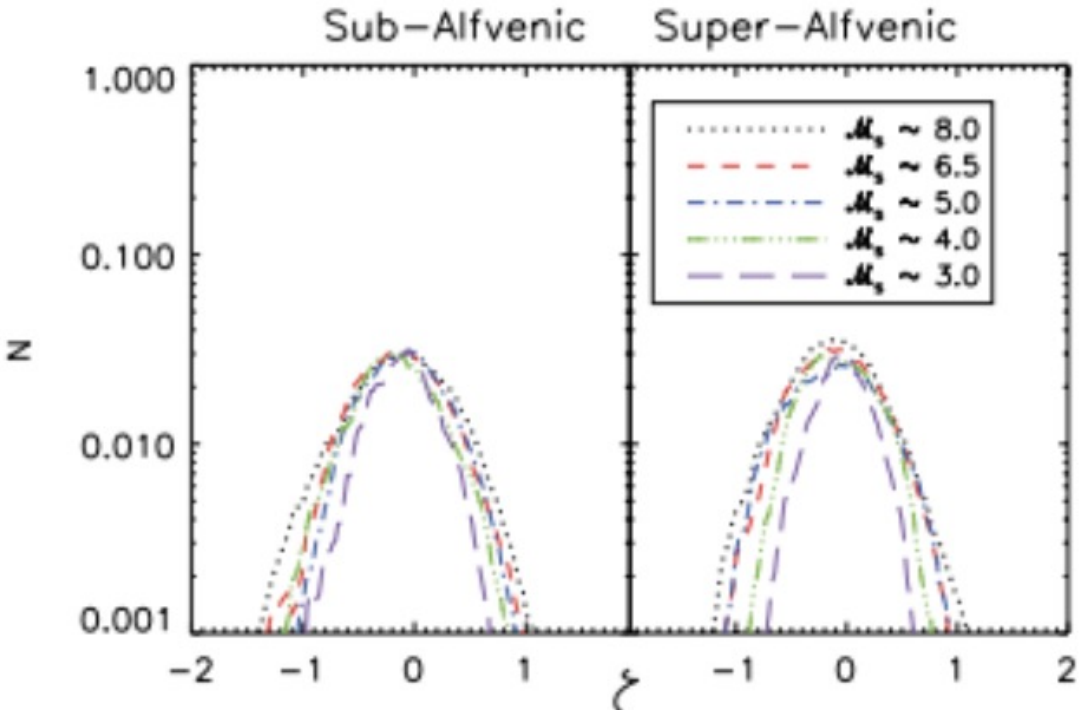
$$\sigma_s^2 = ln(1+b^2\mathcal{M}_s^2)$$

Skewness=A*M_s+b

Kurtosis=A*M_s+b



Linear Column Density PDF

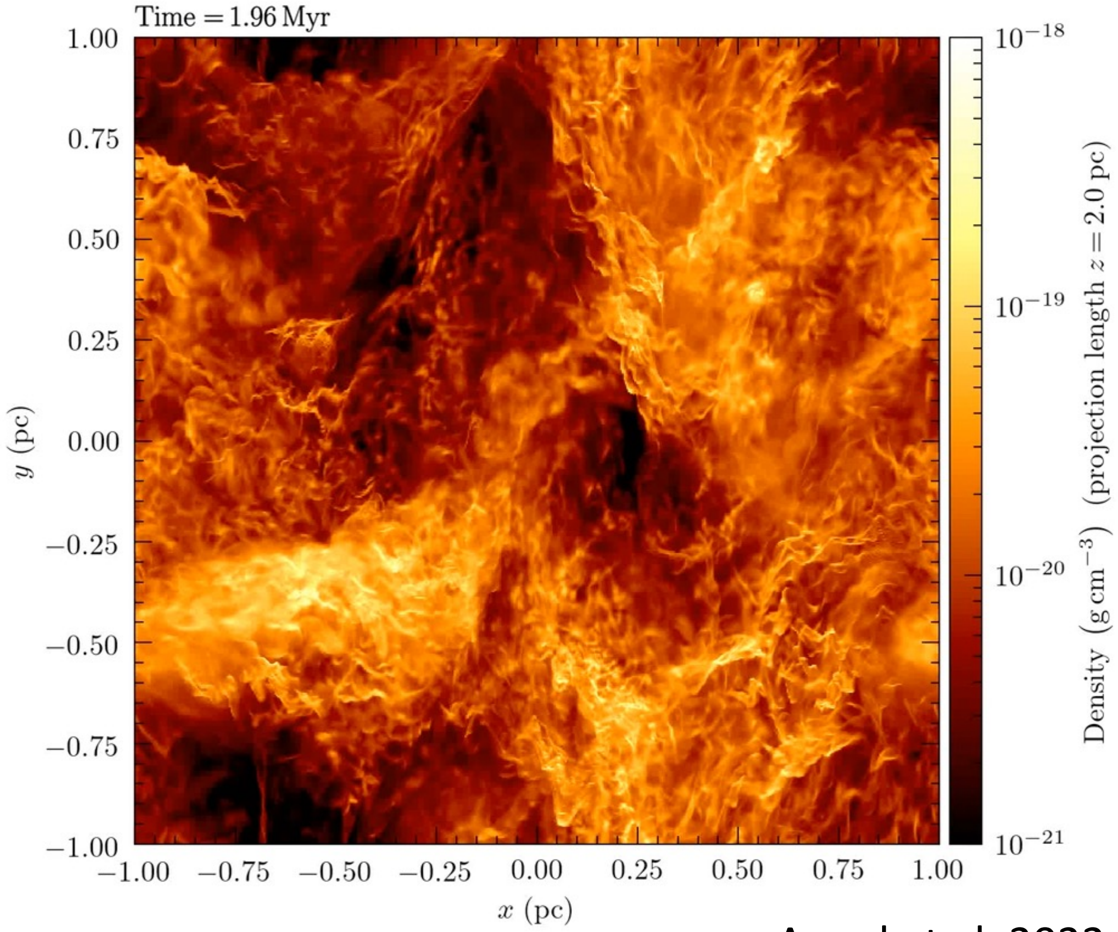


Fundamental parameters for dynamics of star formation:

Measuring these parameters can allow us to distinguish between different star formation models

1) $E(k)_{KE}$
2) $M_A = v/v_A$
3) $M_s = v/c_s$
4) $$\beta_0 = \frac{8\pi c_s^2 \rho_0}{B_0^2}$$
5) $$\alpha \equiv \frac{2E_K}{|E_g|} = \frac{5\sigma_v^2 R}{GM}$$ $$\lambda_C \equiv \frac{\left(\frac{M}{\Phi}\right)_{obs}}{\left(\frac{M}{\Phi}\right)_{crit}}$$

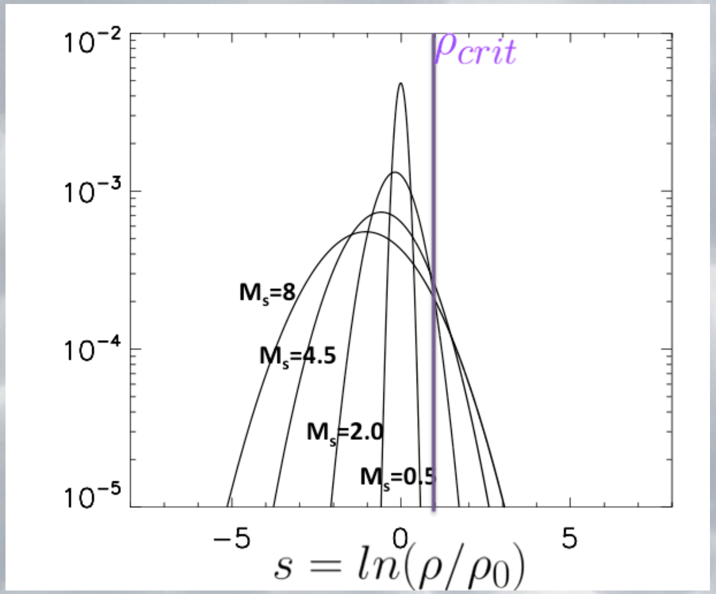
1) How is turbulence developed and what are the 'scalings'?
2) How strong is the magnetic pressure relative to turbulence?
3) How strong is turbulence relative to gas pressure?
4) Ratio of 2 and 3 describes gas pressure to magnetic pressure (i.e., plasma beta)
5) The role of gravity (Virial parameter and Mass to Flux ratio)



Appel et al. 2022

Supersonic Turbulence *enhances* rate of star formation
due to density fluctuations

Star Formation



Supersonic Turbulence *decreases* rate of star formation
due to bulk pressure support

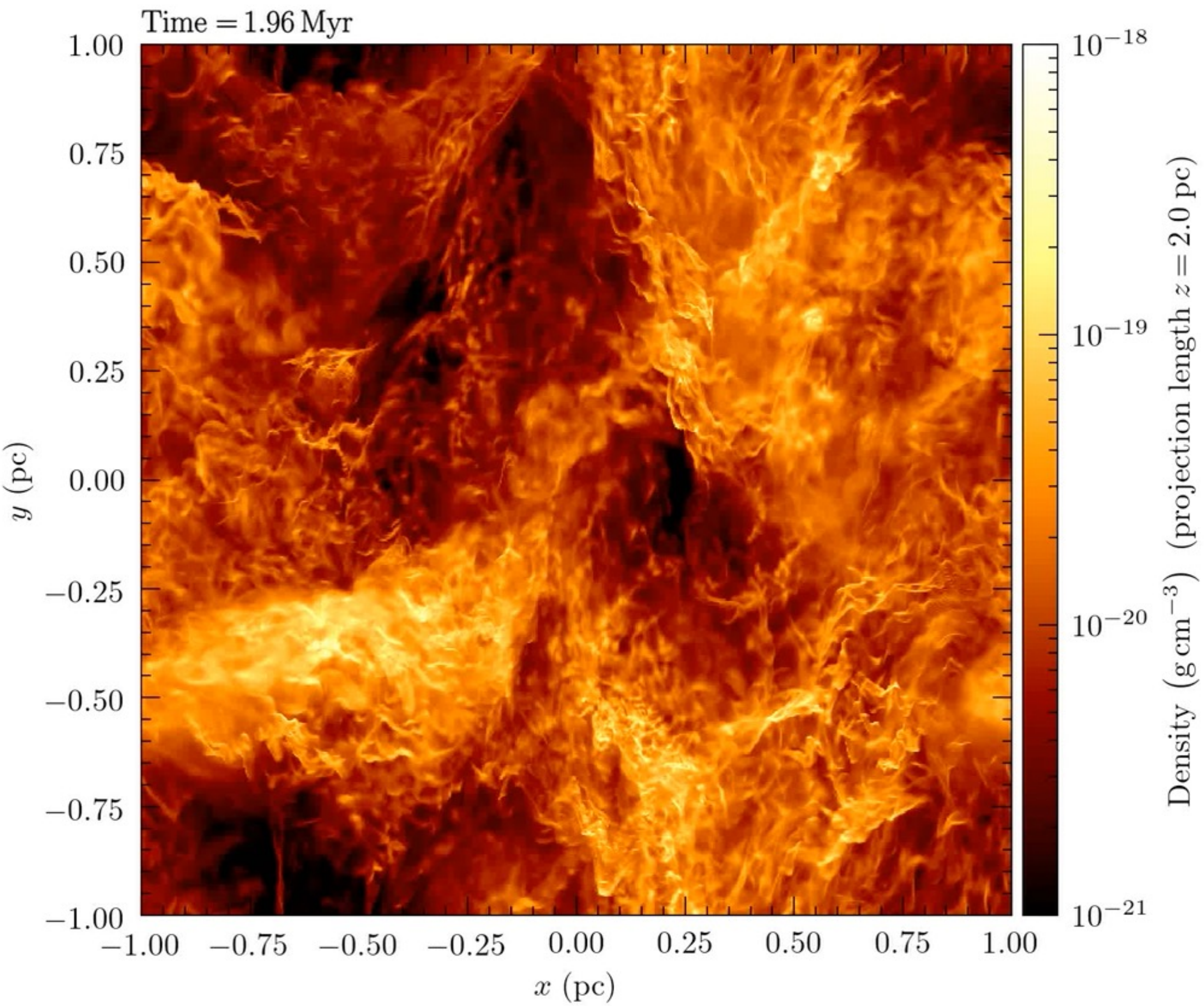
Sonic Mach number

Effect 1: Supersonic Turbulence *enhances* rate of star formation in shocked regions due to density fluctuations.

Christoph Federrath

Effect 1 Visualized: Supersonic turbulence seed density field

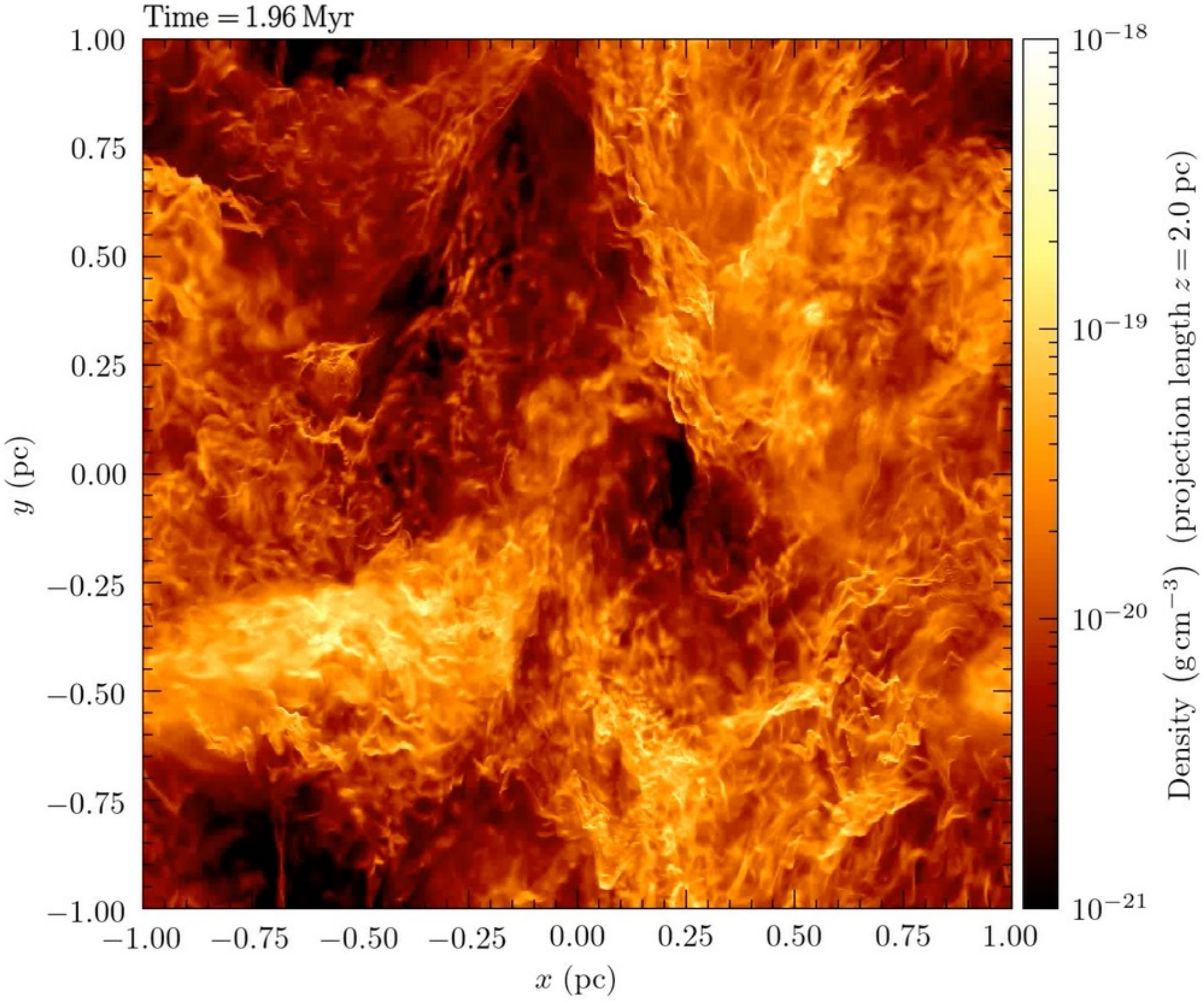
Gravity turned on, turbulence off.



Sonic Mach number

Effect 2 Visualized: Supersonic turbulence seed density field and keeps pressure high

Gravity turned on, turbulence on

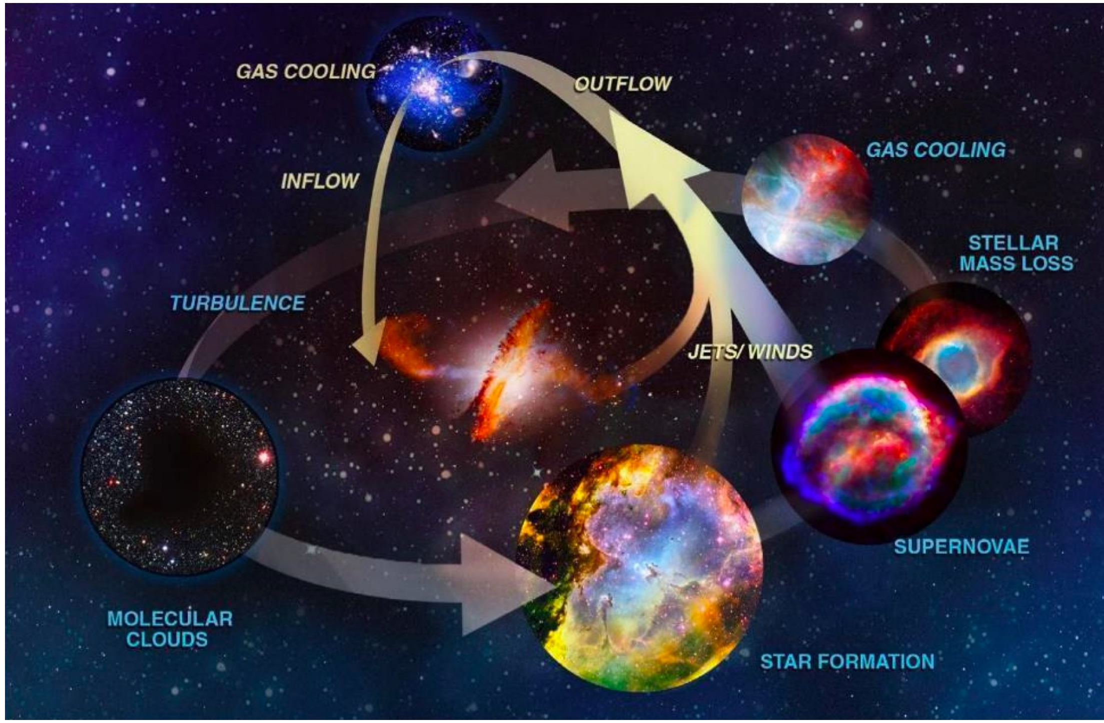


Effect 2: Supersonic Turbulence *decreases* global rate of star formation in low density regions due to pressure support.

Christoph Federrath

MHD Turbulence..

- What is turbulence?
- Hydro: Kolmogorov 41
- MHD (1995): GS95 and Critical Balance
- MHD (2006): Dynamic Alignment
- Intermittency
- Compressibility
- Diagnostics
- Star formation self-regulation via turbulence and feedback



How well does turbulence theory describe the ISM?

- Our understanding of basic scaling laws of MHD turbulence has advanced tremendously!
- We can apply these scaling laws to observational results and test them with simulations.
- We have developed new techniques for measuring turbulence in the ISM.

Pessimism

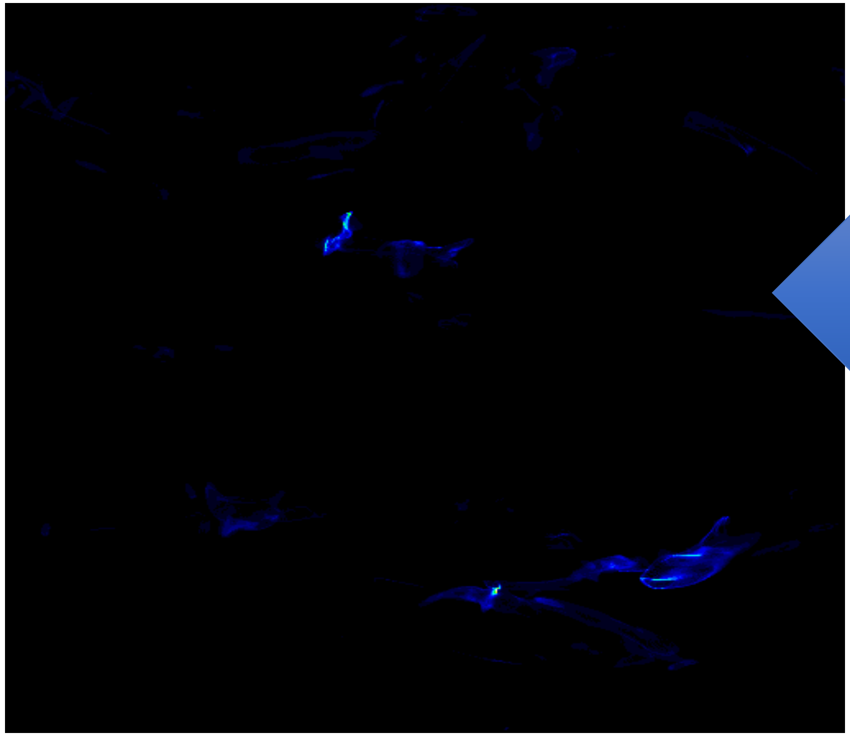
- The Reynolds numbers ($Re=VL/\nu$) of the ISM are as high as 10^{10} while simulations can only achieve $\sim 10^4$.
- Numerical simulations do not resolve necessary scales and/or do not include necessary physics.
- Observations are polluted with noise, instrumentation effects, and are limited to the LOS
- No complete theory of turbulence exists.

Numerics vs. Observations

What are the limitations?

Full information…density (PPP), velocity, magnetic fields etc…

Synthetic observations (PPV) MHD 512^3 M_s=7

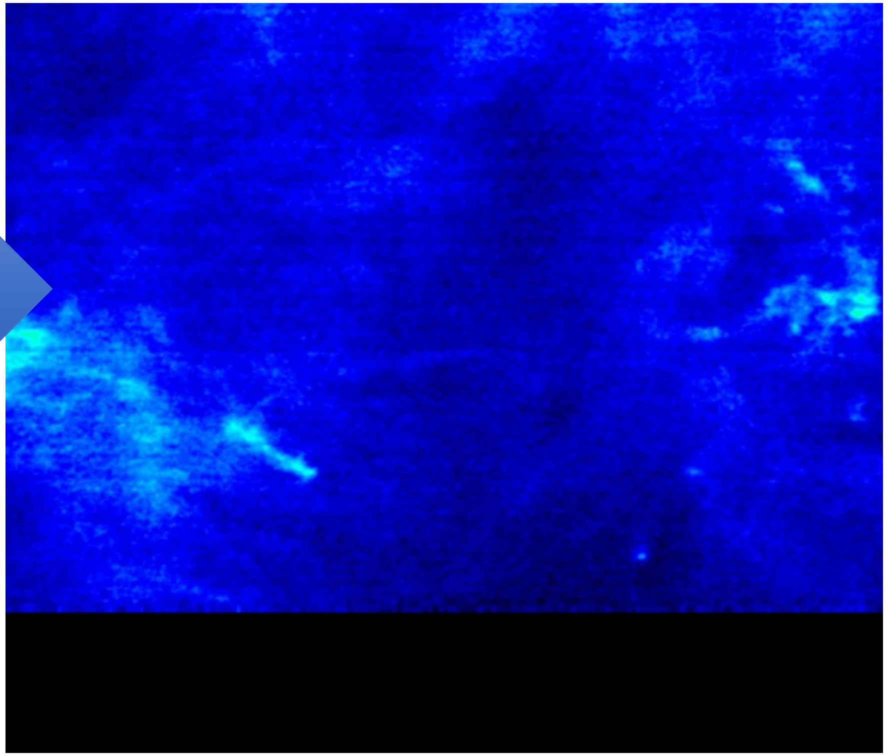


Very Idealized environment
Spatial scales do not match the real world
Currently we can get max Re of order $<10^4$

Statistical tools

Partial Picture… column density (PP), velocity + density fluctuations (PPV), some magnetic fields…

Galactic Arecibo HI (PPV) data



Can only get column density….noise and instrument effects are contaminants

Re ~VL/ν ~10^{10}

Why be an optimist?

- Larson laws (Larson 1981)- Power law correlations between Molecular cloud sizes and linewidths (e.g. Myers 83; Dame et al. 86; Solomon 87; Dickey 85; Scalo 87)

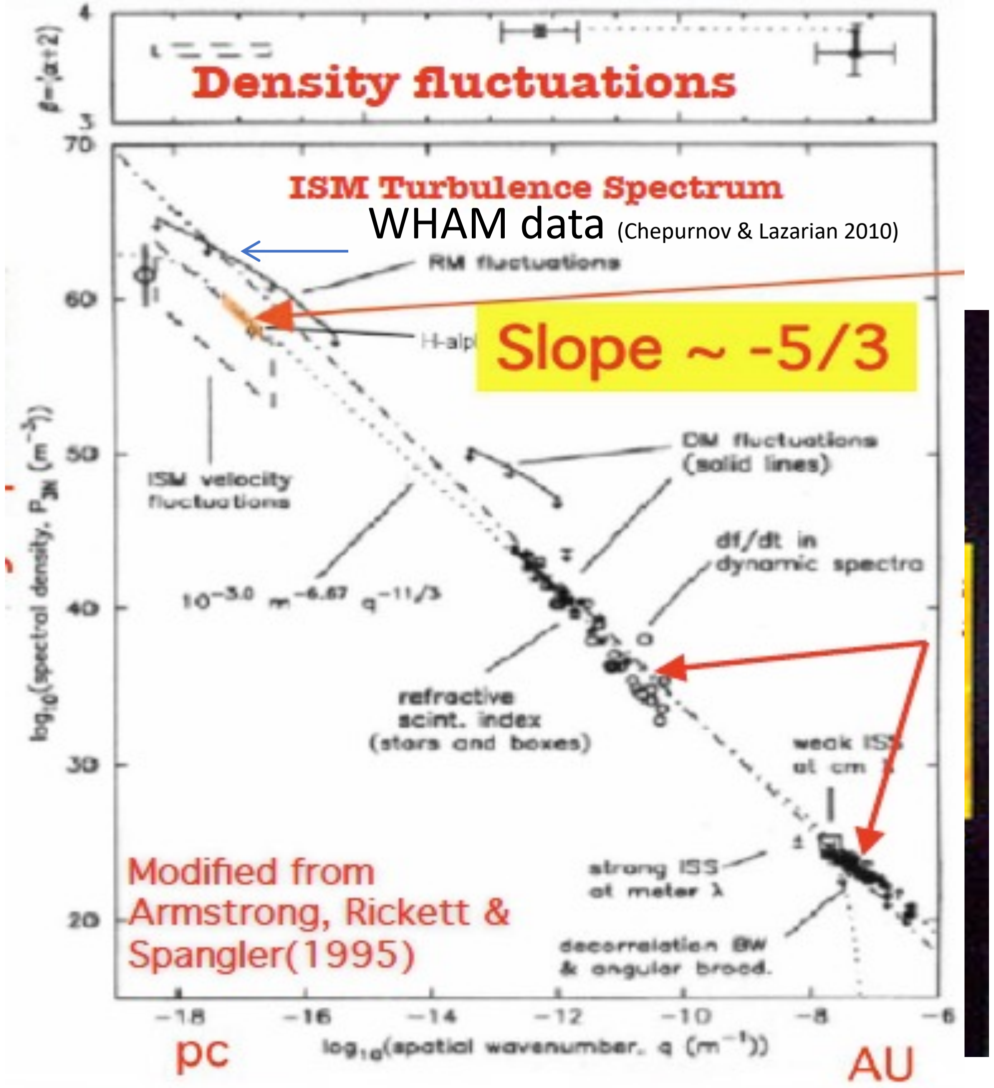
-Emission line broadening (e.g Heiles & Troland 03)

-Morphological confirmation with IRAS in 80s. Revealed full beautiful complexity!

-Spectrum of CO and HI corresponding to compressible/incompressible turbulence (e.g. Lazarian & Pogosyan 2000; Dickey et al. 2001; Chepurnov et al. 2009; Padoan et al. 2009; Stanimirovic & Lazarian 2001)

-log-normal PDFs (e.g. Vazquez-Semadeni 94; Kainulainen et al. 09)

-The big power law (e.g. Armstrong, Rickett & Spangler 1995; Chepurnov & Lazarian 2010)

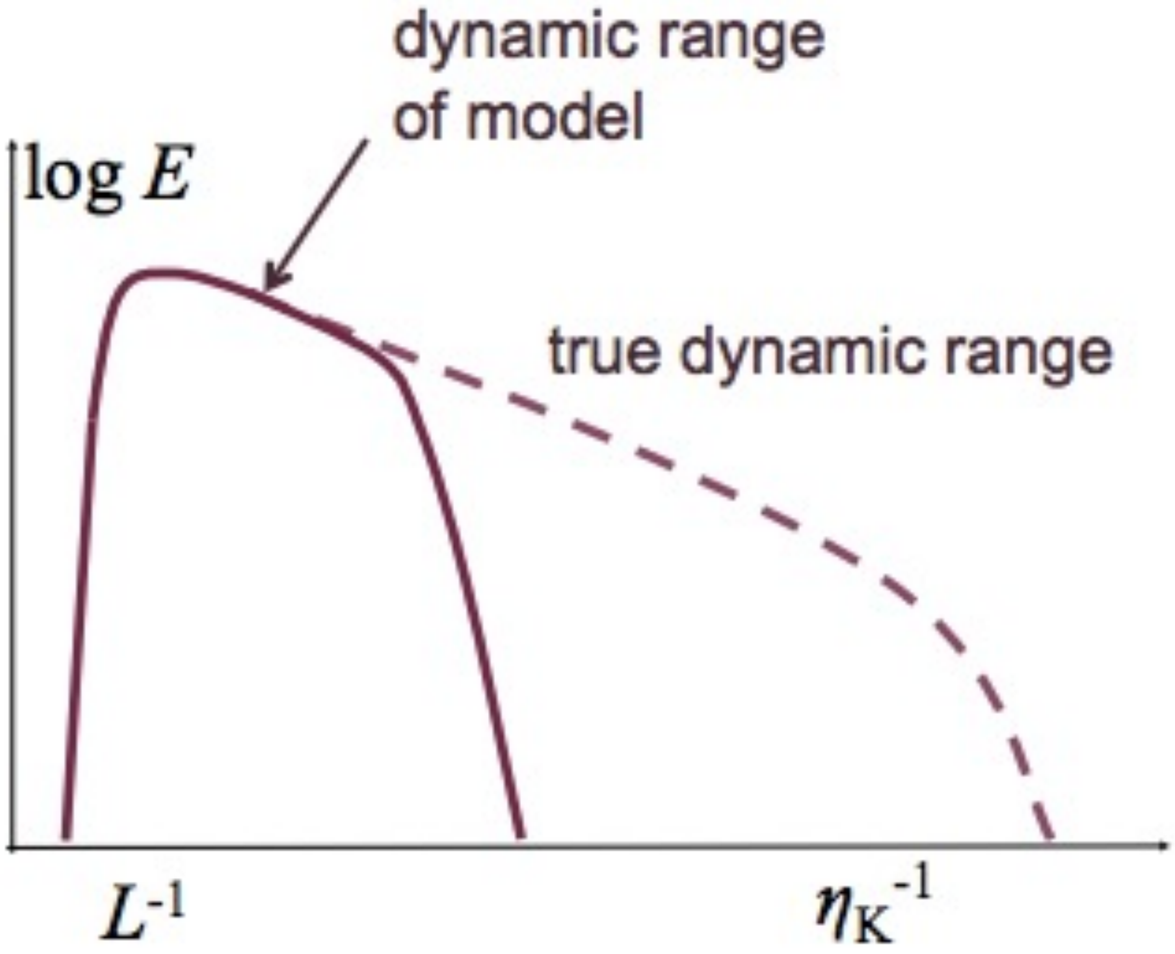


WHAM data (Chepurnov & Lazarian 2010)

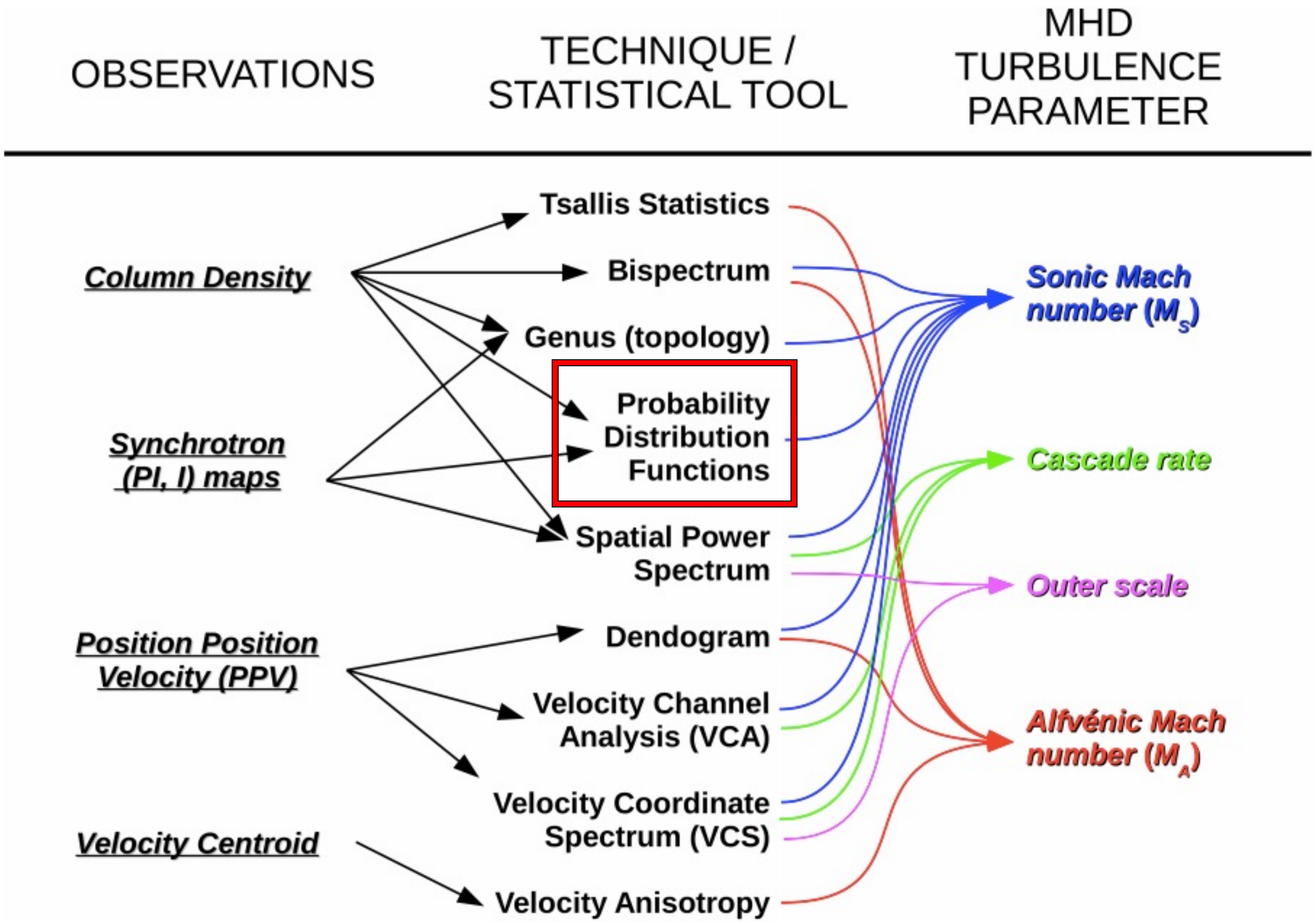
Slope ~ -5/3

Modified from Armstrong, Rickett & Spangler(1995)

Limitations of simulations

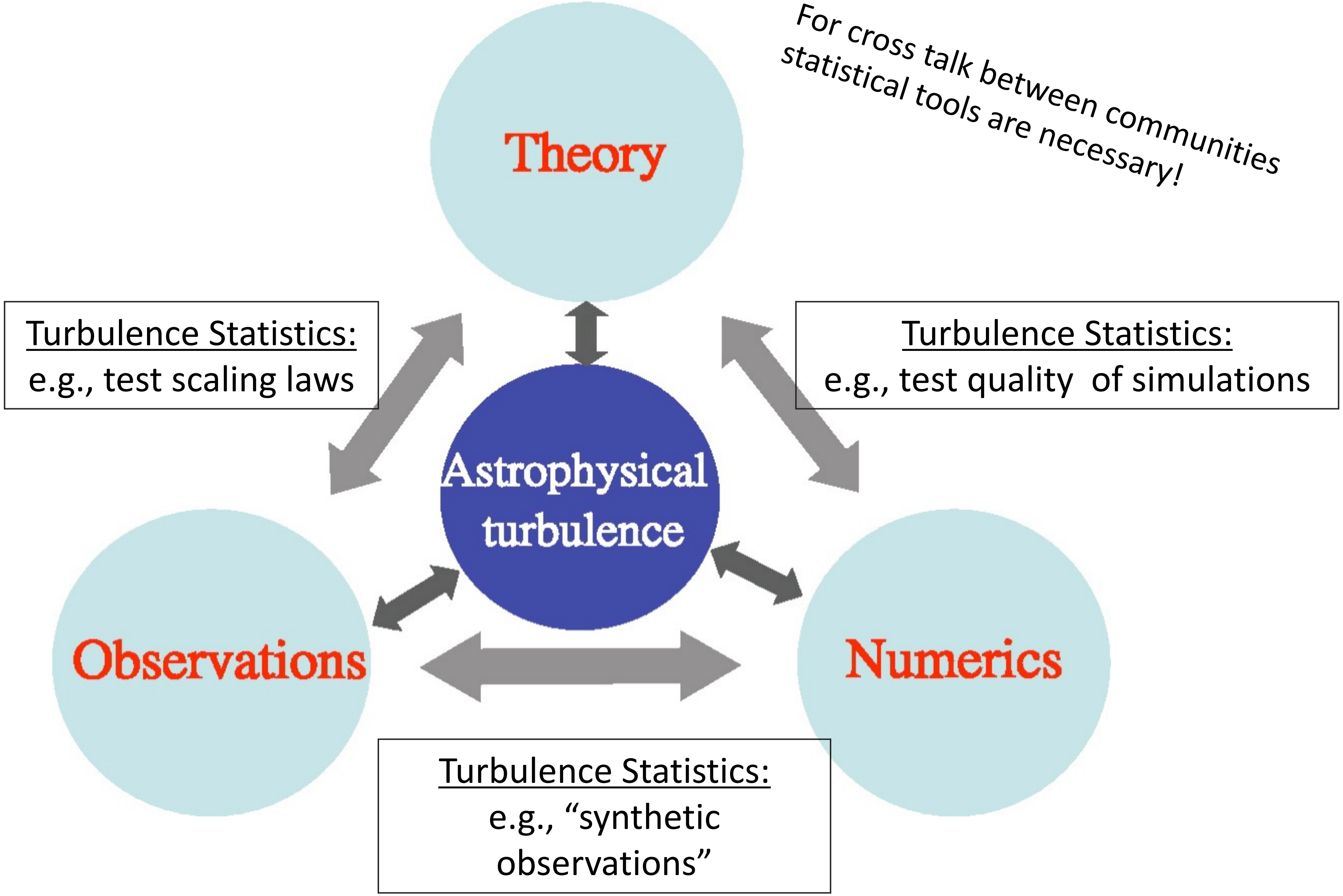


Turbulence Statistics



How to Study MHD Turbulence in Galaxies?

For cross talk between communities statistical tools are necessary!



Theory

Astrophysical turbulence

Observations

Numerics

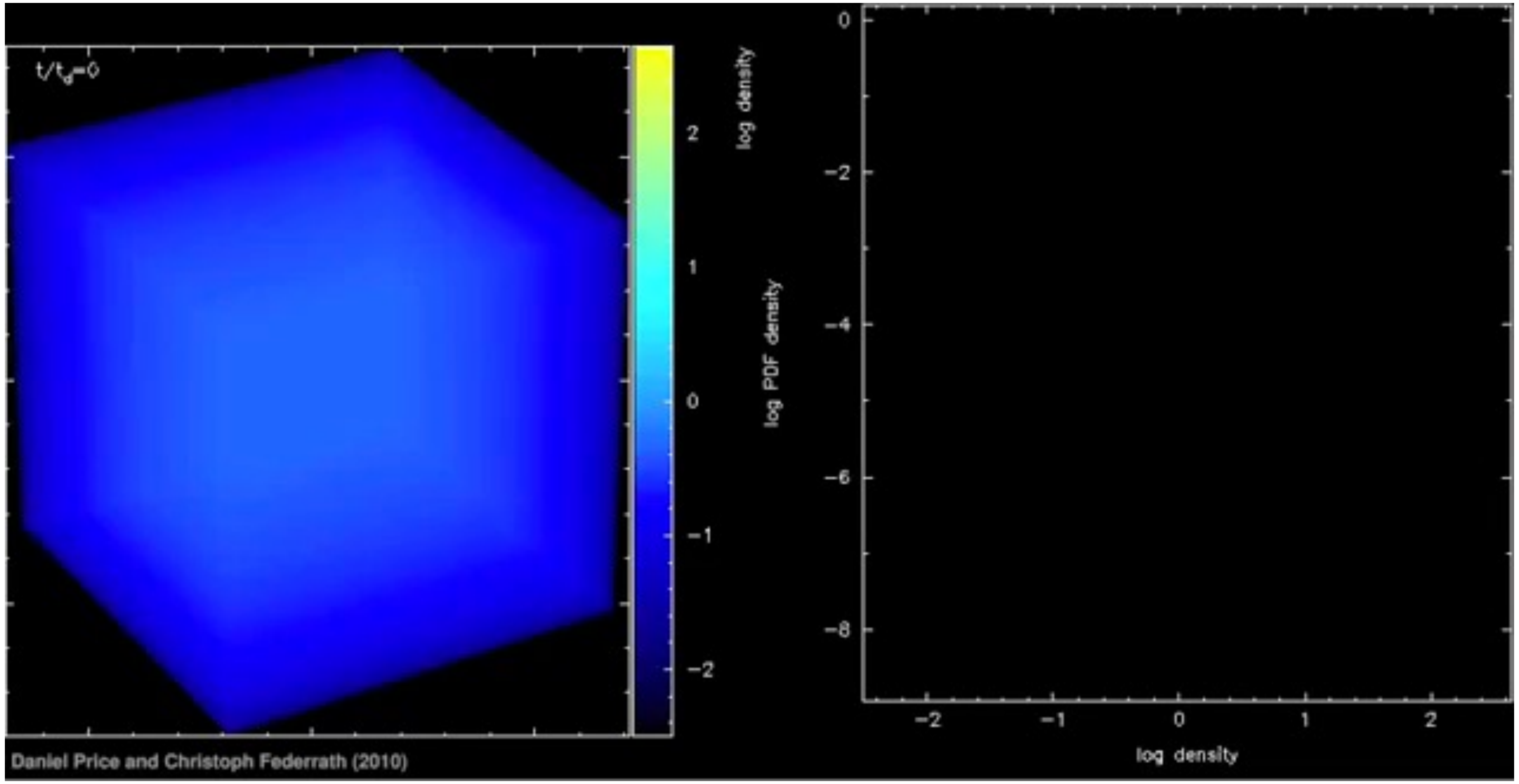
Turbulence Statistics:
e.g., test scaling laws

Turbulence Statistics:
e.g., test quality of simulations

Turbulence Statistics:
e.g., “synthetic observations”

Are one and two point statistics enough to describe turbulence?

Example: Density Probability distribution of supersonic isothermal turbulence is lognormal



PDF moments (variance, skewness, and kurtosis) of the column density PDF are related to the sonic Mach number

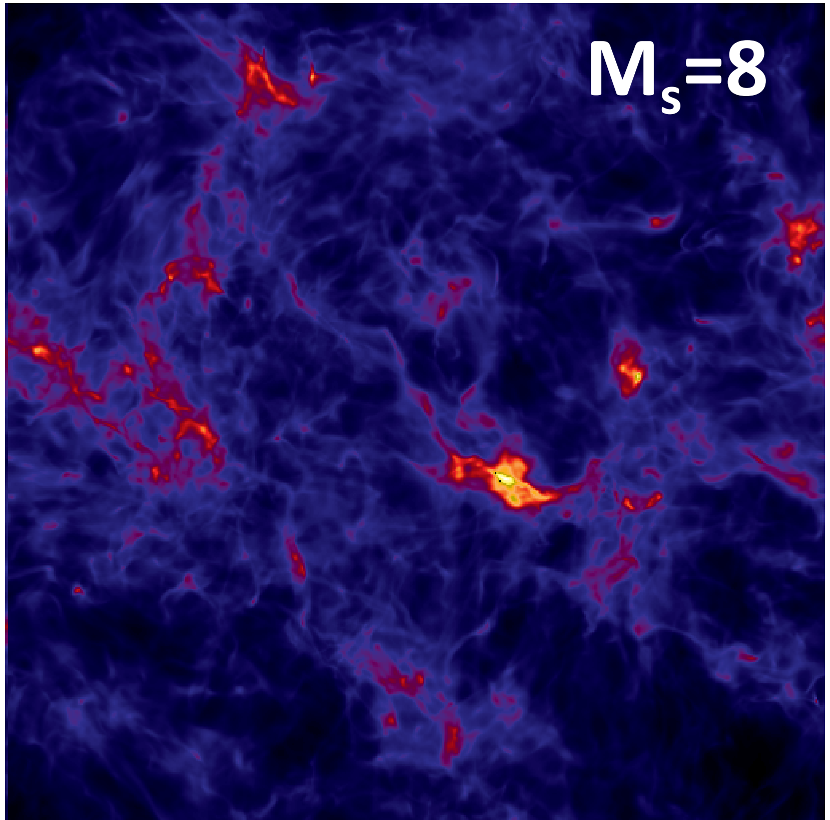
$$\sigma_s^2 = ln(1 + b^2 M_s^2)$$

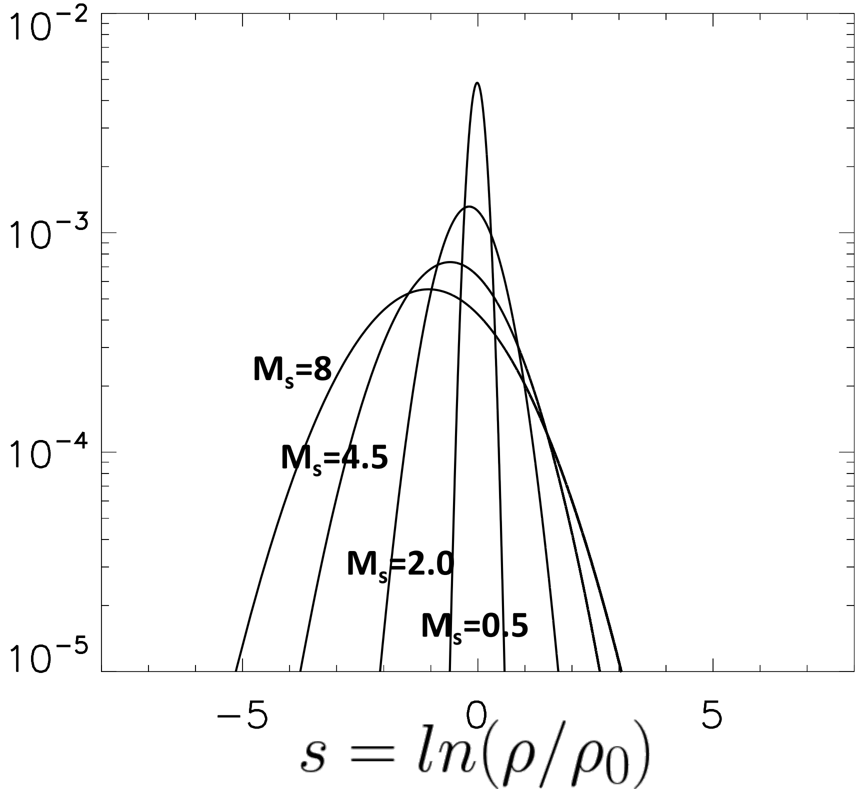
$$\boxed{\sigma^2_{\rho/\rho_0} = b^2 M^2}$$

3D normalised density variance

numerically determined parameter

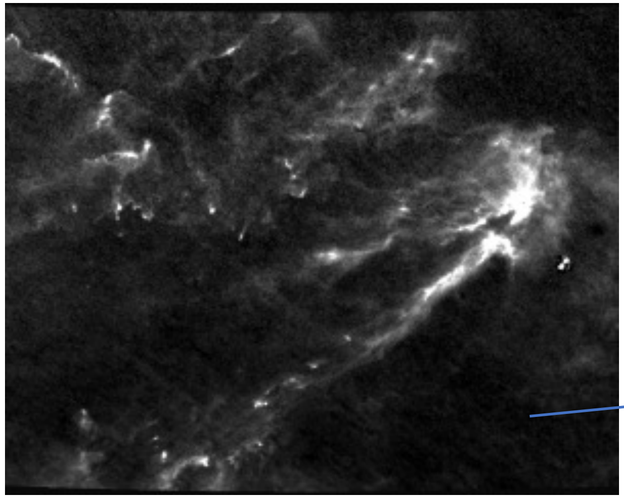
3D rms Mach number

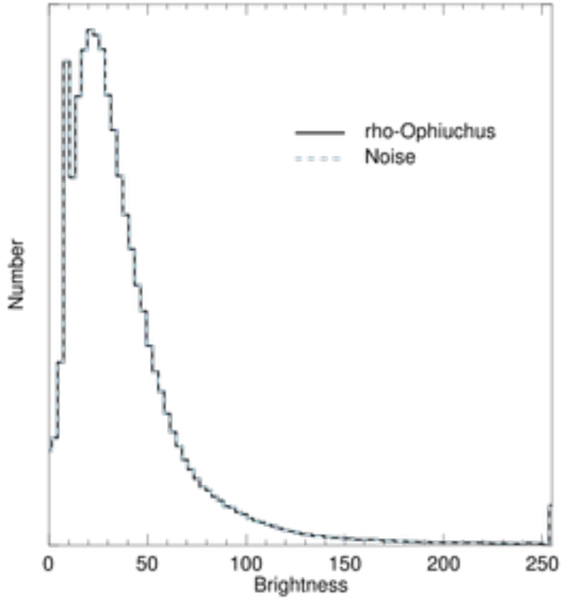




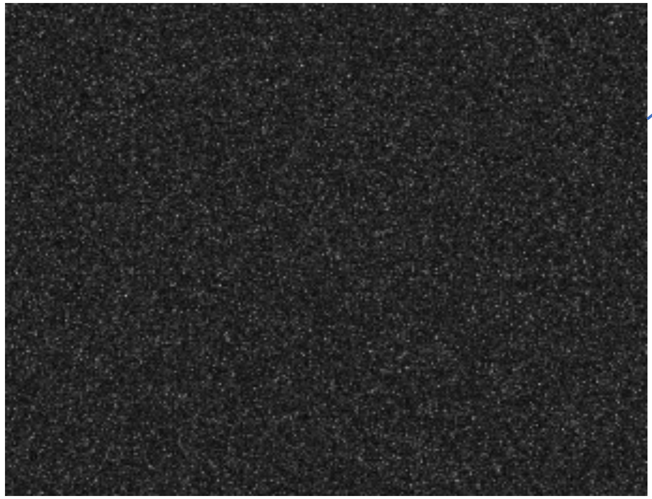
PDFs (1-point function) are limited; contains no spatial information

Rho-Ophiucus





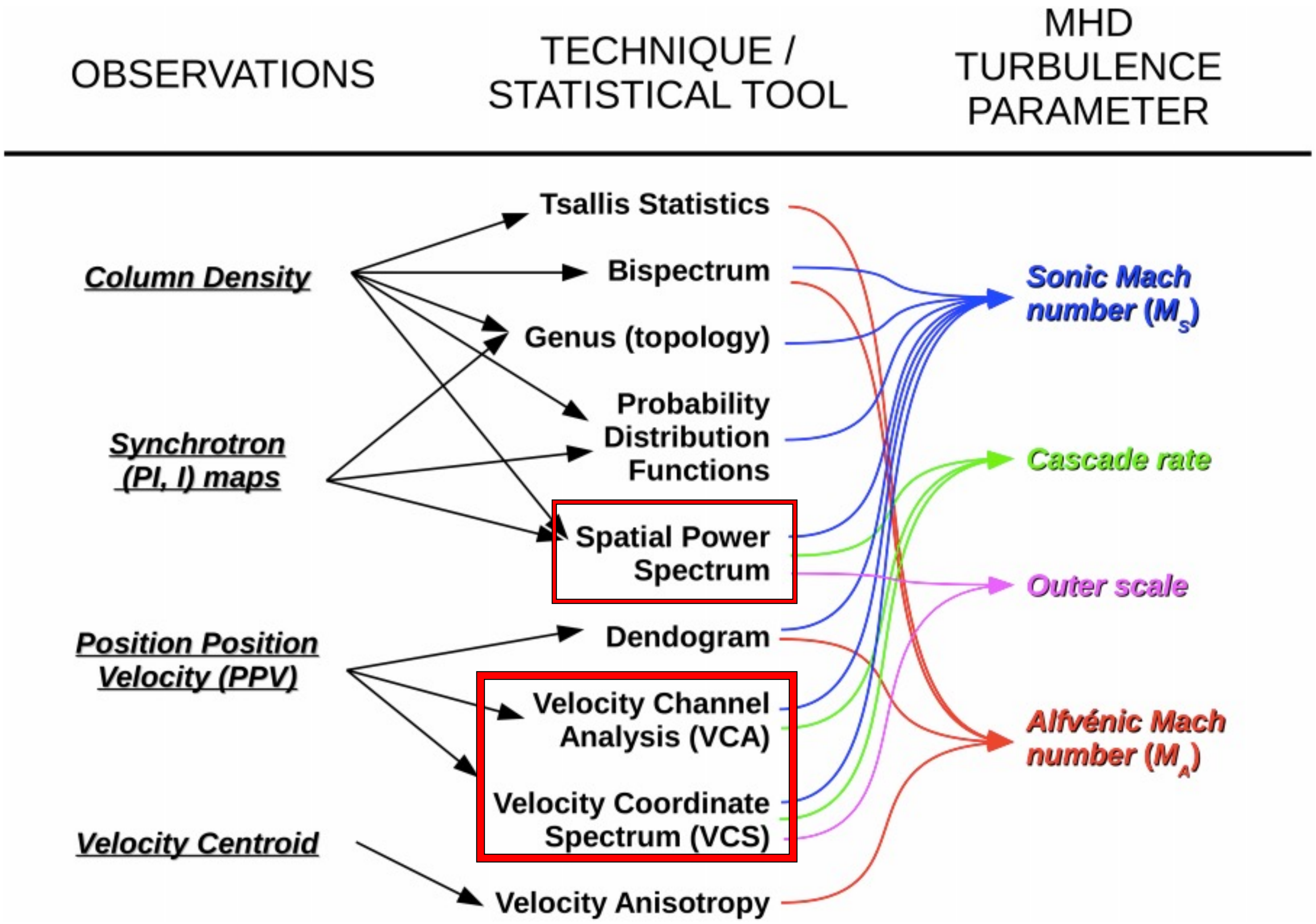
Rho-Ophiucus (scrambled to noise)



Rho-Ophiucus (Bieber)

Chris Beaumont

Turbulence Statistics



OBSERVATIONS

TECHNIQUE / STATISTICAL TOOL

MHD TURBULENCE PARAMETER

Column Density

Synchrotron (PI, I) maps

Position Position Velocity (PPV)

Velocity Centroid

Tsallis Statistics

Bispectrum

Genus (topology)

Probability Distribution Functions

Spatial Power Spectrum

Dendogram

Velocity Channel Analysis (VCA)

Velocity Coordinate Spectrum (VCS)

Velocity Anisotropy

Sonic Mach number (M_S)

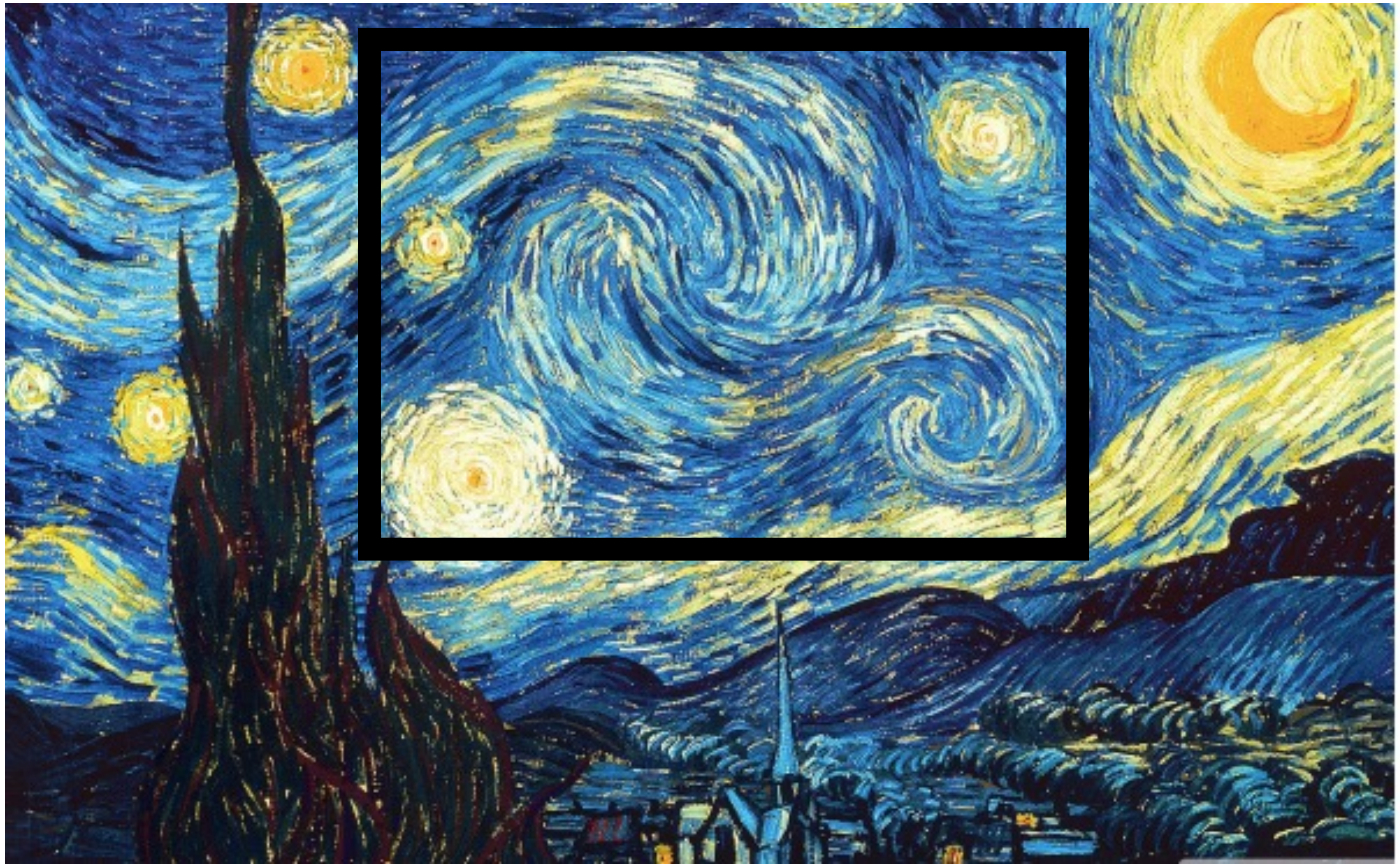
Cascade rate

Outer scale

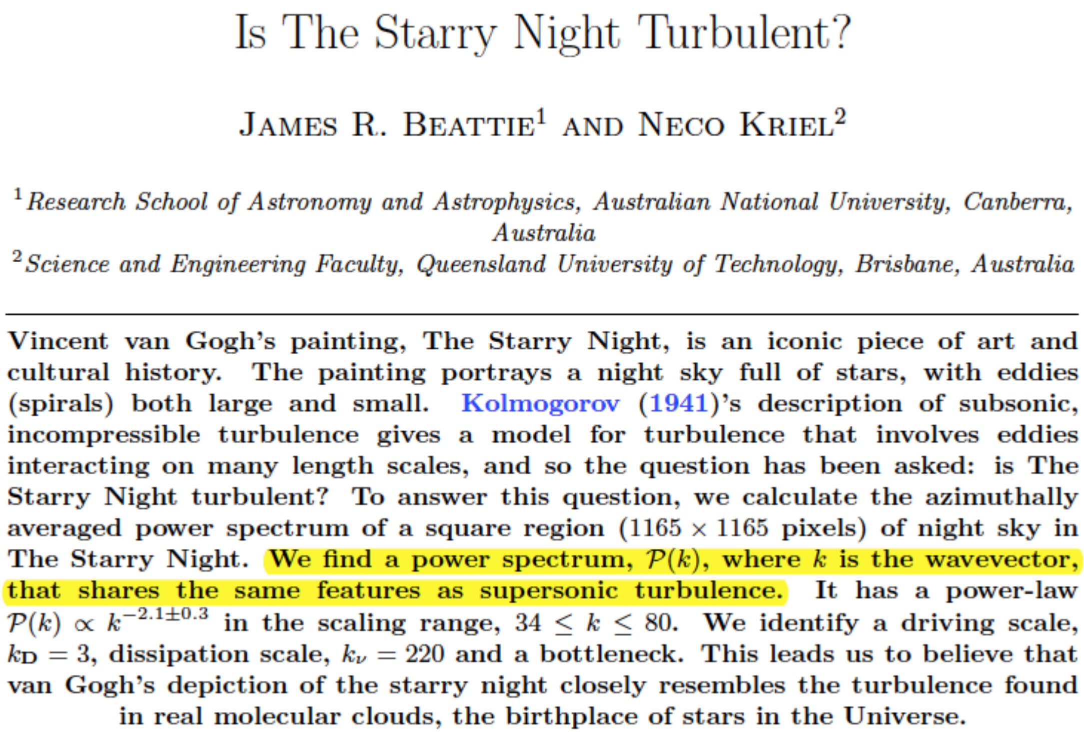
Alfvénic Mach number (M_A)

How robust a statistic is the Fourier power spectrum for turbulence studies?

Vincent van Gogh's
The Starry Night



How robust a statistic is the Fourier power spectrum for turbulence studies?

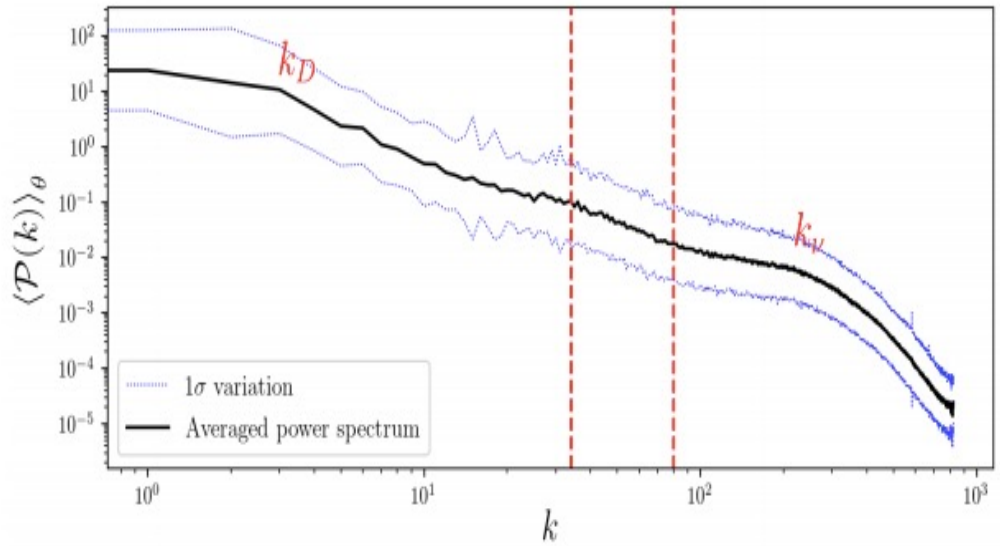


Is The Starry Night Turbulent?

JAMES R. BEATTIE[1] AND NECO KRIEL[2]

[1] *Research School of Astronomy and Astrophysics, Australian National University, Canberra, Australia*
[2] *Science and Engineering Faculty, Queensland University of Technology, Brisbane, Australia*

Vincent van Gogh's painting, The Starry Night, is an iconic piece of art and cultural history. The painting portrays a night sky full of stars, with eddies (spirals) both large and small. Kolmogorov (1941)'s description of subsonic, incompressible turbulence gives a model for turbulence that involves eddies interacting on many length scales, and so the question has been asked: is The Starry Night turbulent? To answer this question, we calculate the azimuthally averaged power spectrum of a square region (1165 × 1165 pixels) of night sky in The Starry Night. We find a power spectrum, $\mathcal{P}(k)$, where k is the wavevector, that shares the same features as supersonic turbulence. It has a power-law $\mathcal{P}(k) \propto k^{-2.1\pm0.3}$ in the scaling range, $34 \leq k \leq 80$. We identify a driving scale, $k_{\rm D} = 3$, dissipation scale, $k_\nu = 220$ and a bottleneck. This leads us to believe that van Gogh's depiction of the starry night closely resembles the turbulence found in real molecular clouds, the birthplace of stars in the Universe.



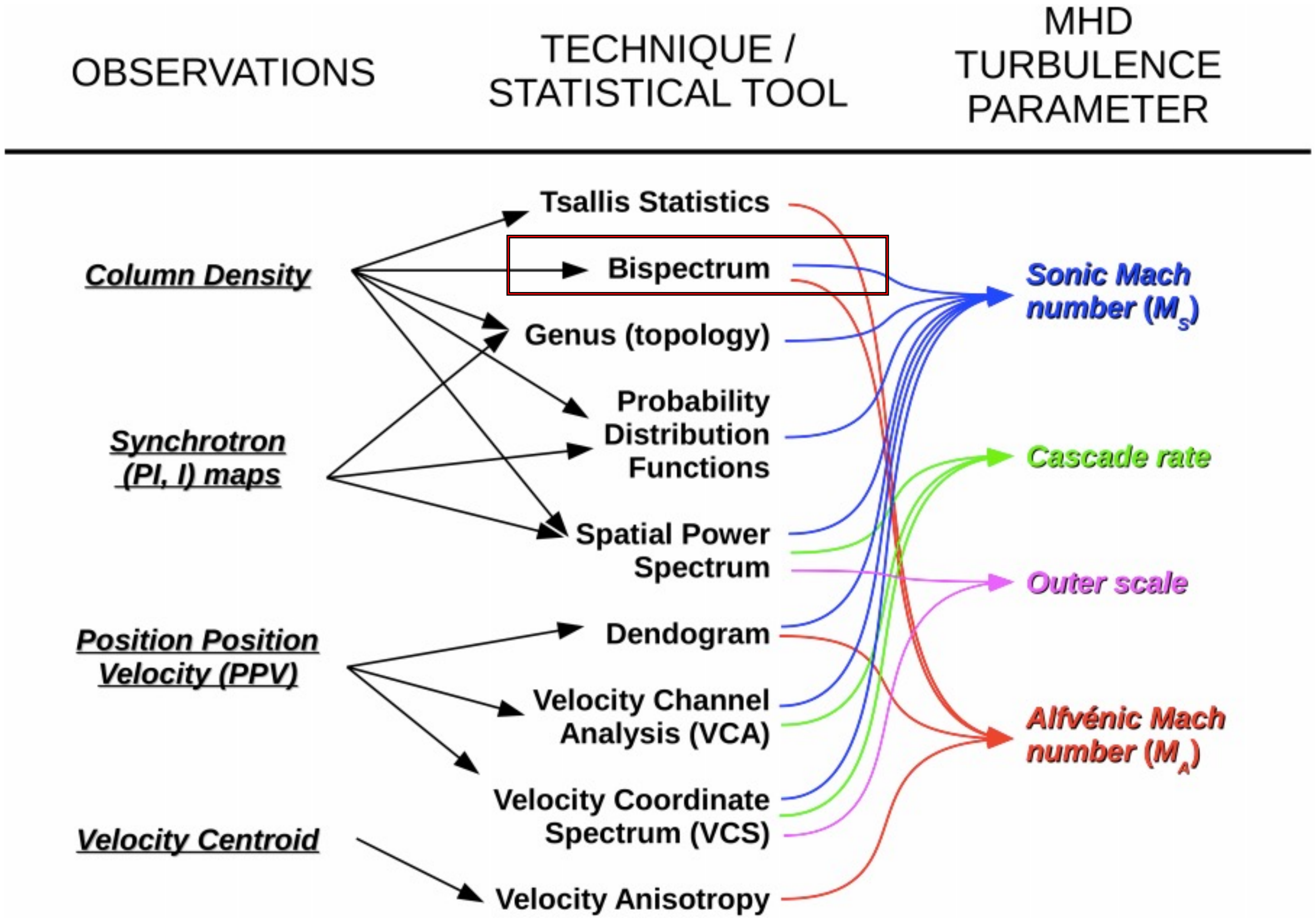
James Beattie

Conclusions:

We need tools beyond the power spectrum to robustly characterize astrophysical turbulence!

Fourier power spectrum lacks information on phases and therefore misses structural information in turbulent flows.

Need higher order statistics (or ML) that capture phase information



Bispectrum

Three point correlation function in Fourier space.

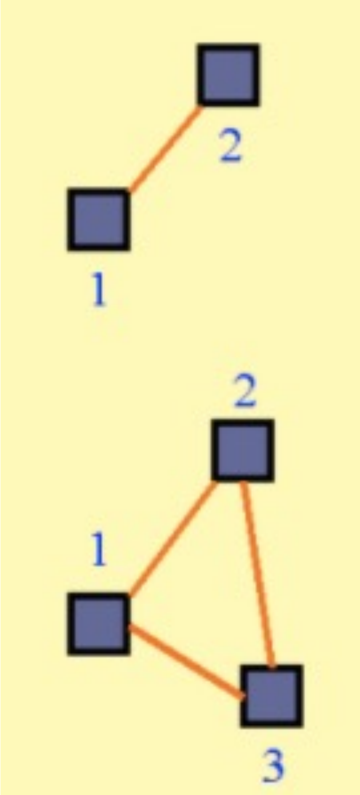
Preserves amplitude and phase information (is complex quantity).

Sensitive to non-linear fluctuations/non-Gaussianity

Is zero for Gaussian field

$$P(\vec{k}) = \sum_{\vec{k}} F(\vec{k}) F^*(\vec{k})$$

$$B(\vec{k_1}, \vec{k_2}) = \sum_{\vec{k_1}} \sum_{\vec{k_2}} F(\vec{k_1}) F(\vec{k_2}) F^*(\vec{k_1} + \vec{k_2})$$



Has been applied in the fields of:

CMB non-Gaussianity (e.g. Spergel & Goldberg 1999)

BAO detection (e.g. Slepian & Eisenstein 2016)

Galaxy distributions (e.g. Scoccimarro et al. 1998)

Neuroscience EEG (e.g. Bullock et al. 1997)

Anesthesiology (e.g. Johansen 2000)

ISM Turbulence (e.g. Burkhart et al 2009)

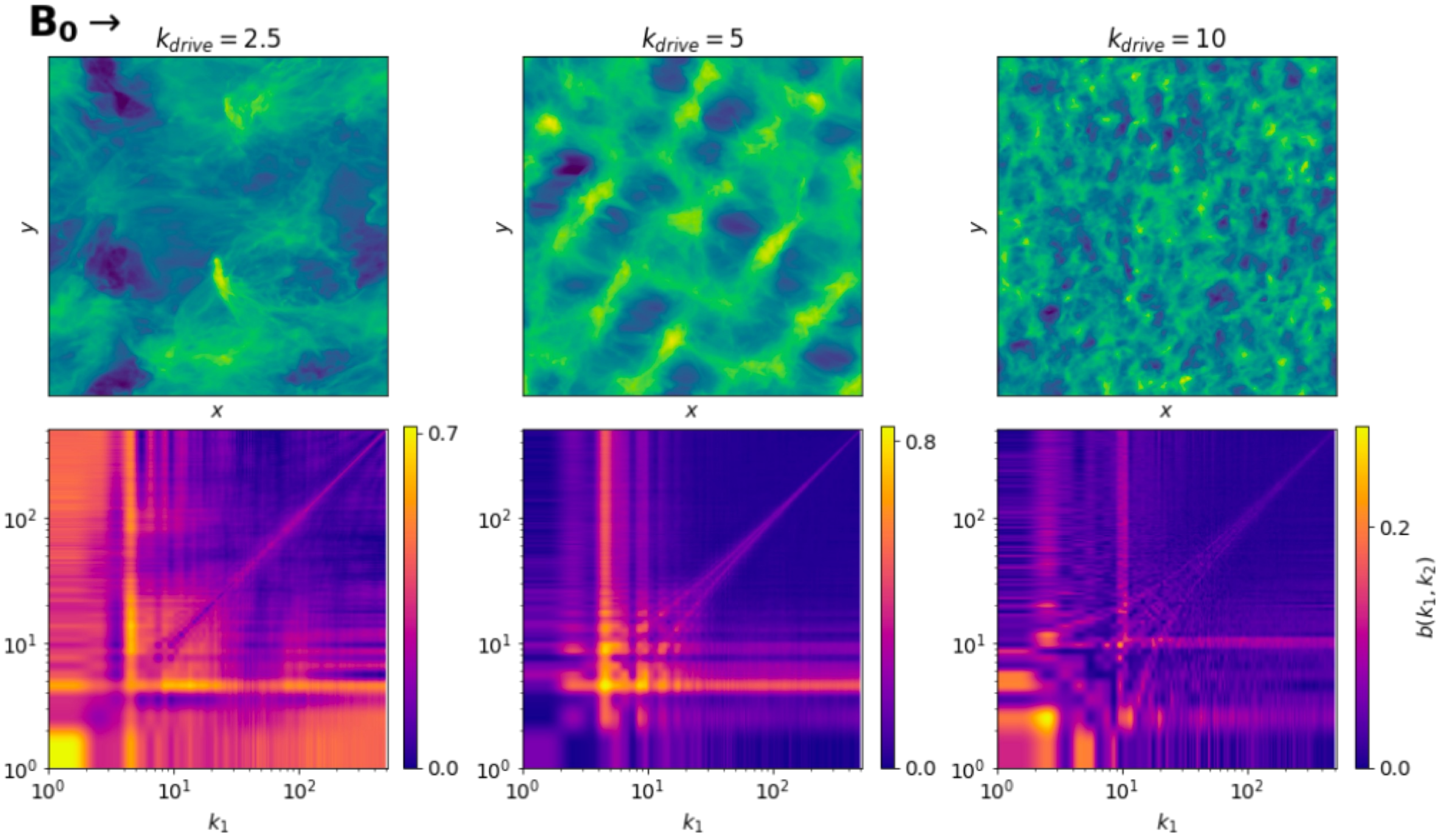
The Bispectrum

We study the bispectrum $B(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$, evaluating 3-point correlations for $\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 = 0$ and reducing B to a function of scalar wave numbers $(k_1,\ k_2)$ and triangle angle θ.

$$B(k_1, k_2, \theta) = \langle \tilde{f}(\mathbf{k}_1)\tilde{f}(\mathbf{k}_2)\tilde{f}^*(\mathbf{k}_1 + \mathbf{k}_2)\rangle$$

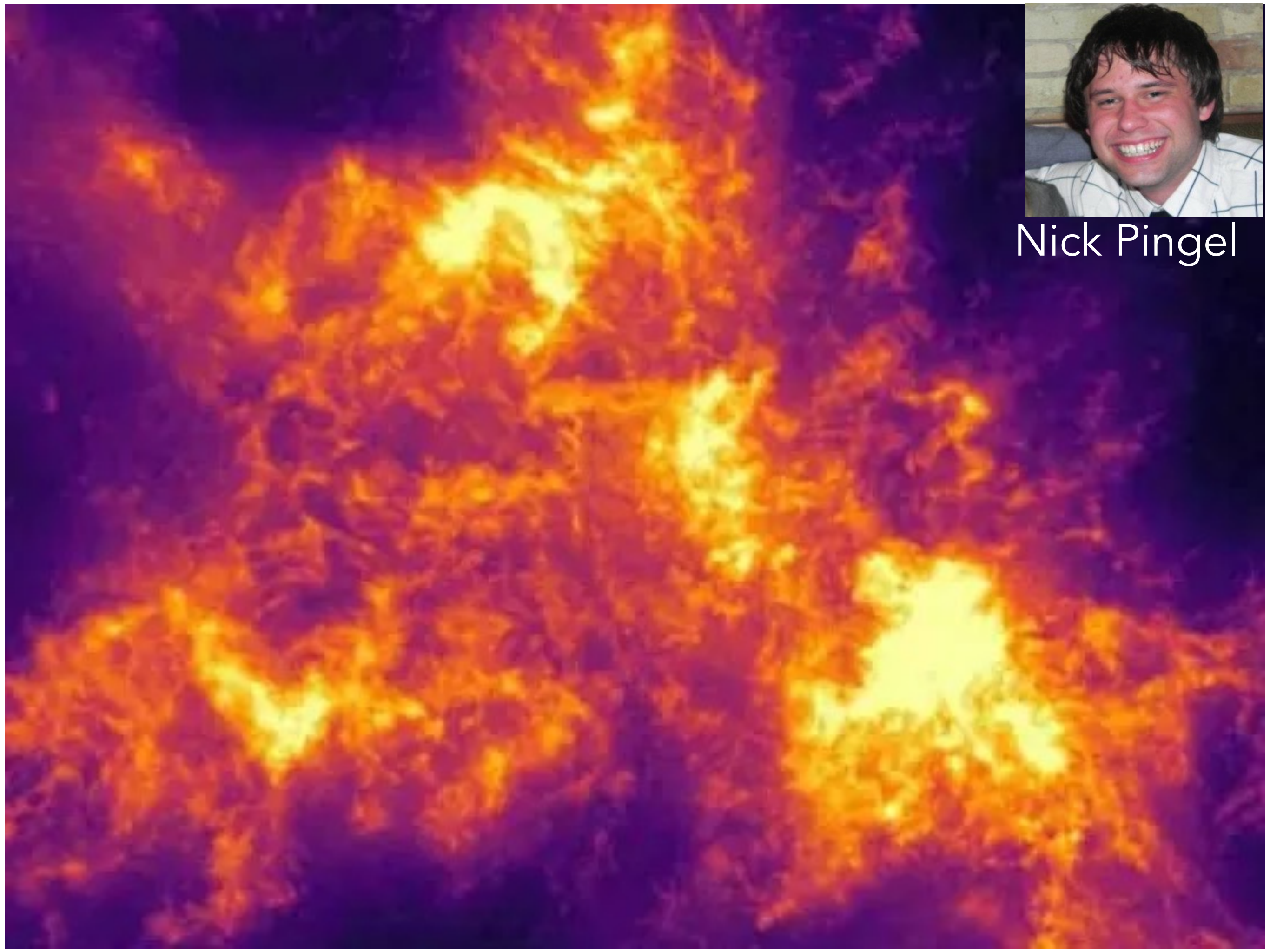
Michael O'Brien

(a)



Bispectrum is sensitive to turbulence driving scale in column density maps!

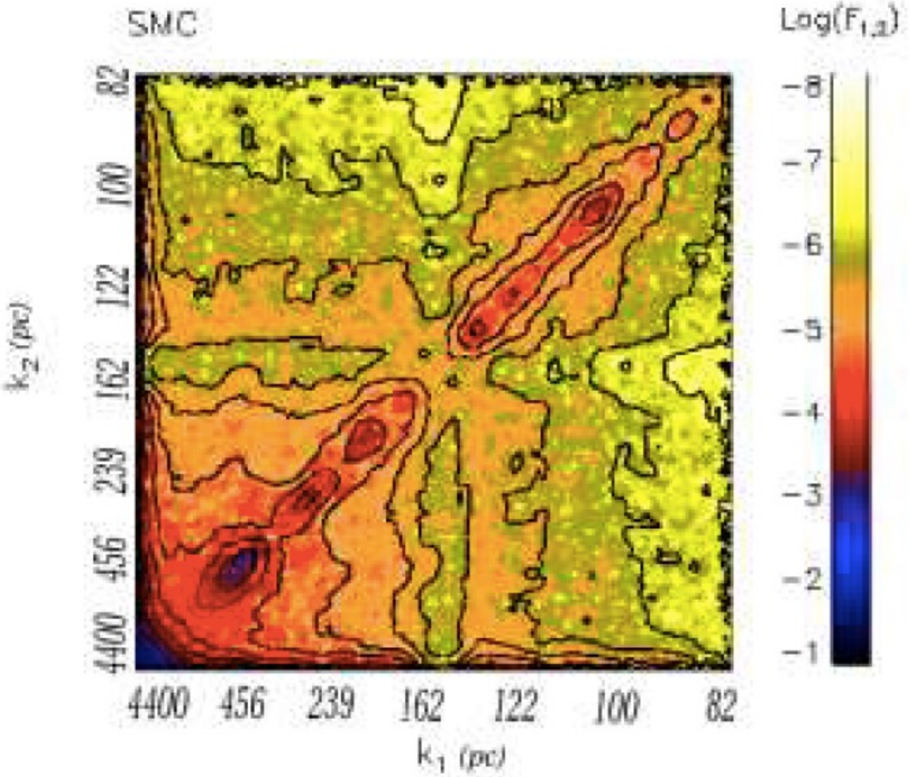
Bispectrum: Application to the Small Magellanic Cloud (SMC)



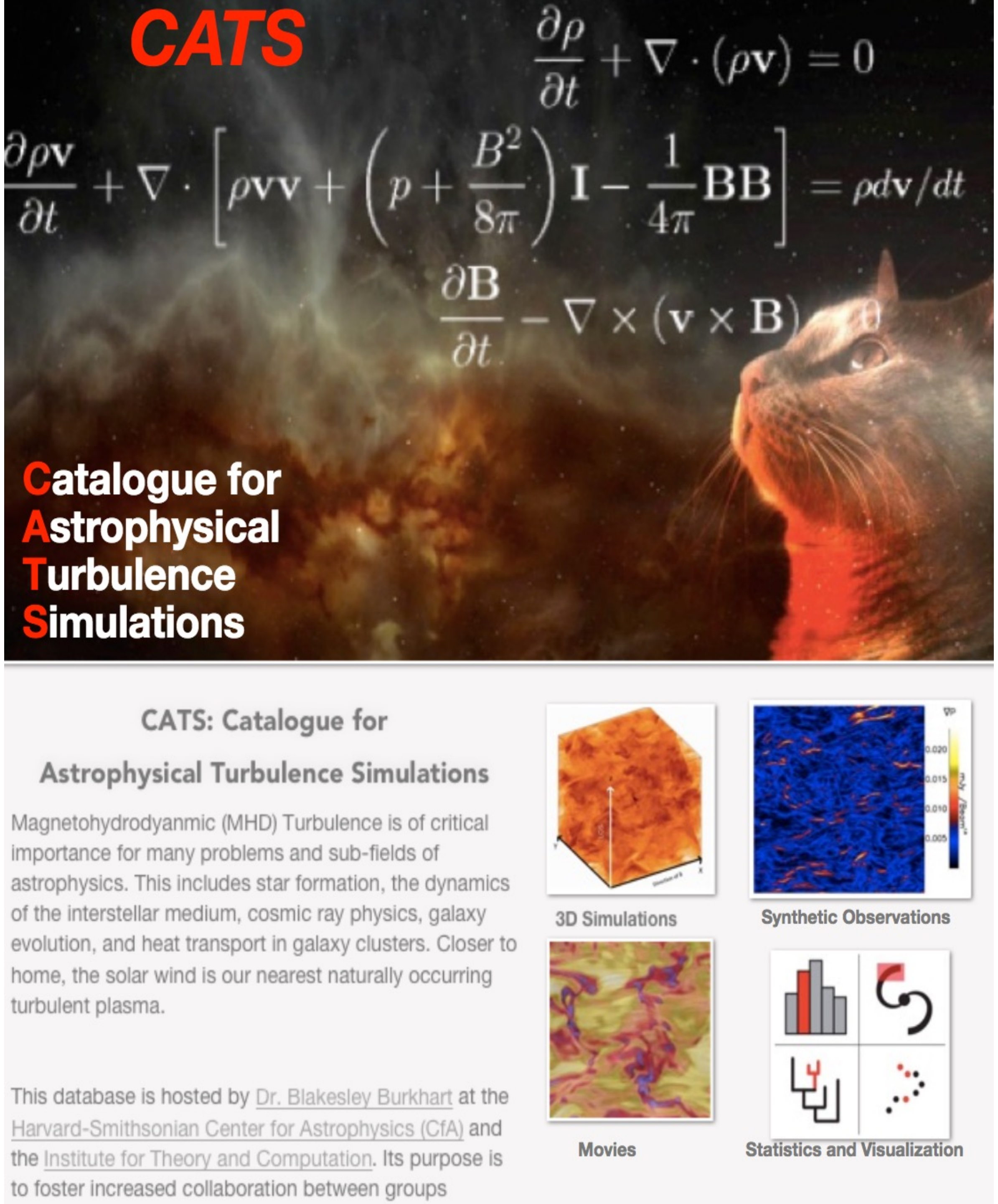
Nick Pingel

SMC in 21 cm emission

- SMC has large and intermediate scale injection features.
- Multiple drivers of turbulence
- Intermediate scale corresponds to mean HI supershell radius



Burkhart et al. 2010



$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

$$\frac{\partial \rho \mathbf{v}}{\partial t} + \nabla \cdot \left[\rho \mathbf{v}\mathbf{v} + \left(p + \frac{B^2}{8\pi} \right) \mathbf{I} - \frac{1}{4\pi} \mathbf{B}\mathbf{B} \right] = \rho d\mathbf{v}/dt$$

$$\frac{\partial \mathbf{B}}{\partial t} - \nabla \times (\mathbf{v} \times \mathbf{B}) = 0$$

CATS: Catalogue for Astrophysical Turbulence Simulations

Magnetohydrodyanmic (MHD) Turbulence is of critical importance for many problems and sub-fields of astrophysics. This includes star formation, the dynamics of the interstellar medium, cosmic ray physics, galaxy evolution, and heat transport in galaxy clusters. Closer to home, the solar wind is our nearest naturally occurring turbulent plasma.

This database is hosted by Dr. Blakesley Burkhart at the Harvard-Smithsonian Center for Astrophysics (CfA) and the Institute for Theory and Computation. Its purpose is to foster increased collaboration between groups

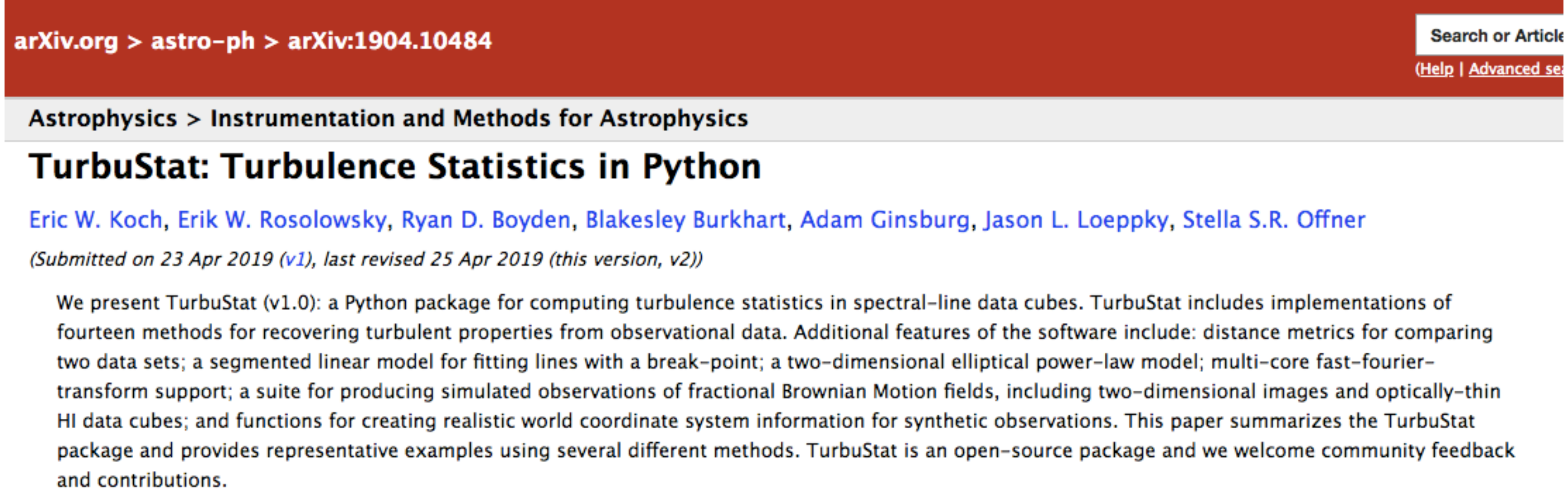
The Catalogue for Astrophysical Turbulence Simulations

www.MHDturbulence.com

Includes simulations & simulated observations from codes:

- AREPO
- Enzo
- Godunov
- Athena++
- FLASH

Links to visualization and statistical tools for studies of turbulence that can be applied to turbulence in astrophysical environments (observations and simulations).



arXiv.org > astro-ph > arXiv:1904.10484

Astrophysics > Instrumentation and Methods for Astrophysics

TurbuStat: Turbulence Statistics in Python

Eric W. Koch, Erik W. Rosolowsky, Ryan D. Boyden, Blakesley Burkhart, Adam Ginsburg, Jason L. Loeppky, Stella S.R. Offner

(Submitted on 23 Apr 2019 (v1), last revised 25 Apr 2019 (this version, v2))

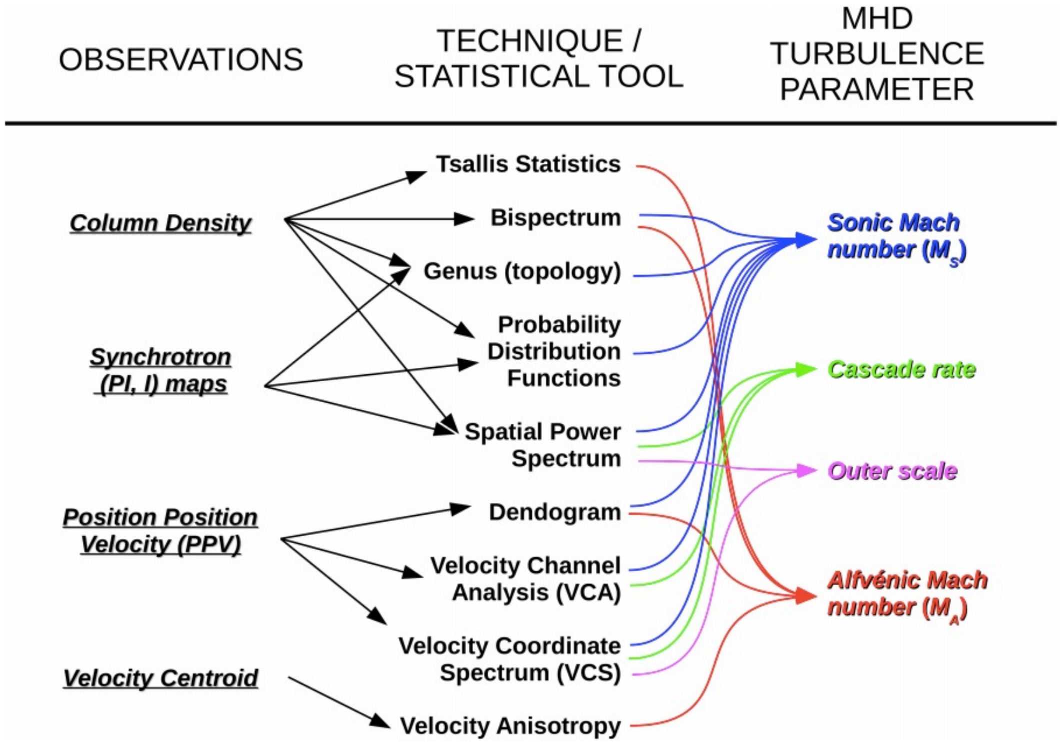
We present TurbuStat (v1.0): a Python package for computing turbulence statistics in spectral-line data cubes. TurbuStat includes implementations of fourteen methods for recovering turbulent properties from observational data. Additional features of the software include: distance metrics for comparing two data sets; a segmented linear model for fitting lines with a break-point; a two-dimensional elliptical power-law model; multi-core fast-fourier-transform support; a suite for producing simulated observations of fractional Brownian Motion fields, including two-dimensional images and optically-thin HI data cubes; and functions for creating realistic world coordinate system information for synthetic observations. This paper summarizes the TurbuStat package and provides representative examples using several different methods. TurbuStat is an open-source package and we welcome community feedback and contributions.

Comments:	Accepted in AJ. 21 pages, 8 figures
Subjects:	**Instrumentation and Methods for Astrophysics (astro-ph.IM)**
Cite as:	arXiv:1904.10484 [astro-ph.IM]
	(or arXiv:1904.10484v2 [astro-ph.IM] for this version)

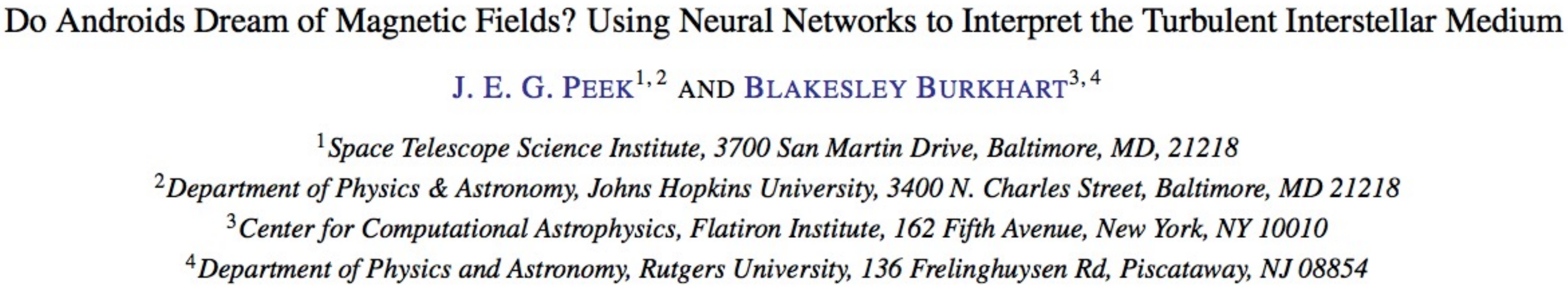
Submission history

From: Eric Koch [view email]

turbustat.readthedocs.io



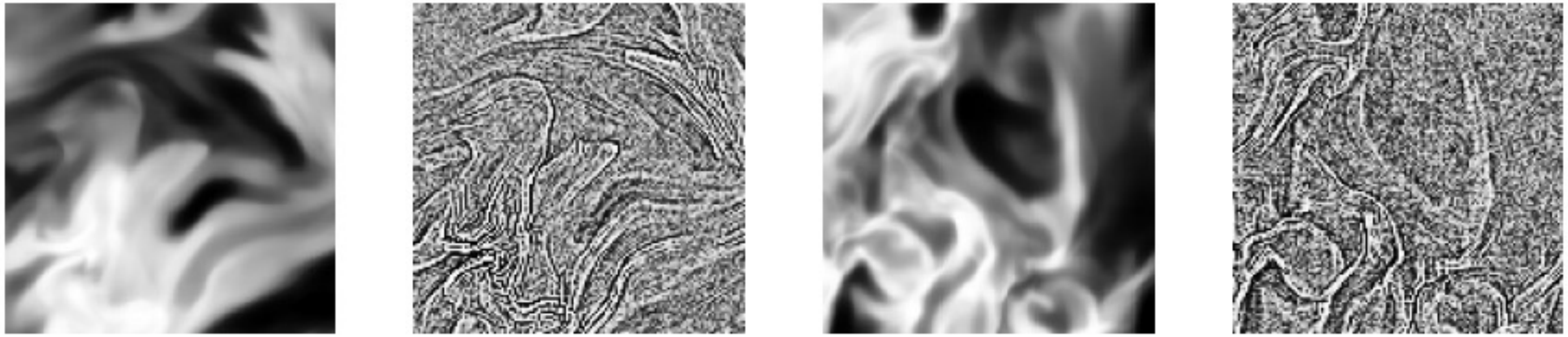
Using Convolutional Neural Networks (CNNs): interpret magnetic fields and phase information in turbulent flows



Do Androids Dream of Magnetic Fields? Using Neural Networks to Interpret the Turbulent Interstellar Medium

J. E. G. PEEK[1,2] AND BLAKESLEY BURKHART[3,4]

[1] *Space Telescope Science Institute, 3700 San Martin Drive, Baltimore, MD, 21218*
[2] *Department of Physics & Astronomy, Johns Hopkins University, 3400 N. Charles Street, Baltimore, MD 21218*
[3] *Center for Computational Astrophysics, Flatiron Institute, 162 Fifth Avenue, New York, NY 10010*
[4] *Department of Physics and Astronomy, Rutgers University, 136 Frelinghuysen Rd, Piscataway, NJ 08854*



sub-Alfvénic

sub-Alfvénic
Fixed Fourier Power

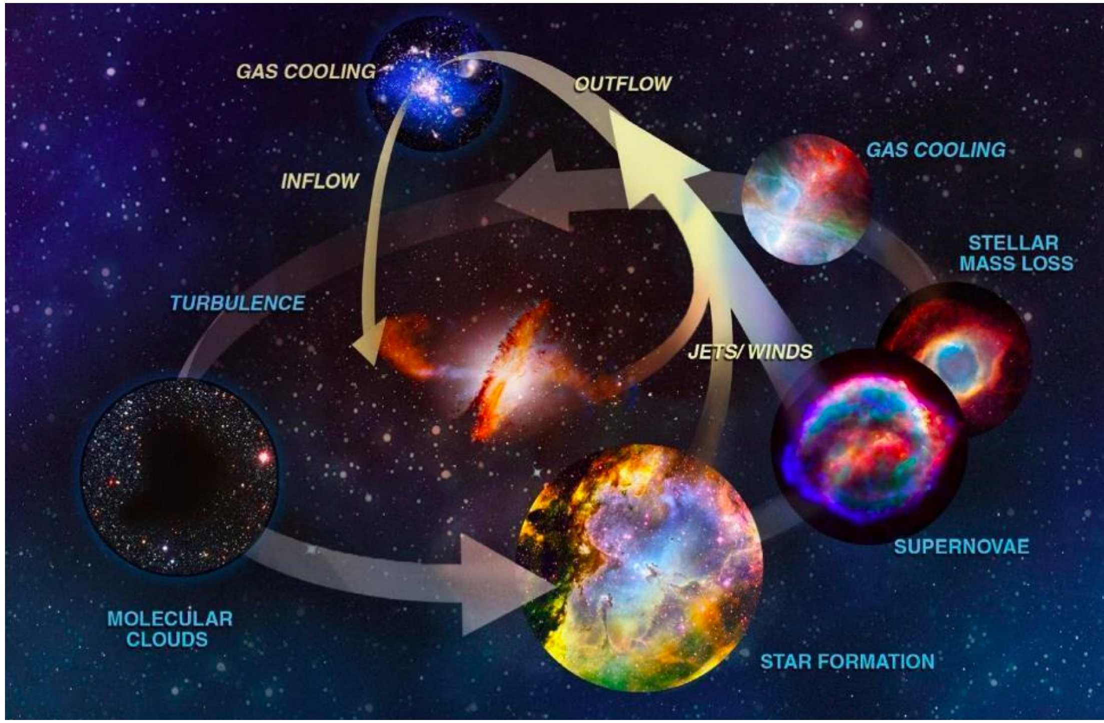
super-Alfvénic

super-Alfvénic
Fixed Fourier Power

Github.com/deepskies/deepmhd

MHD Turbulence..

- What is turbulence?
- Hydro: Kolmogorov 41
- MHD (1995): GS95 and Critical Balance
- MHD (2006): Dynamic Alignment
- Intermittency
- Reconnection
- Compressibility
- Diagnostics
- Star formation self-regulation via turbulence and feedback

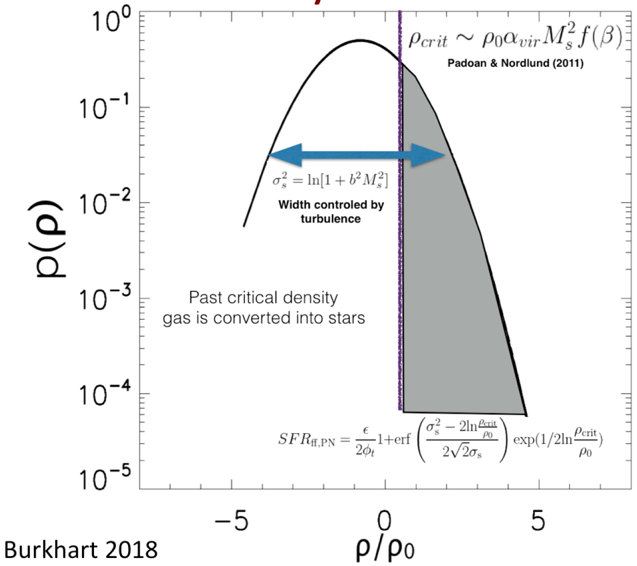


The turbulent **density Probability Distribution Function (PDF)** is key aspect of analytic theories of the cloud scale SFE

- Initial mass function — Padoan & Nordlund02, Hennebelle & Chabrier08, 09, 12, Elmegreen11, Hopkins 12, Veltchev+12,
- Star formation efficiency — Elmegreen08, Federrath & Klessen13, Girichidis+14
- Star formation rate — Krumholz & Mckee05, Padoan & Nordlund11, Renaud12, Fedderath & Klessen12, Gribel+17
- The Kennicutt-Schmidt relation — Elmegreen02, Krumholz & Mckee05, Tassis07, Zamora-Aviles12,14 Fedderath13

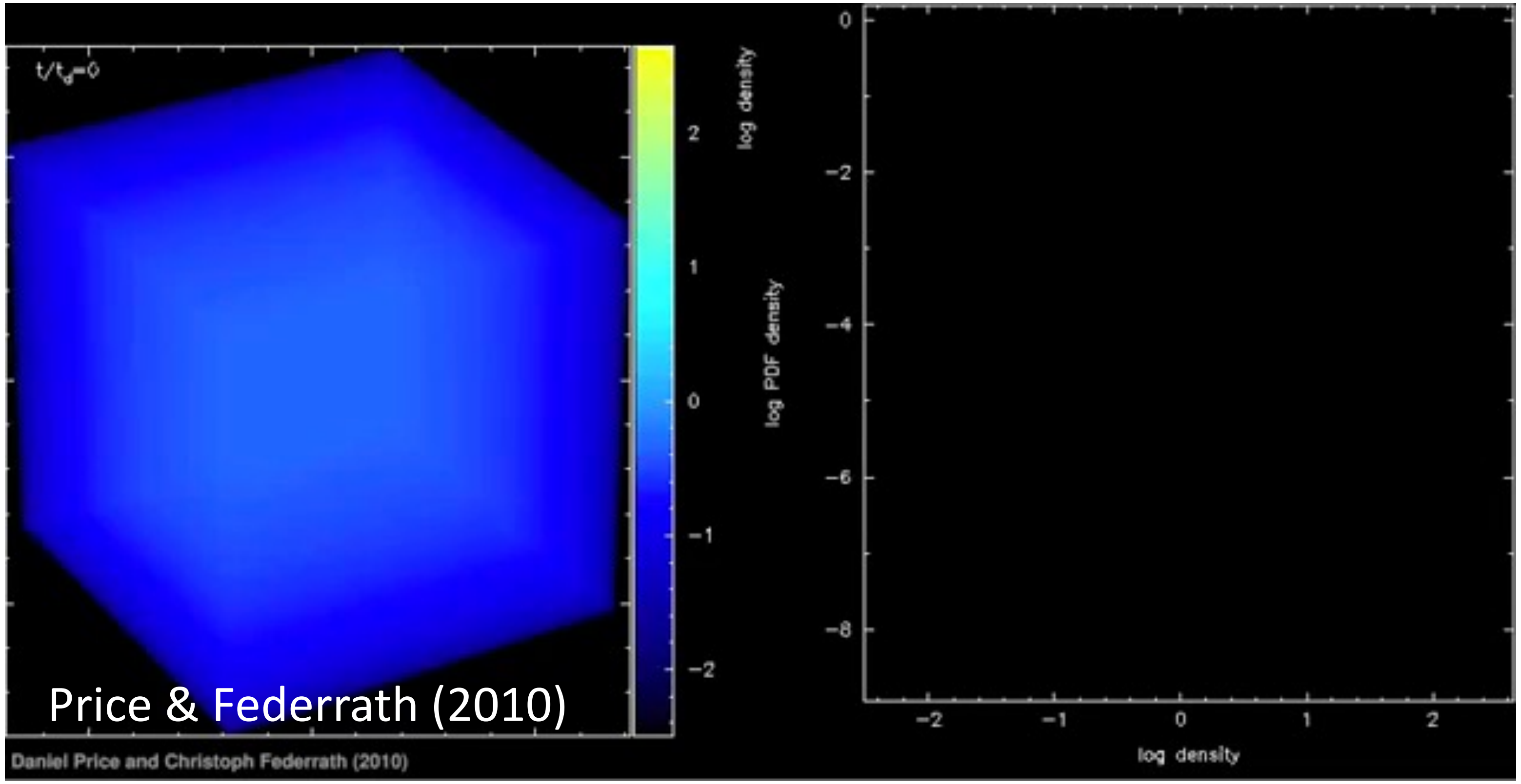
Are all based on integrals over the lognormal turbulent density PDF

$$\boxed{SFR_{ff} = \frac{\epsilon}{\phi_t}\int_{scrit}^{\infty}\frac{t_{ff}(\rho_0)}{t_{ff}(\rho)}\frac{\rho}{\rho_0}p(s)ds}$$



Burkhart 2018

The Probability Distribution Function (PDF) of turbulence is lognormal



Price & Federrath (2010)

The interaction of multiple shocks in supersonic turbulence leads naturally to a log-normal probability distribution of density.

Width of the lognormal density PDF is related to the sonic Mach number (M_s)

$$\sigma_s^2 = ln(1 + b^2 M_s^2)$$

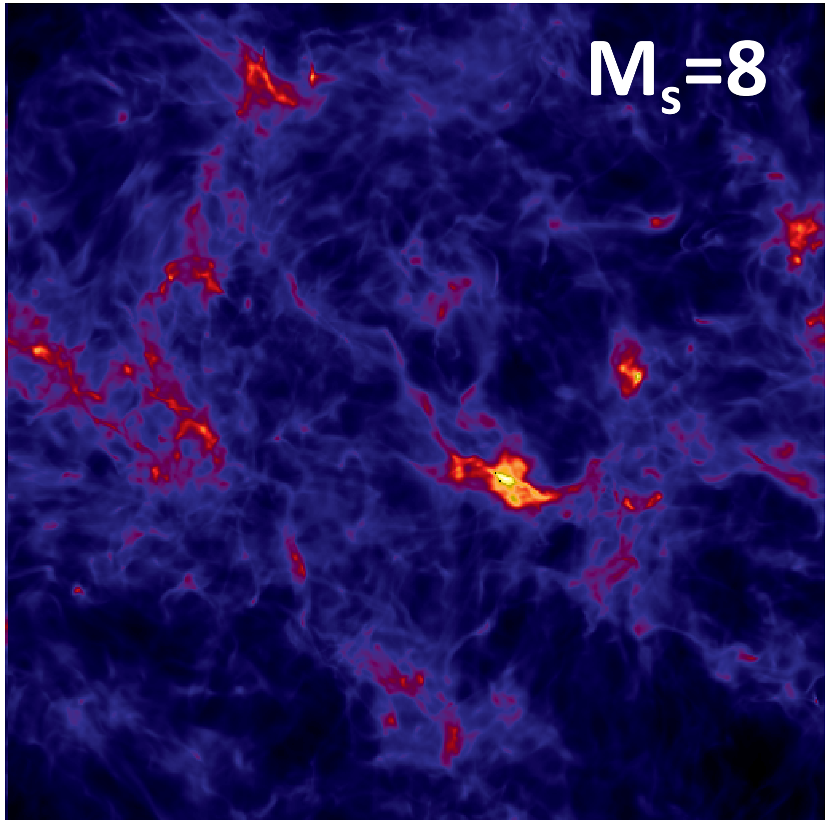
$$\boxed{\sigma^2_{\rho/\rho_0} = b^2 M^2}$$

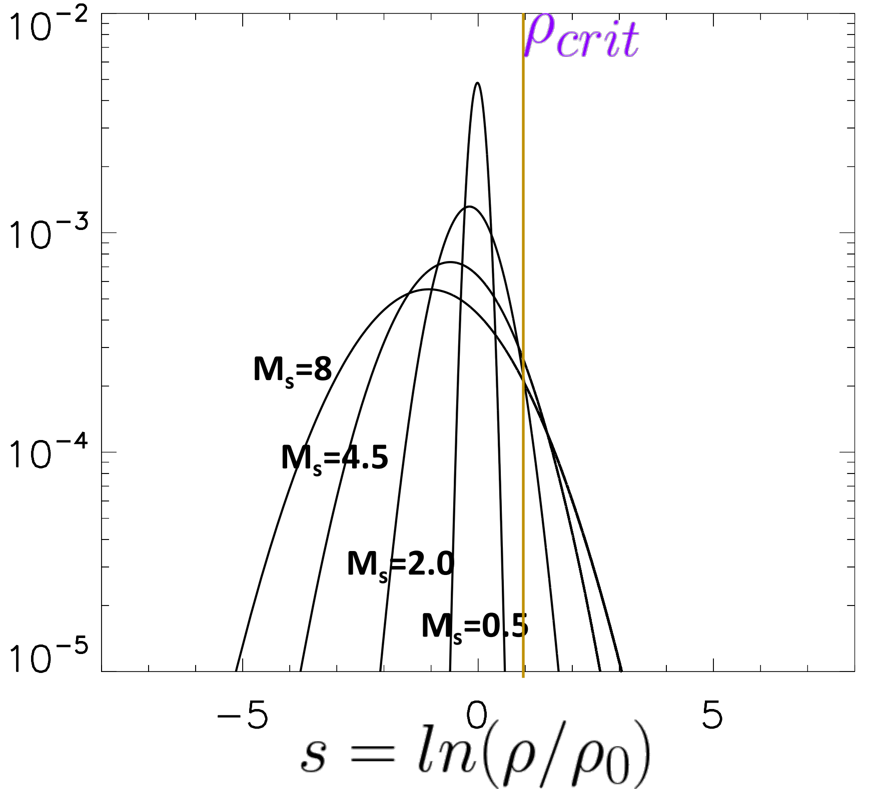
3D normalised density variance

numerically determined parameter

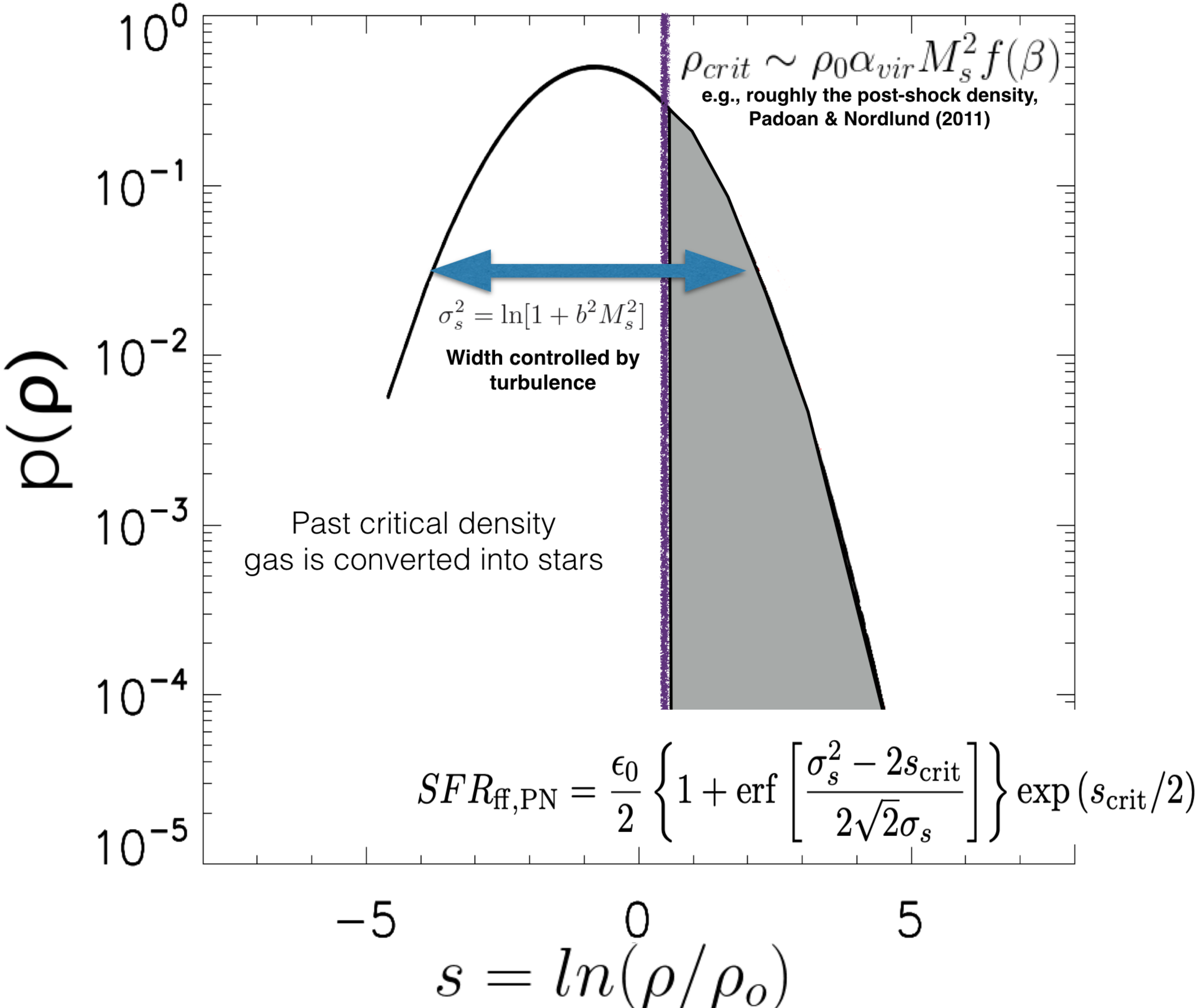
3D rms Mach number

Federrath et al. 2008; Burkhart & Lazarian 2012





Turbulence Regulated Star Formation Theories



$\rho_{crit} \sim \rho_0 \alpha_{vir} M_s^2 f(\beta)$

e.g., roughly the post-shock density, Padoan & Nordlund (2011)

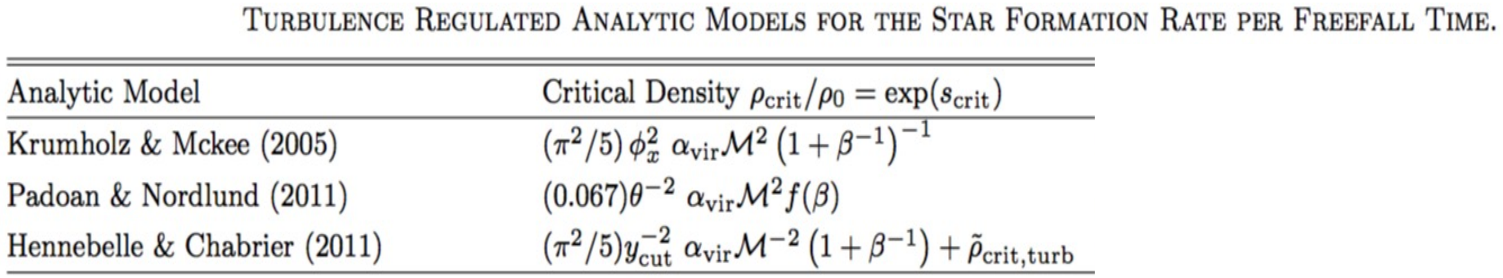
$\sigma_s^2 = \ln[1 + b^2 M_s^2]$

Width controlled by turbulence

Past critical density gas is converted into stars

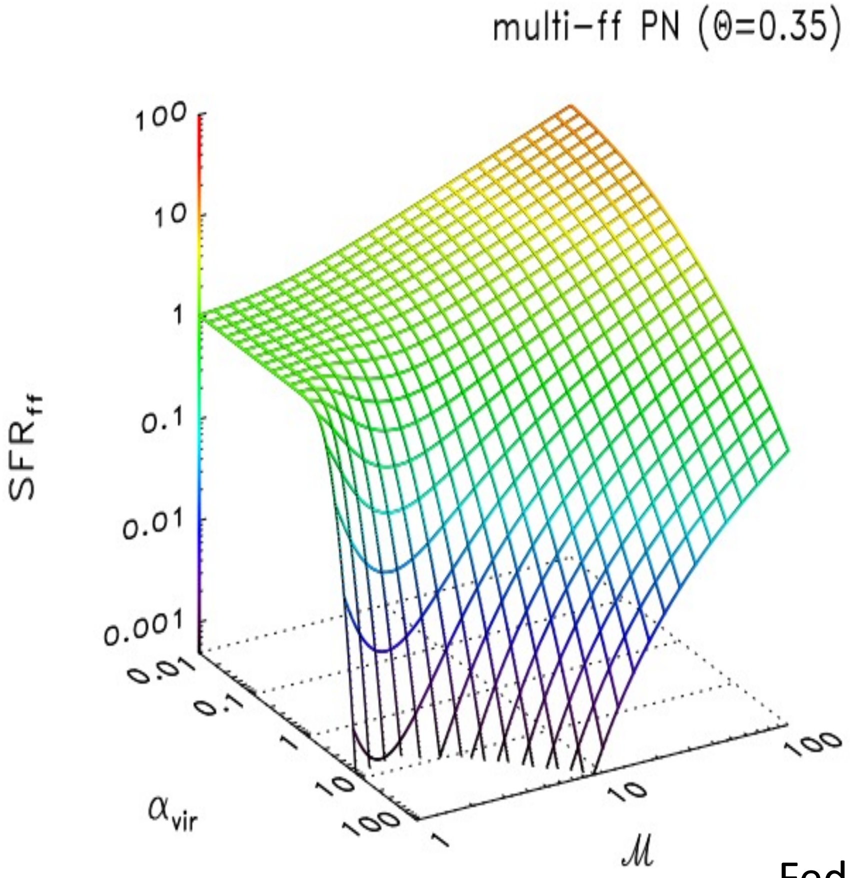
$$SFR_{\mathrm{ff,PN}} = \frac{\epsilon_0}{2}\left\{1+\mathrm{erf}\left[\frac{\sigma_s^2 - 2s_{\mathrm{crit}}}{2\sqrt{2}\sigma_s}\right]\right\}\exp\left(s_{\mathrm{crit}}/2\right)$$

Turbulence Regulated Star Formation Theories



TURBULENCE REGULATED ANALYTIC MODELS FOR THE STAR FORMATION RATE PER FREEFALL TIME.

Analytic Model	Critical Density $\rho_{\rm crit}/\rho_0 = \exp(s_{\rm crit})$
Krumholz & Mckee (2005)	$(\pi^2/5)\,\phi_x^2\,\alpha_{\rm vir}\mathcal{M}^2\,(1+\beta^{-1})^{-1}$
Padoan & Nordlund (2011)	$(0.067)\theta^{-2}\,\alpha_{\rm vir}\mathcal{M}^2 f(\beta)$
Hennebelle & Chabrier (2011)	$(\pi^2/5)y_{\rm cut}^{-2}\,\alpha_{\rm vir}\mathcal{M}^{-2}\,(1+\beta^{-1})+\tilde{\rho}_{\rm crit,turb}$



multi−ff PN (Θ=0.35)

Federrath & Klessen 2012

Predictions and features:

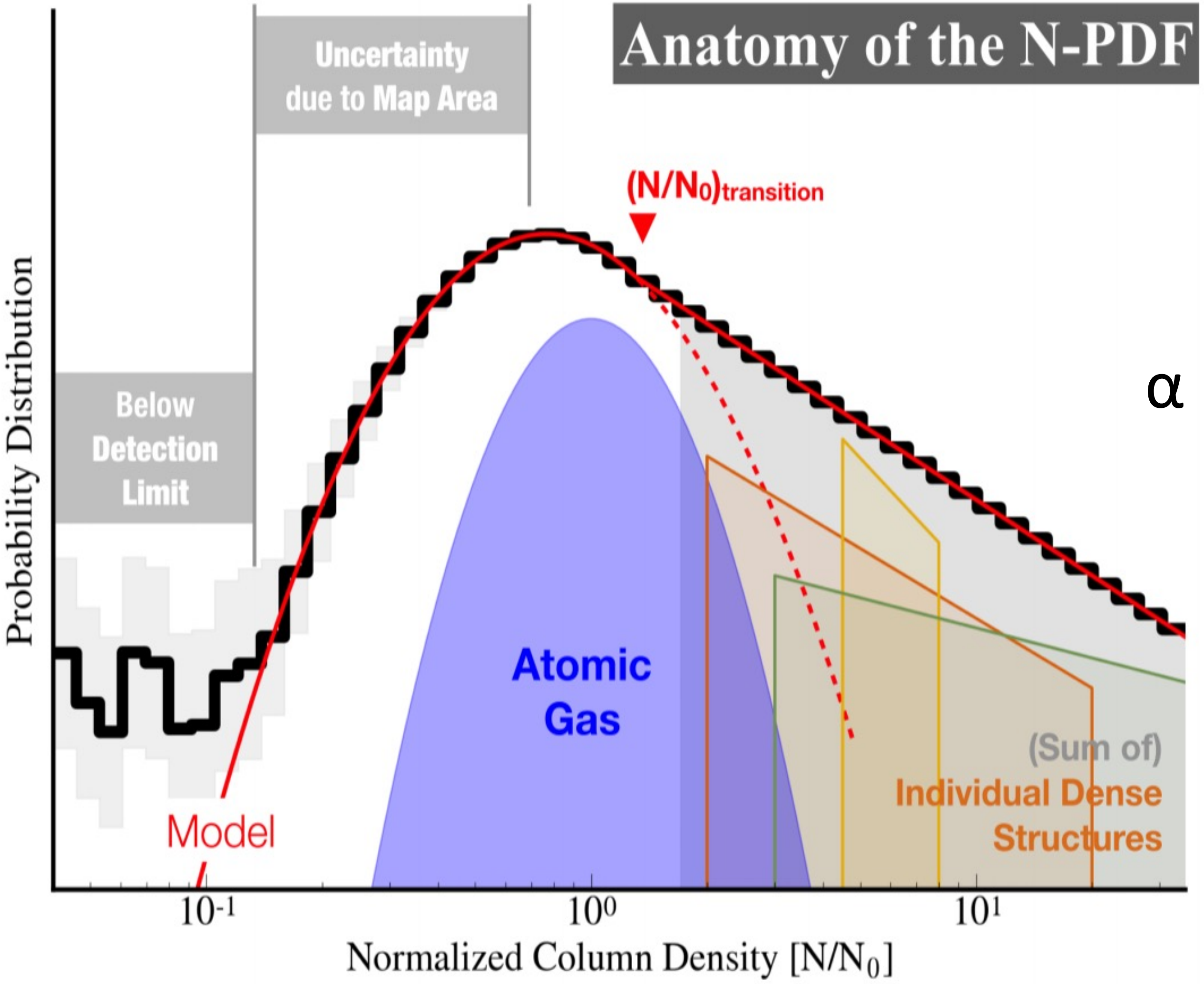
Higher SFR$_{\rm ff}$ with increased turbulence.

Constant SFR$_{\rm ff}$ if turbulent parameters stay constant.

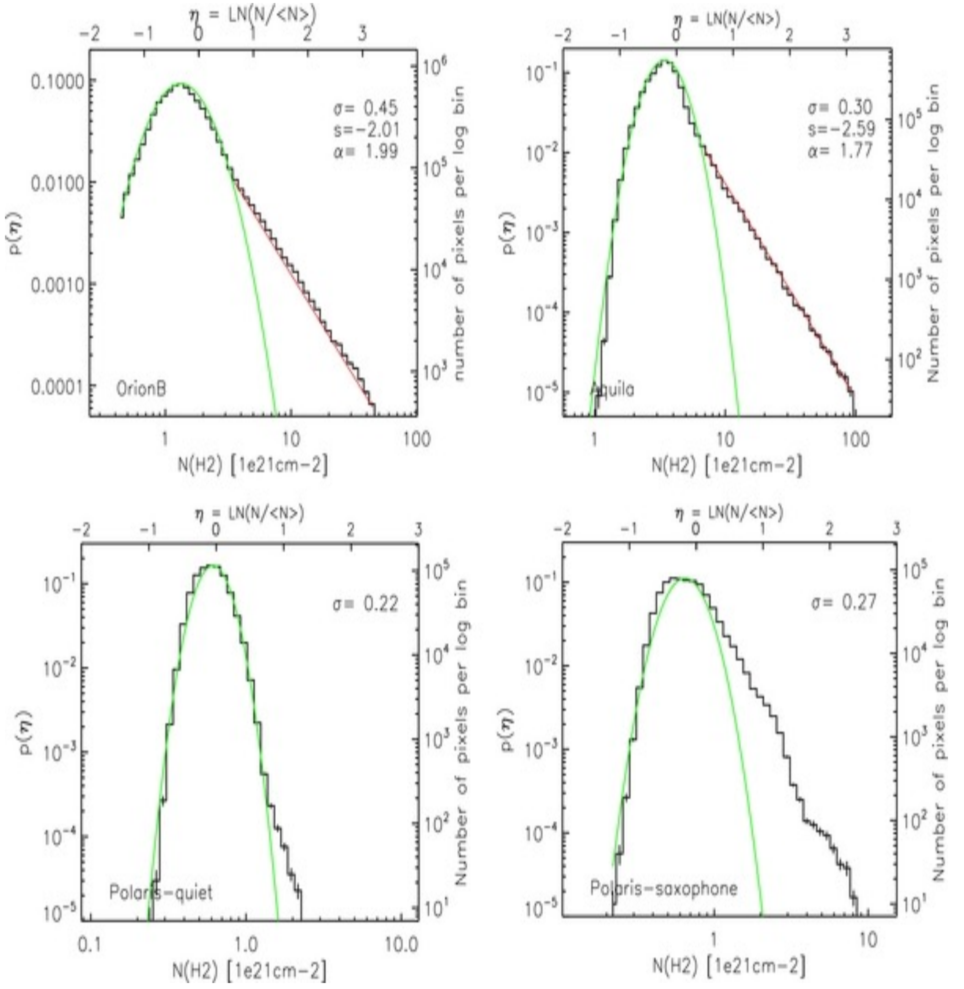
Critical density for collapse depends on a number of parameters of order unity.

The density PDF in star forming regions has power law which traces collapsing gas

Chen, Burkhart, Goodman & Collins 2018



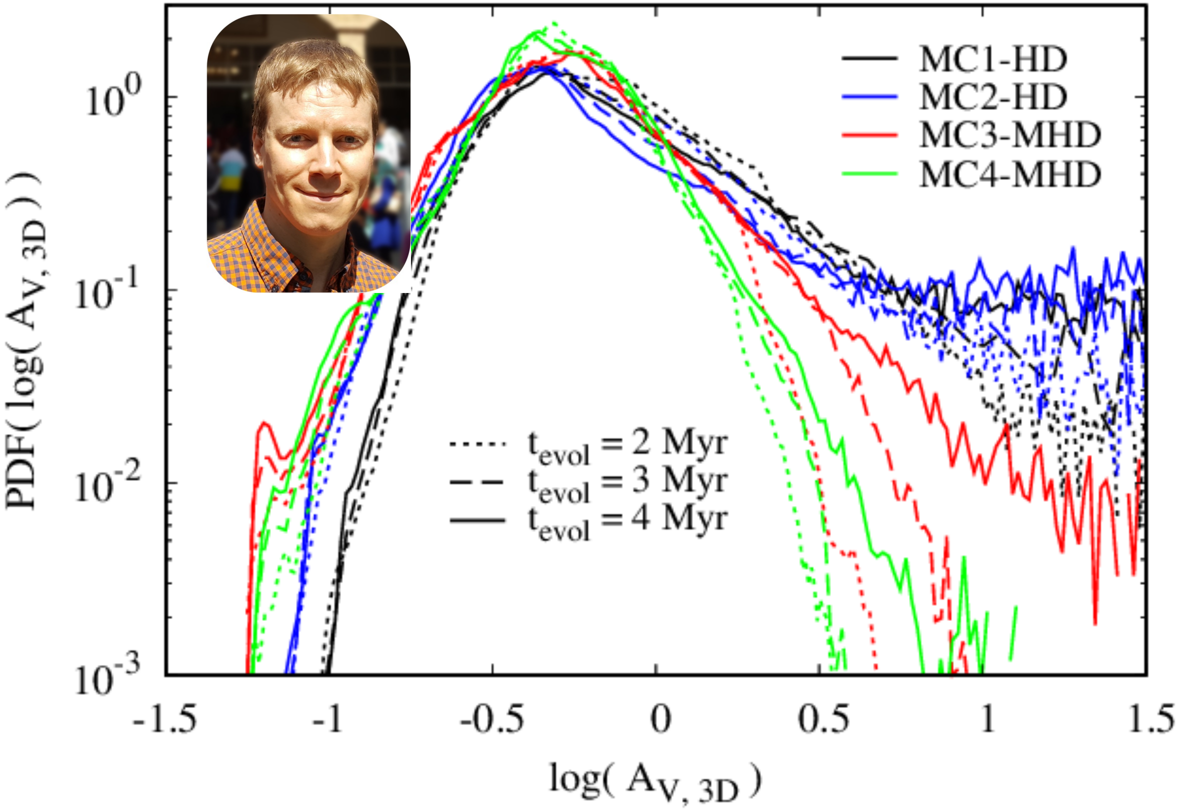
α



Herschel observations of Schneider et al. 2014, 2015

PDF: collapse vs turbulence

Daniel Seifried



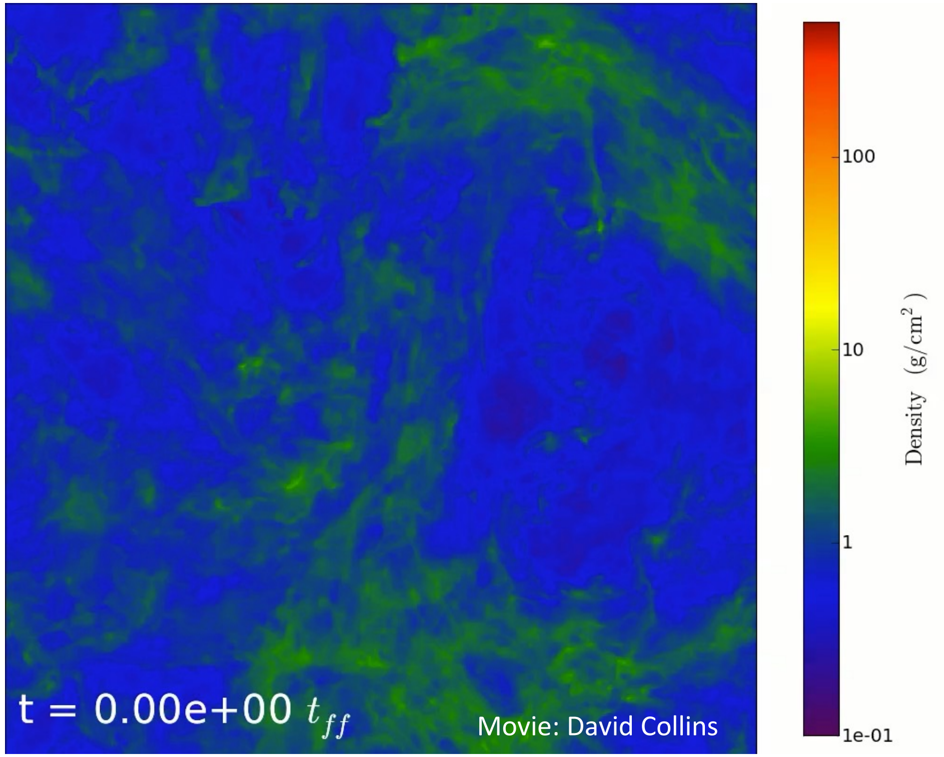
Philipp Girichidis

Power law slopes become more flat (shallower) as the cloud collapses

t=0 supersonic turbulence

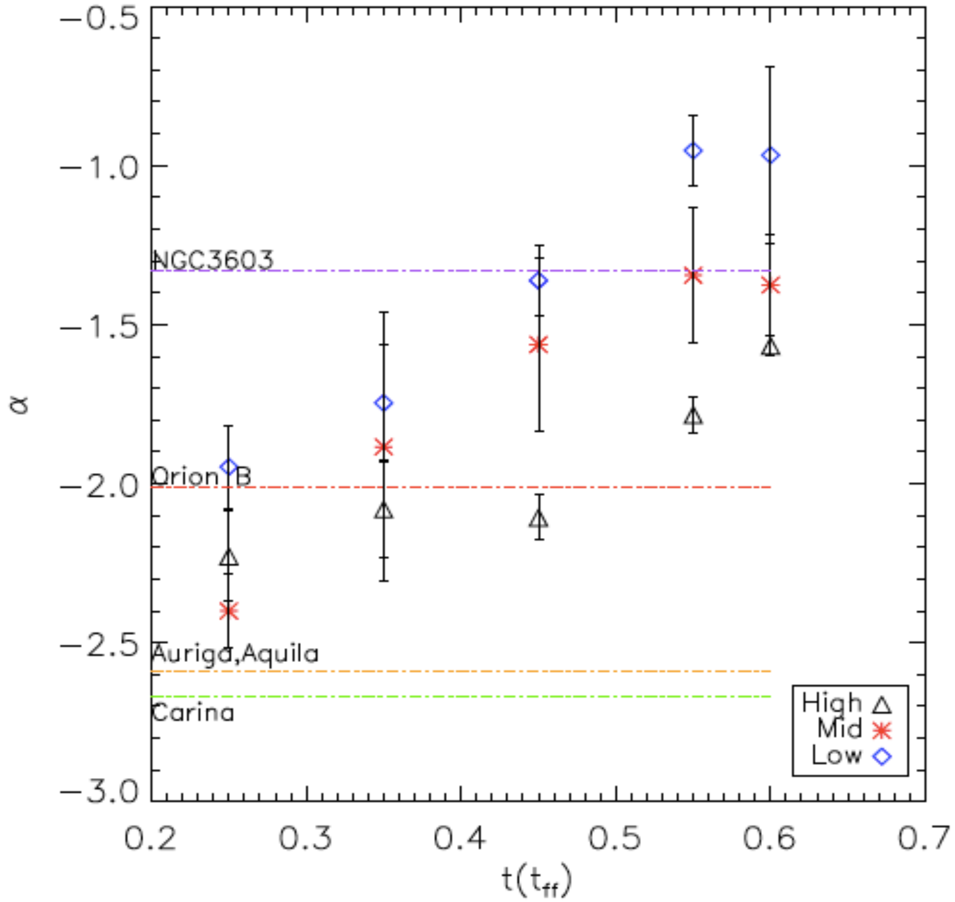
t>0 includes self-gravity

Collins et al. 2012;
Burkhart, Collins & Lazarian 2015



Movie: David Collins

Power law PDF slopes vs. time



Let’s update the turbulence regulated star formation theories to include gravity/feedback! Consider a piecewise density PDF….

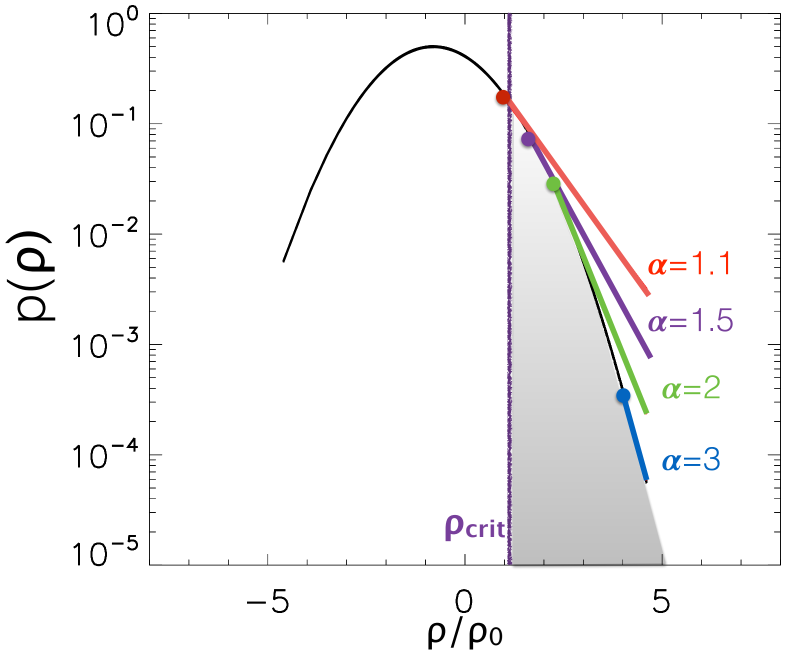
Burkhart, Collins & Stalpes 2017

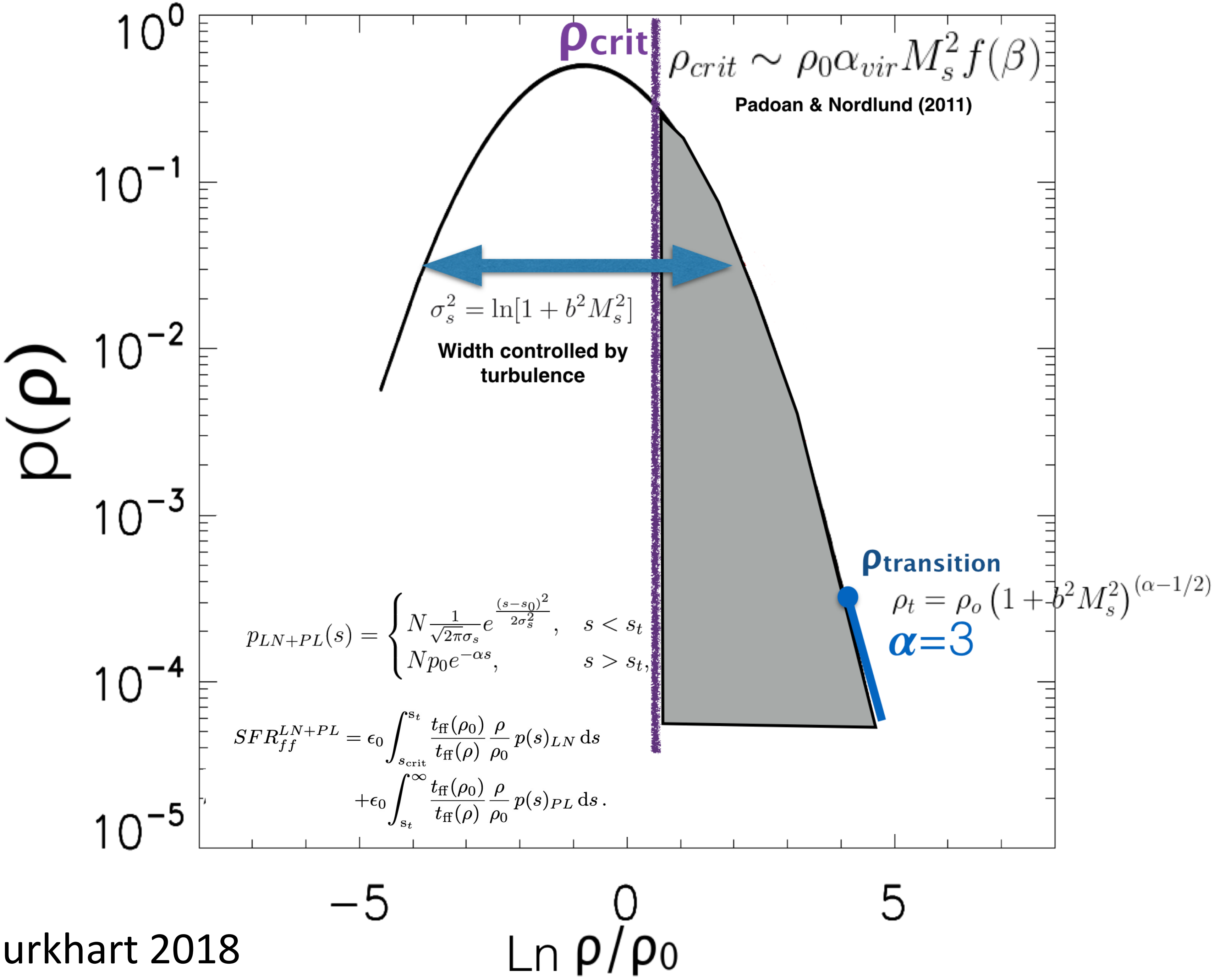
$$p_{LN+PL}(s) = \begin{cases} N\frac{1}{\sqrt{2\pi}\sigma_s}e^{\frac{(s-s_0)^2}{2\sigma_s^2}}, & s < s_t \\ Np_0e^{-\alpha s}, & s > s_t, \end{cases} \qquad s_t = ln(\rho_t/\rho_0)$$

The conditions of continuity and differentiability allow us to solve for:

$$p_0 = \frac{e^{1/2(\alpha-1)\alpha\sigma_s^2}}{\sigma_s\sqrt{2\pi}}$$

$$s_t = (\alpha - 1/2)\sigma_s^2$$





$$\rho_{crit} \sim \rho_0 \alpha_{vir} M_s^2 f(\beta)$$

Padoan & Nordlund (2011)

$$\sigma_s^2 = \ln[1 + b^2 M_s^2]$$

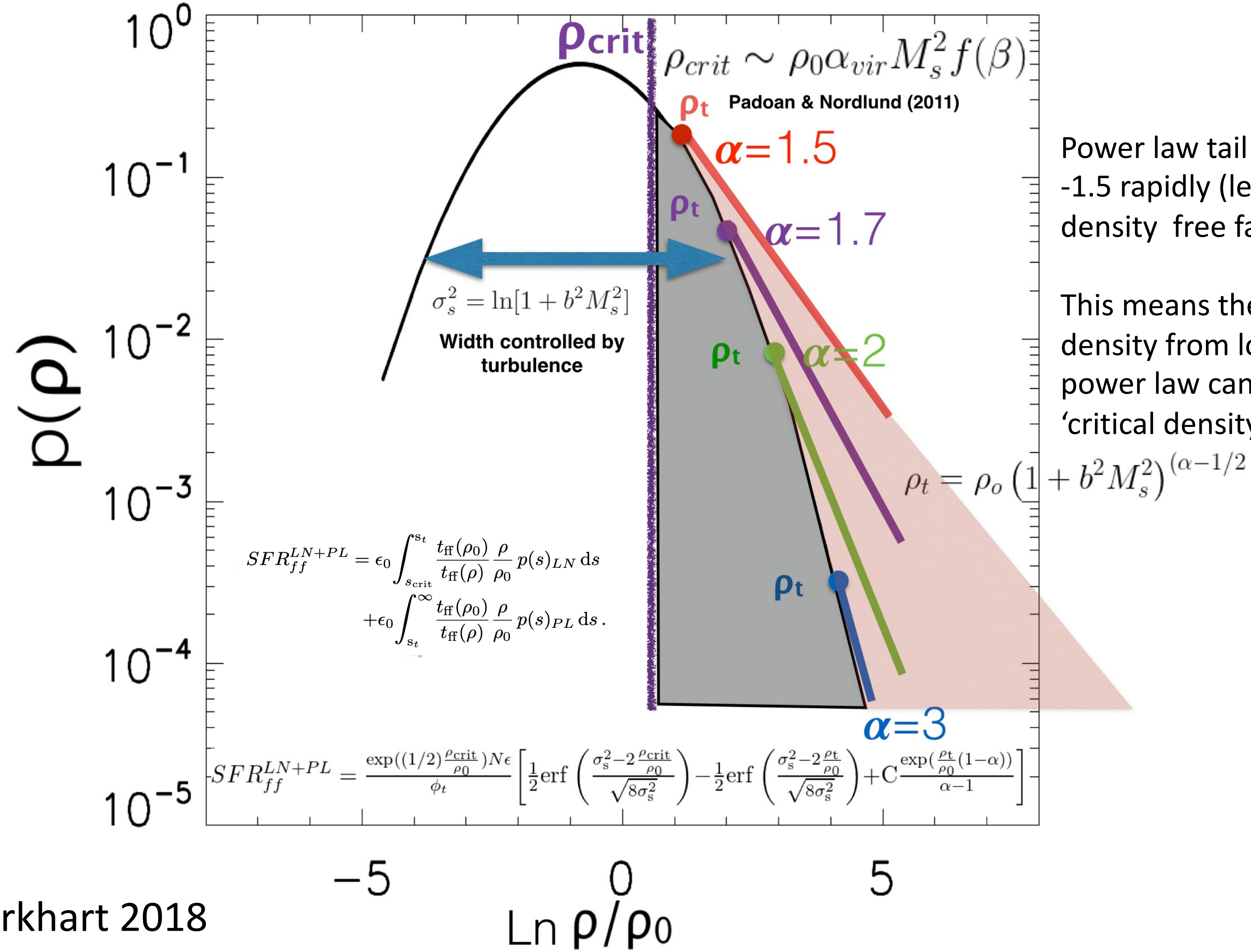
Width controlled by turbulence

$$\rho_t = \rho_o \left(1 + b^2 M_s^2\right)^{(\alpha - 1/2)}$$

$$p_{LN+PL}(s) = \begin{cases} N \frac{1}{\sqrt{2\pi}\sigma_s} e^{\frac{(s-s_0)^2}{2\sigma_s^2}}, & s < s_t \\ N p_0 e^{-\alpha s}, & s > s_t, \end{cases}$$

$$SFR_{ff}^{LN+PL} = \epsilon_0 \int_{s_{\rm crit}}^{\rm s_t} \frac{t_{\rm ff}(\rho_0)}{t_{\rm ff}(\rho)} \frac{\rho}{\rho_0} p(s)_{LN} \, {\rm d}s$$
$$+ \epsilon_0 \int_{\rm s_t}^{\infty} \frac{t_{\rm ff}(\rho_0)}{t_{\rm ff}(\rho)} \frac{\rho}{\rho_0} p(s)_{PL} \, {\rm d}s \,.$$

Burkhart 2018



$$\rho_{crit} \sim \rho_0 \alpha_{vir} M_s^2 f(\beta)$$

$$\sigma_s^2 = \ln[1 + b^2 M_s^2]$$

$$\rho_t = \rho_o \left(1 + b^2 M_s^2\right)^{(\alpha - 1/2)}$$

$$SFR_{ff}^{LN+PL} = \epsilon_0 \int_{s_{\mathrm{crit}}}^{s_t} \frac{t_{\mathrm{ff}}(\rho_0)}{t_{\mathrm{ff}}(\rho)} \frac{\rho}{\rho_0} p(s)_{LN}\,\mathrm{d}s + \epsilon_0 \int_{s_t}^{\infty} \frac{t_{\mathrm{ff}}(\rho_0)}{t_{\mathrm{ff}}(\rho)} \frac{\rho}{\rho_0} p(s)_{PL}\,\mathrm{d}s\,.$$

$$SFR_{ff}^{LN+PL} = \frac{\exp((1/2)\frac{\rho_{\mathrm{crit}}}{\rho_0})N\epsilon}{\phi_t}\left[\frac{1}{2}\mathrm{erf}\left(\frac{\sigma_s^2 - 2\frac{\rho_{\mathrm{crit}}}{\rho_0}}{\sqrt{8\sigma_s^2}}\right) - \frac{1}{2}\mathrm{erf}\left(\frac{\sigma_s^2 - 2\frac{\rho_t}{\rho_0}}{\sqrt{8\sigma_s^2}}\right) + \mathrm{C}\frac{\exp(\frac{\rho_t}{\rho_0}(1-\alpha))}{\alpha - 1}\right]$$

Power law tail reaches value of -1.5 rapidly (less than one mean density free fall time).

This means the transition density from lognormal to power law can be used as a 'critical density'.

Burkhart 2018

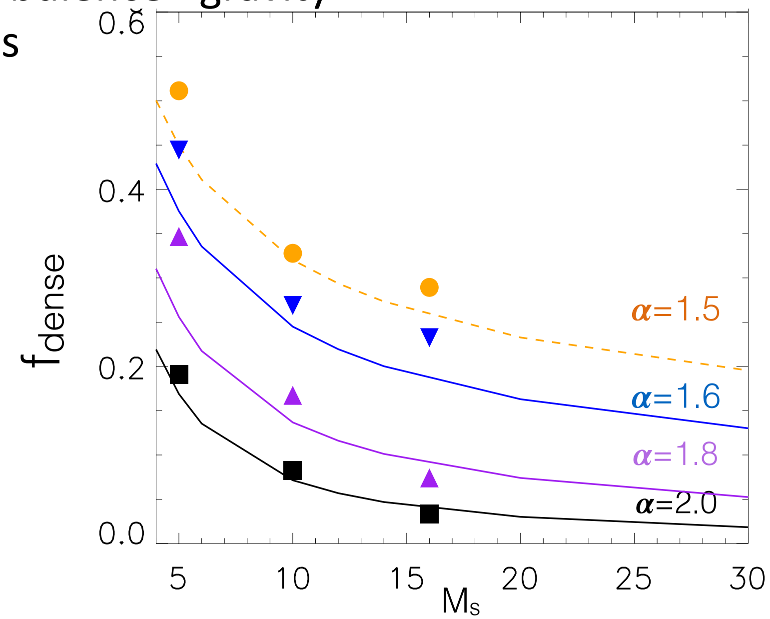
Model for dense gas fraction vs. comparison with simulations

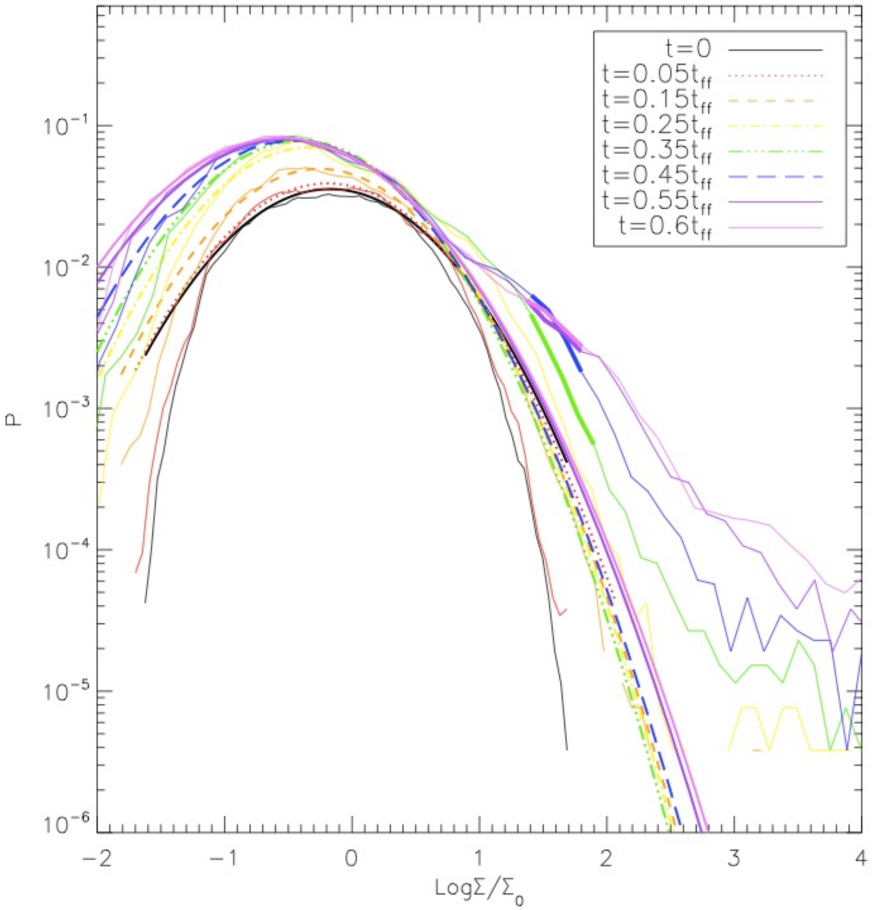
$$f_{\rm dense} = \frac{\int_{\rm s_t}^{\infty} \exp(\rm s)P_{PL}(s)\,ds}{\int_{-\infty}^{s_t}\exp(\rm s)P_{LN}(s)\,ds + \int_{\rm s_t}^{\infty}\exp(\rm s)P_{PL}(s)\,ds}$$

$$f_{\rm dense}(\alpha=1.5,\sigma_s) = \frac{e^{-\sigma_s^2/8}}{\sqrt{\frac{\pi}{8}}\sigma_s\left(1+\mathrm{erf}\left(\frac{\sigma_{\rm s}}{2\sqrt{2}}\right)\right)+e^{-\sigma_s^2/8}}$$

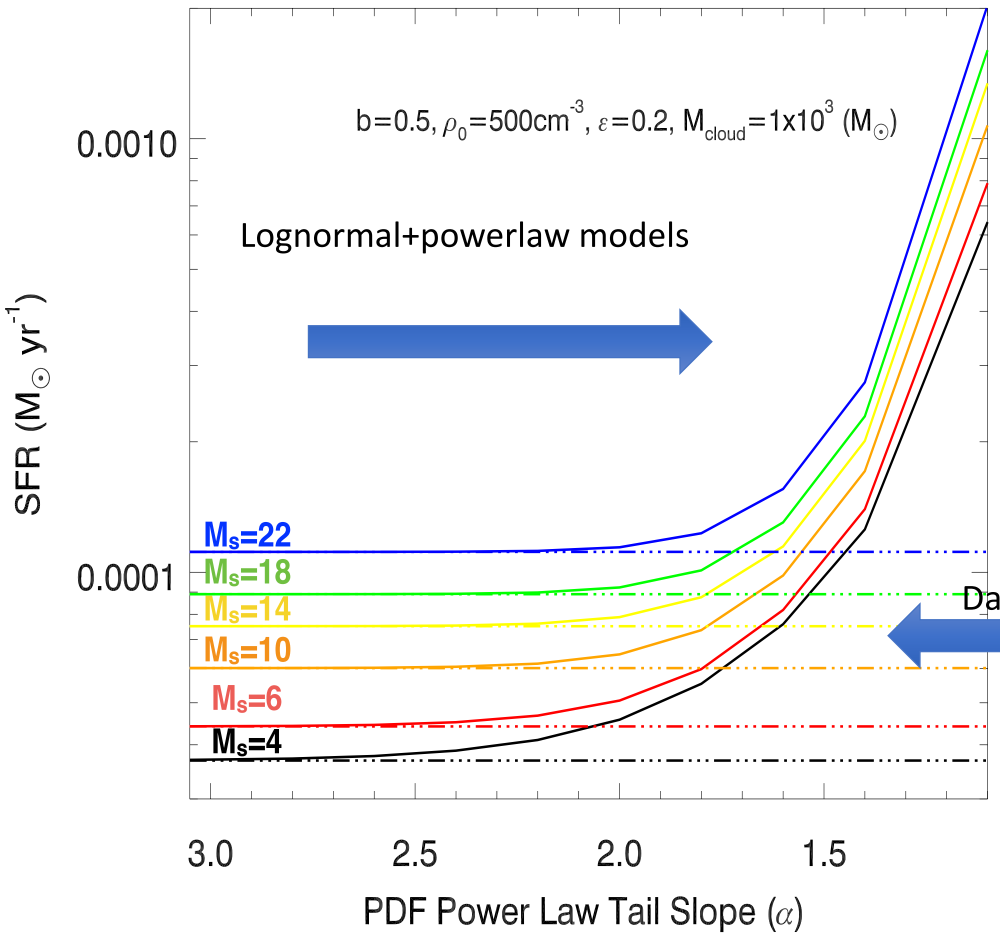
Burkhart, Collins & Lazarian 2015
Burkhart & Mocz 2019

Points: turbulence +gravity simulations





Lognormal-only (turbulence only) vs. lognormal+power law (turbulence + gravity)



Lognormal+powerlaw models

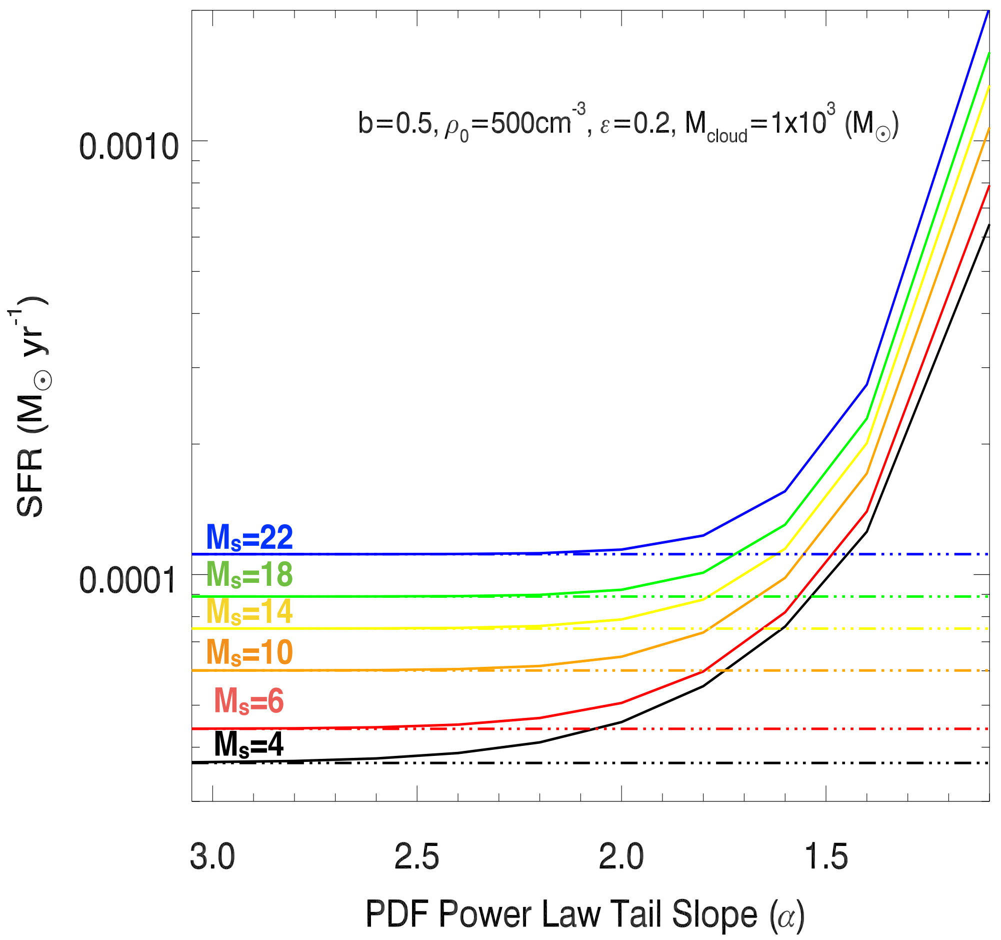
Dashed lines = Lognormal models

$$SFR_{ff}^{LN+PL} = \frac{\exp((1/2)s_{\rm crit})N\epsilon}{\phi_t}\left[\frac{1}{2}\text{erf}\left(\frac{\sigma_s^2-2s_{\rm crit}}{\sqrt{8\sigma_s^2}}\right) - \frac{1}{2}\text{erf}\left(\frac{\sigma_s^2-2s_t}{\sqrt{8\sigma_s^2}}\right) + C\frac{\exp(s_t(1-\alpha))}{\alpha-1}\right]$$

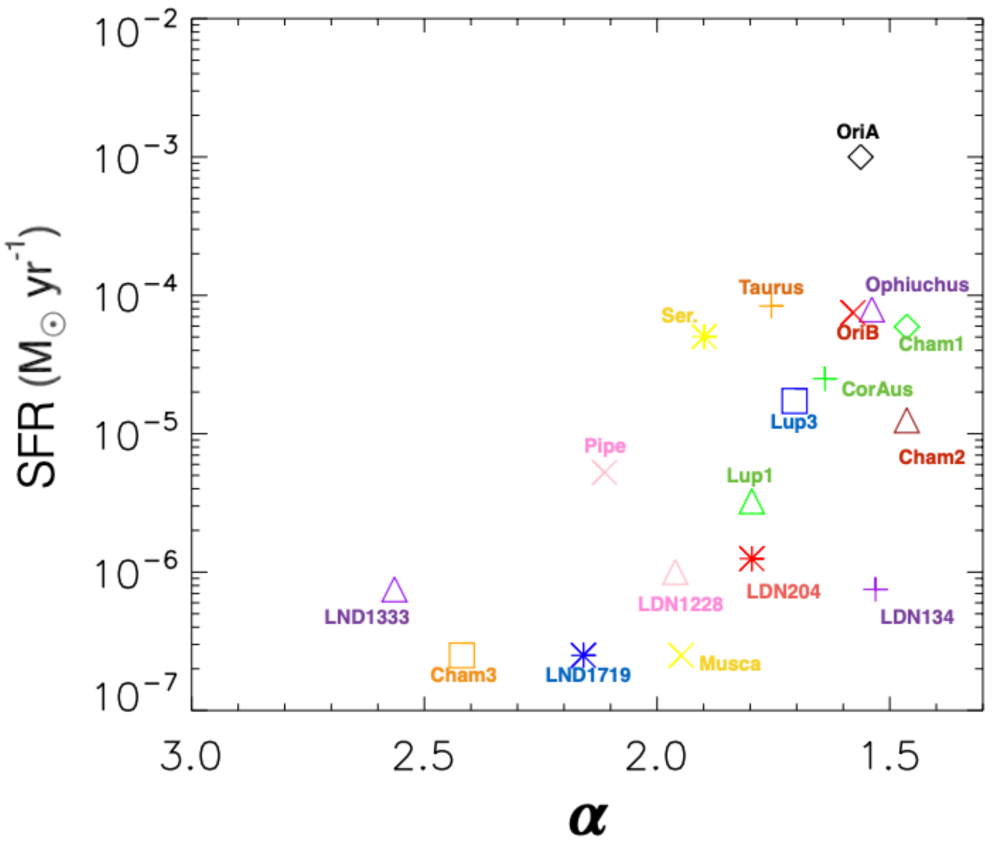
Power law model:
Star formation is inherently time varying
as power law tail evolves!

Burkhart 2018

SFE, SFR, and dens gas are correlated with slope of the density PDF

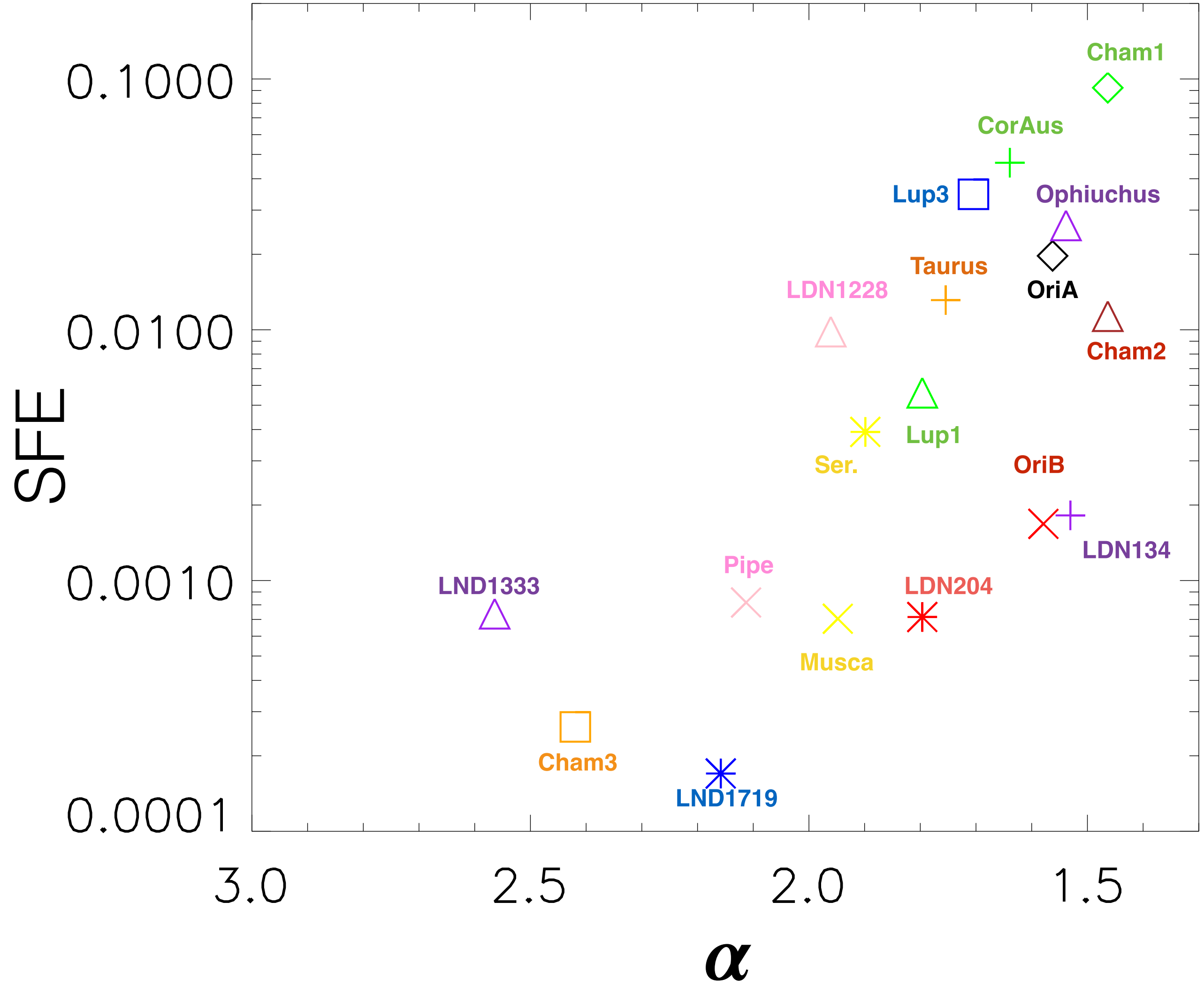


Burkhart 2018



3D Density PDF power law tail slope using reconstruction method of Kainulainen et al. 2014

SFE, SFR, and dense gas are correlated with slope of the density PDF



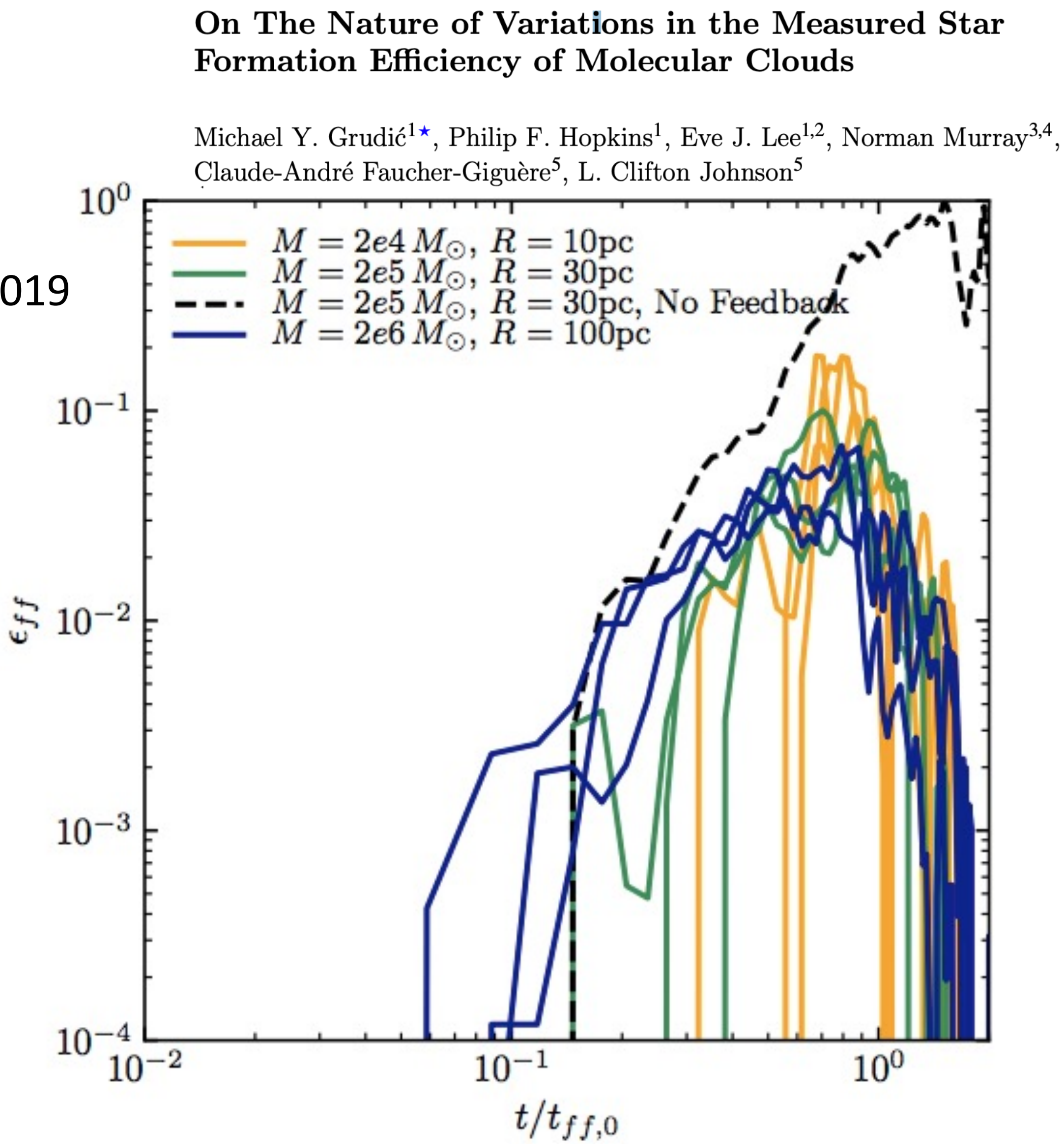
3D Density PDF power law tail slope using reconstruction method of Kainulainen et al. 2014

Appel, Burkhart & Kainulainen 2022, in prep

Feedback regulates density distribution and Star formation

Grudic et al. 2019

Mike Grudic



On The Nature of Variations in the Measured Star Formation Efficiency of Molecular Clouds

Michael Y. Grudić[1]⋆, Philip F. Hopkins[1], Eve J. Lee[1,2], Norman Murray[3,4], Claude-André Faucher-Giguère[5], L. Clifton Johnson[5]

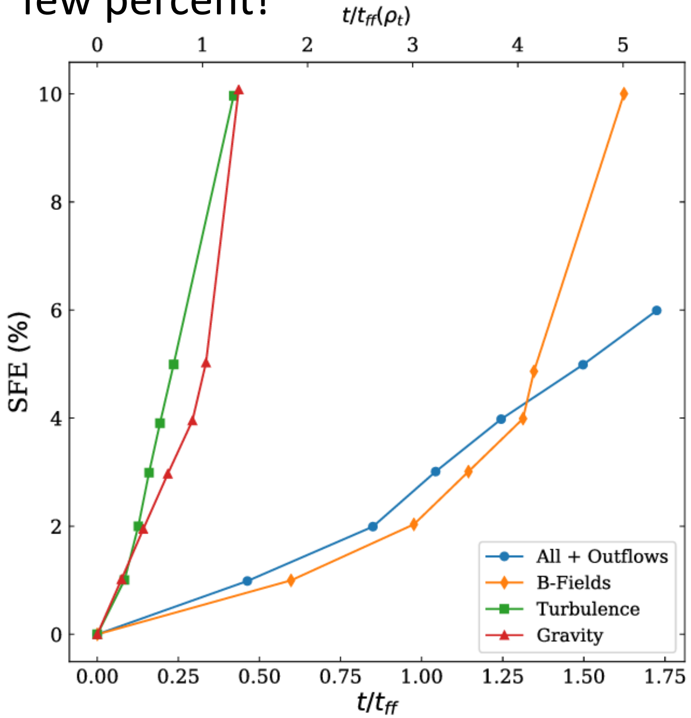
Outflow Feedback + B fields Critical for low SFE

Sabrina Appel
Poster 108

Feedback is an important for setting the star formation efficiency (Wang et al. 2010; Krumholz 2014; Federrath 2015; Grudic et al. 2018)

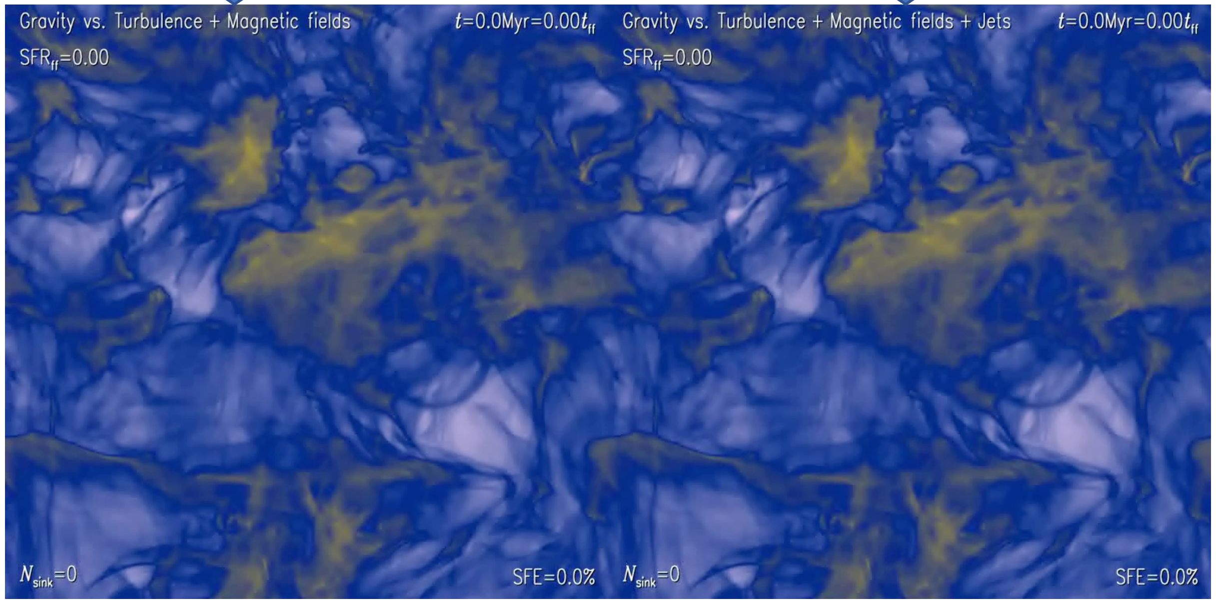
How does Feedback Affect the Density PDF and the SFE in the context of this model?

Only with outflow feedback can SFE per free fall stay around a few percent!



Turbulence +B fields+ Gravity

Feedback+Turbulence +B fields+ Gravity



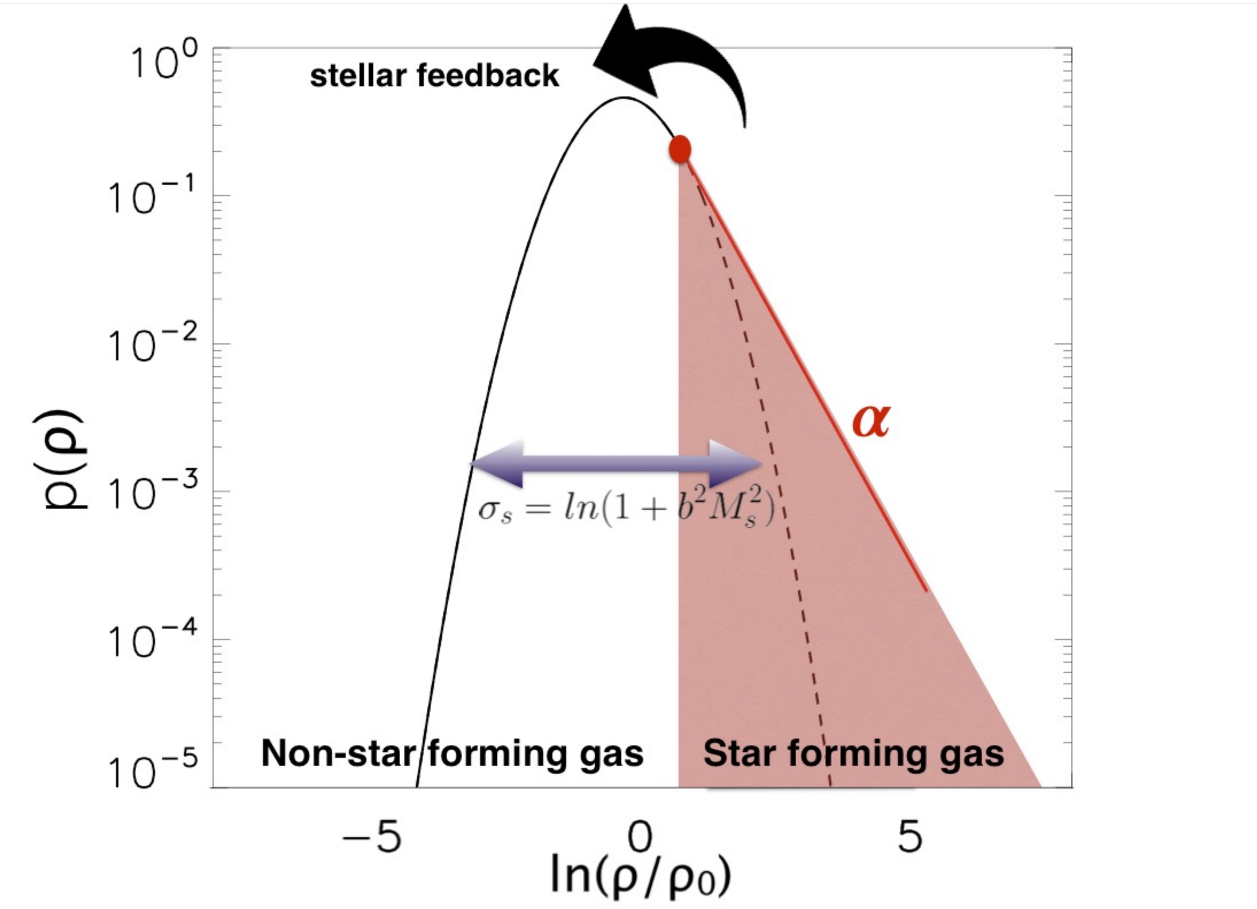
Federrath 2015
Appel et al. 2022

Star forming gas: traced by powerlaw PDF due to dominance of self-gravity

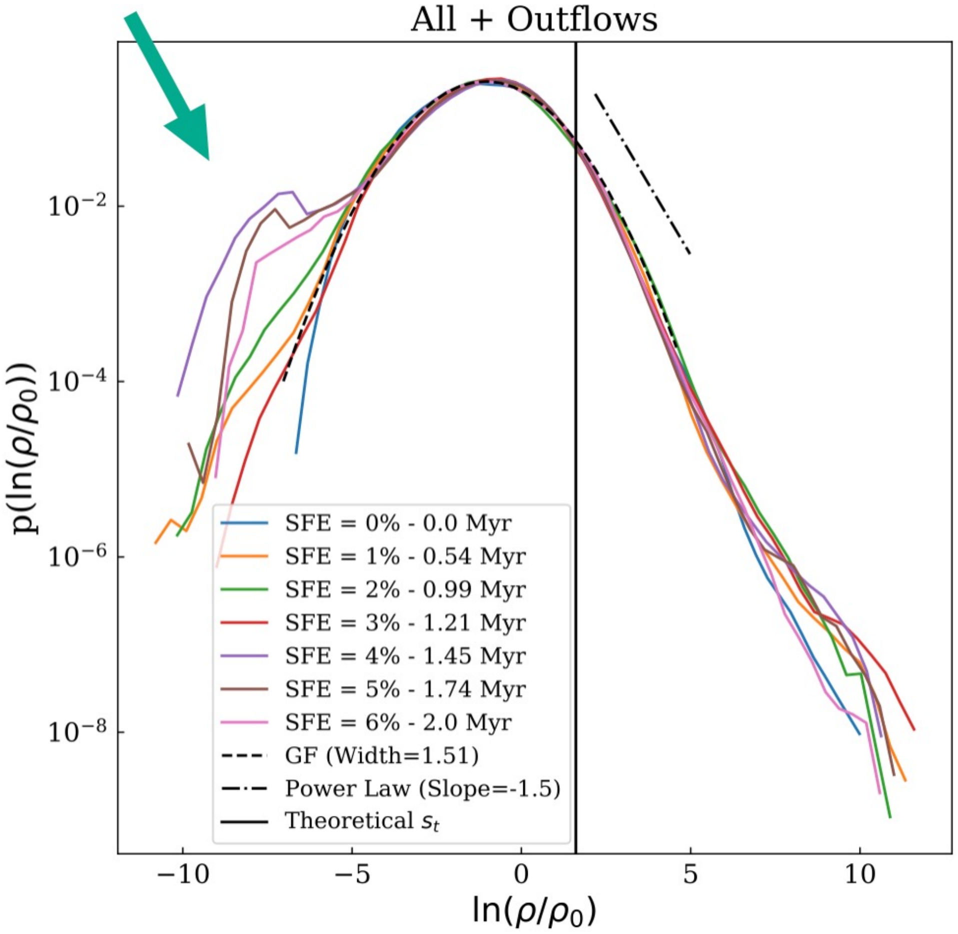
Diffuse gas: traced by lognormal due to supported by turbulence

Gas cycling between states: Stellar feedback (winds/jets) moves gas between states

Gas cycling via Feedback keeps SFE/SFR low

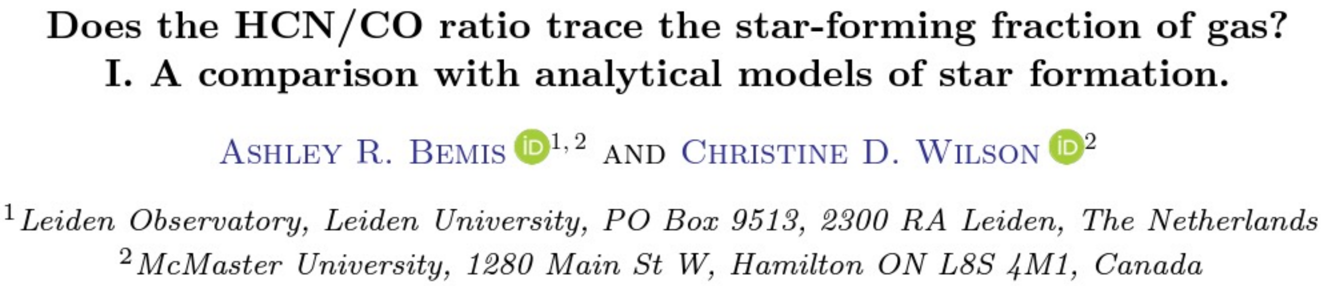


Burkhart & Mocz 2019



Appel et al. 2022

Observational Tests

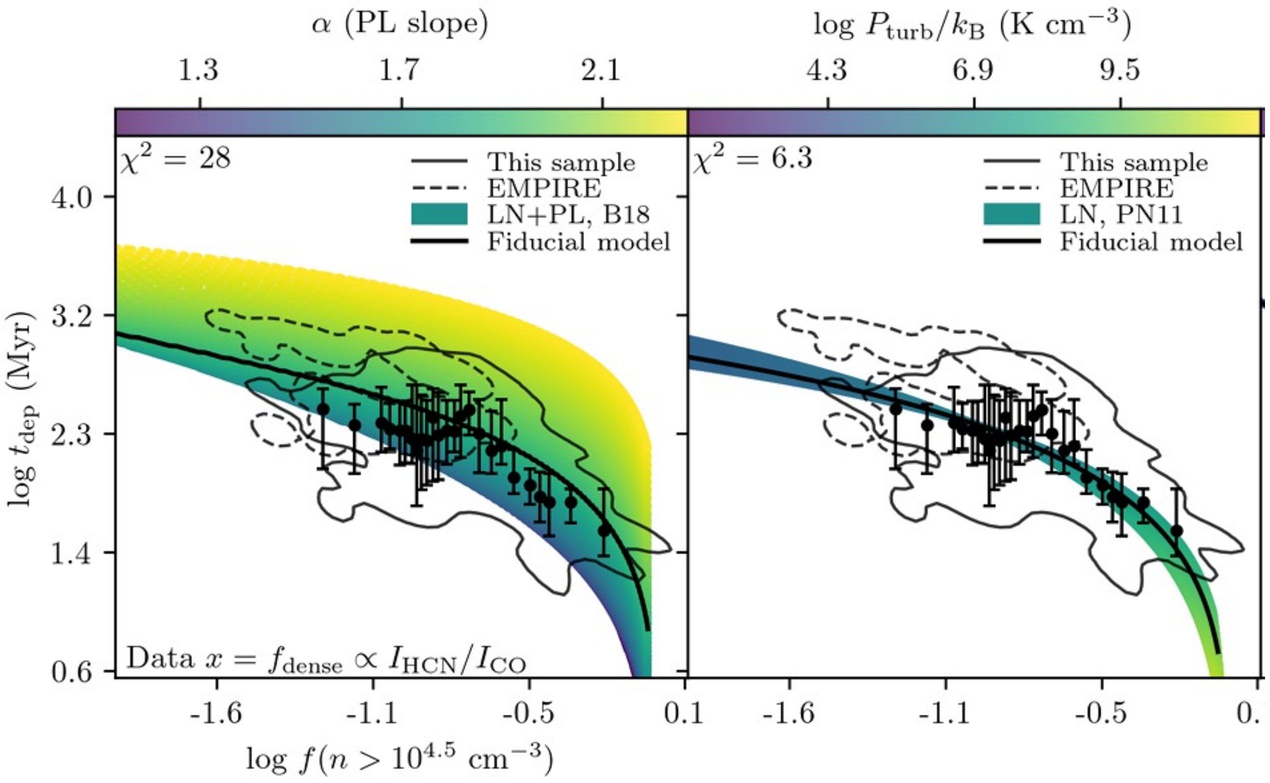


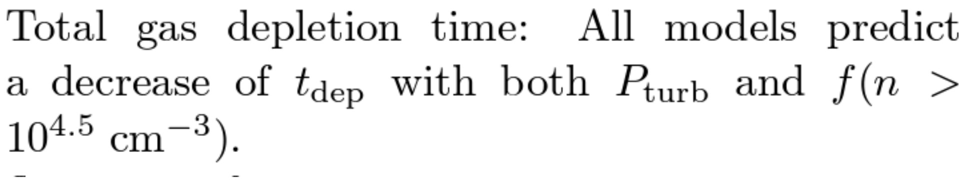
Does the HCN/CO ratio trace the star-forming fraction of gas? I. A comparison with analytical models of star formation.

Ashley R. Bemis[1,2] and Christine D. Wilson[2]

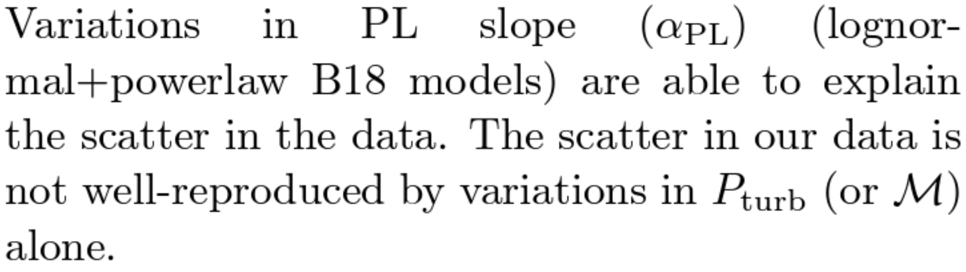
[1] *Leiden Observatory, Leiden University, PO Box 9513, 2300 RA Leiden, The Netherlands*
[2] *McMaster University, 1280 Main St W, Hamilton ON L8S 4M1, Canada*

Powerlaw Model | Lognormal Model



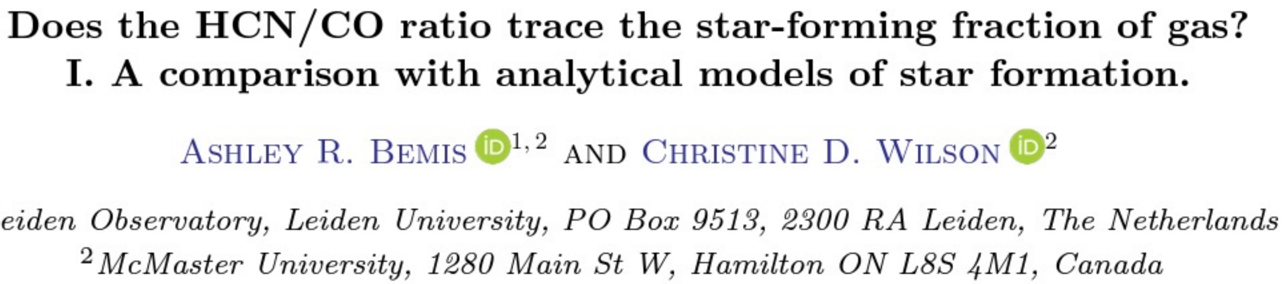


Total gas depletion time: All models predict a decrease of $t_{\rm dep}$ with both $P_{\rm turb}$ and $f(n > 10^{4.5}\ {\rm cm}^{-3})$.



Variations in PL slope ($\alpha_{\rm PL}$) (lognormal+powerlaw B18 models) are able to explain the scatter in the data. The scatter in our data is not well-reproduced by variations in $P_{\rm turb}$ (or $\mathcal{M}$) alone.

Observational Tests



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I. A comparison with analytical models of star formation.

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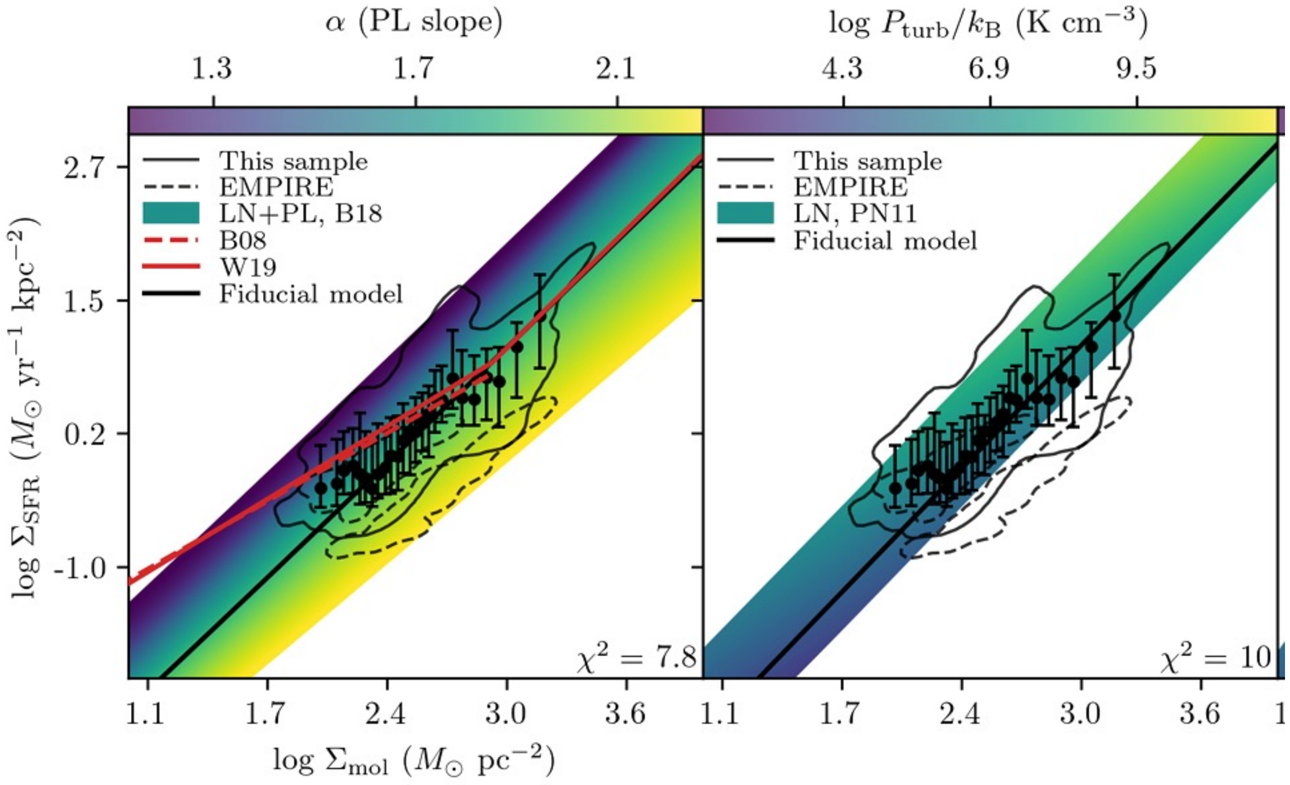
Powerlaw Model

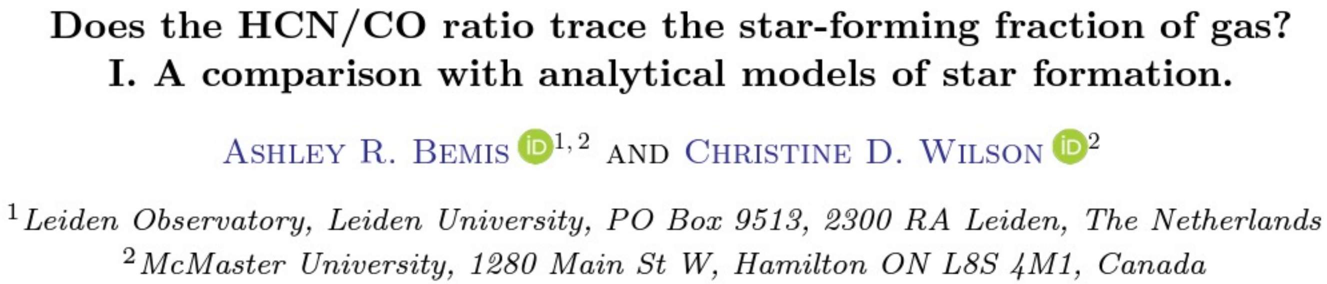
Lognormal Model



Varying power law slopes reproduce the varying slopes of the Kennicutt-Schmidt relation.

Burkhart 2018 model can explain the full range of the data.

Observational Tests



Does the HCN/CO ratio trace the star-forming fraction of gas?
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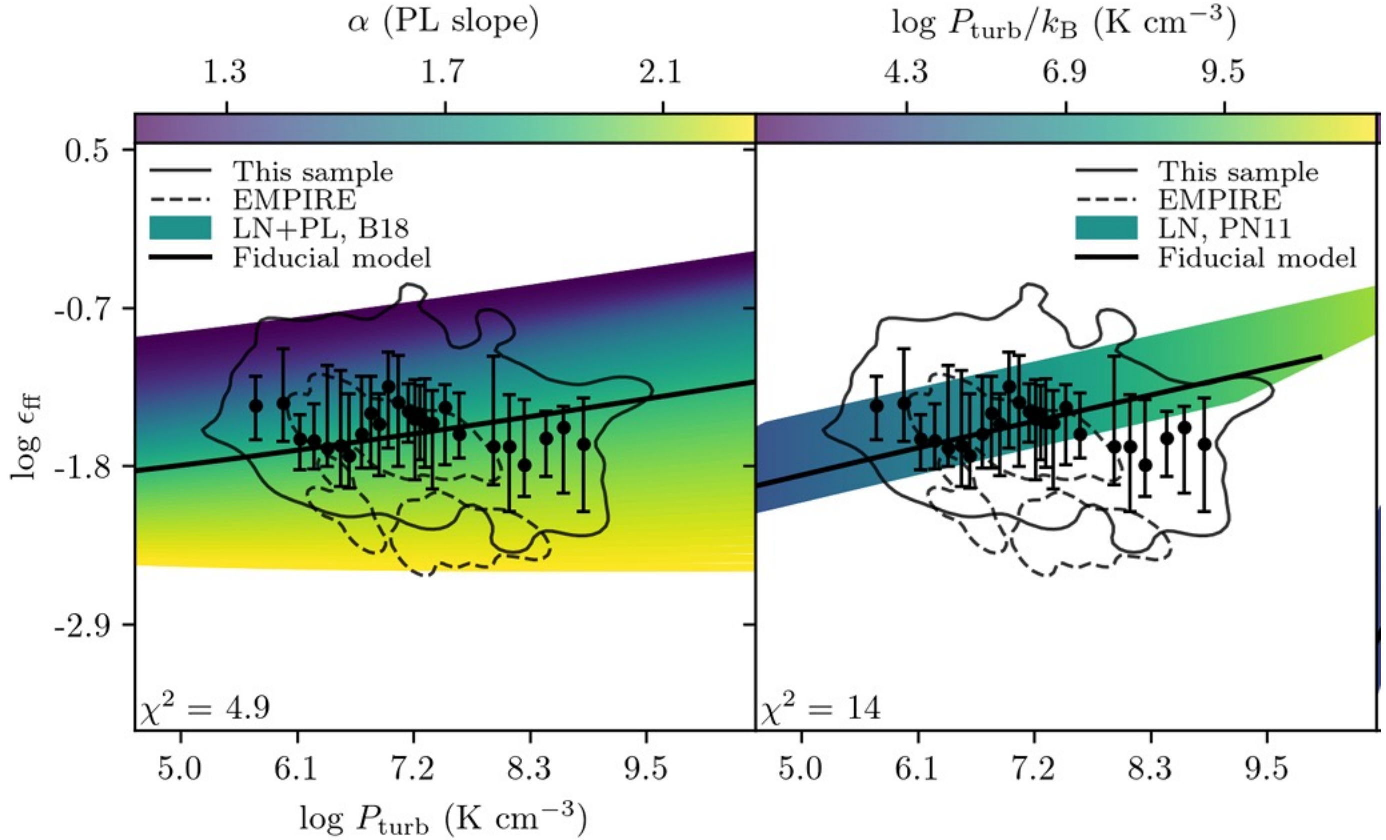
Powerlaw Model

Lognormal Model



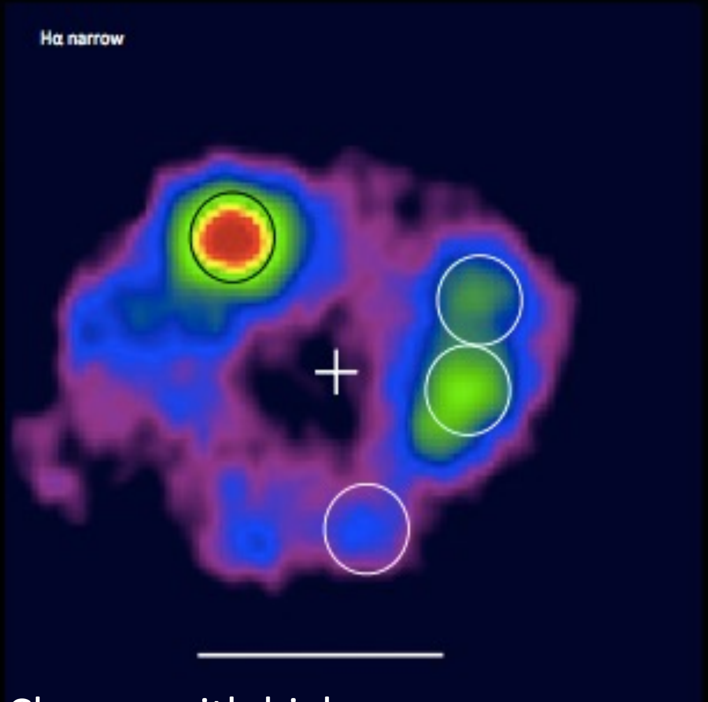
Varying power law slopes reproduce the ~1% efficiency of star formation with additional scatter observed in the KS relation.

Burkhart 2018 model can explain the full range of the data.

A Nearby, Normal Star Forming Disk Galaxy

Smoother with lower:
SFR, mass accretion,
gas velocity dispersion,
gas fraction

A Normal Star Forming Disk Galaxy Far, Far Away



Clumpy with higher:
SFR, mass accretion,
gas velocity dispersion,
gas fraction

Goal: a global analytic model to explain these observations

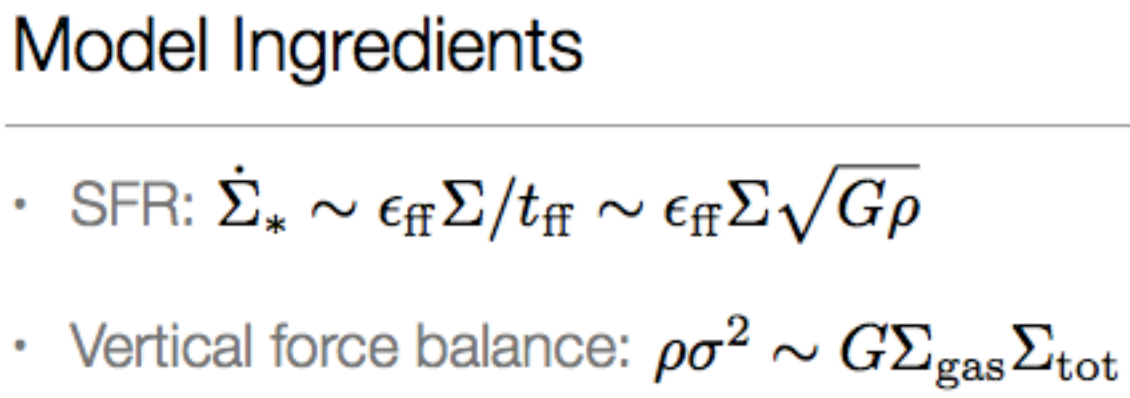
Start by building physical intuition:

$$\left(\frac{dE}{dA}\right)_{\rm turb} \approx \frac{3}{2}\Sigma_{\rm g}\sigma_{\rm g}^2 = 3.1\times10^9\,\Sigma_{{\rm g},10}\sigma_{{\rm g},10}^2\ {\rm erg\ cm}^{-2},$$

$$\begin{aligned}\left(\frac{dE}{dA}\right)_{\rm sf} &\approx \dot{\Sigma}_*\left\langle\frac{p_*}{m_*}\right\rangle\sigma_{\rm g}\frac{r}{v_\phi}\\ &= 3.1\times10^9\,\dot{\Sigma}_{*,-3}\sigma_{{\rm g},10}r_{10}v_{\phi,200}^{-1}\ {\rm erg\ cm}^{-2},\end{aligned}$$

Star formation (supernova feedback) can supply the energy needed for turbulence at the 10 km/s level.

Feedback regulated star formation models take this into account!



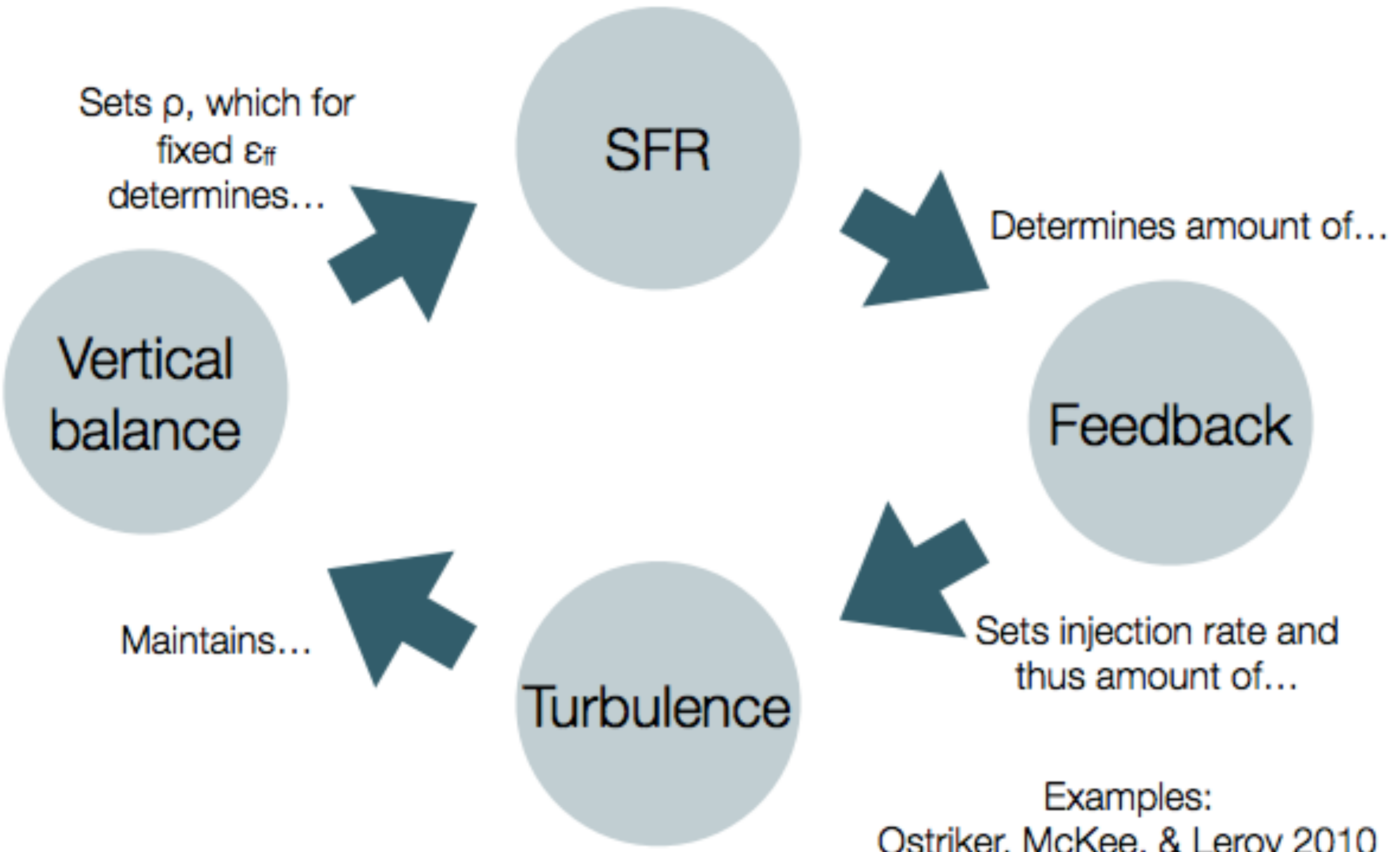
Model Ingredients

- SFR: $\dot{\Sigma}_* \sim \epsilon_{\rm ff} \Sigma / t_{\rm ff} \sim \epsilon_{\rm ff} \Sigma \sqrt{G\rho}$
- Vertical force balance: $\rho\sigma^2 \sim G\Sigma_{\rm gas}\Sigma_{\rm tot}$

e.g.
Krumholz +18
Burkhart +18
Shetty & Ostriker 11
Ostriker+10
Thompson+05
Faucher-Giguere+13
Hayward & Hopkins17
…….

For example. Shetty & Ostriker 11 explains KS relation/ISM/Feedback

Model 1: Feedback + Microphysical SF Law



Sets ρ, which for fixed ε_{ff} determines…

SFR

Determines amount of…

Feedback

Sets injection rate and thus amount of…

Turbulence

Maintains…

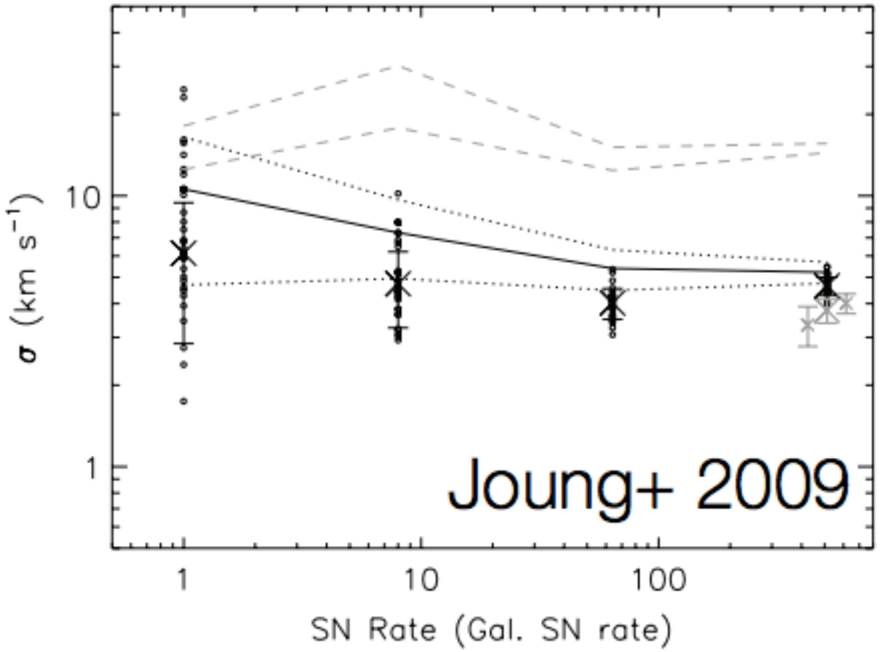
Vertical balance

Examples:
Ostriker, McKee, & Leroy 2010
Shetty & Ostriker 2011

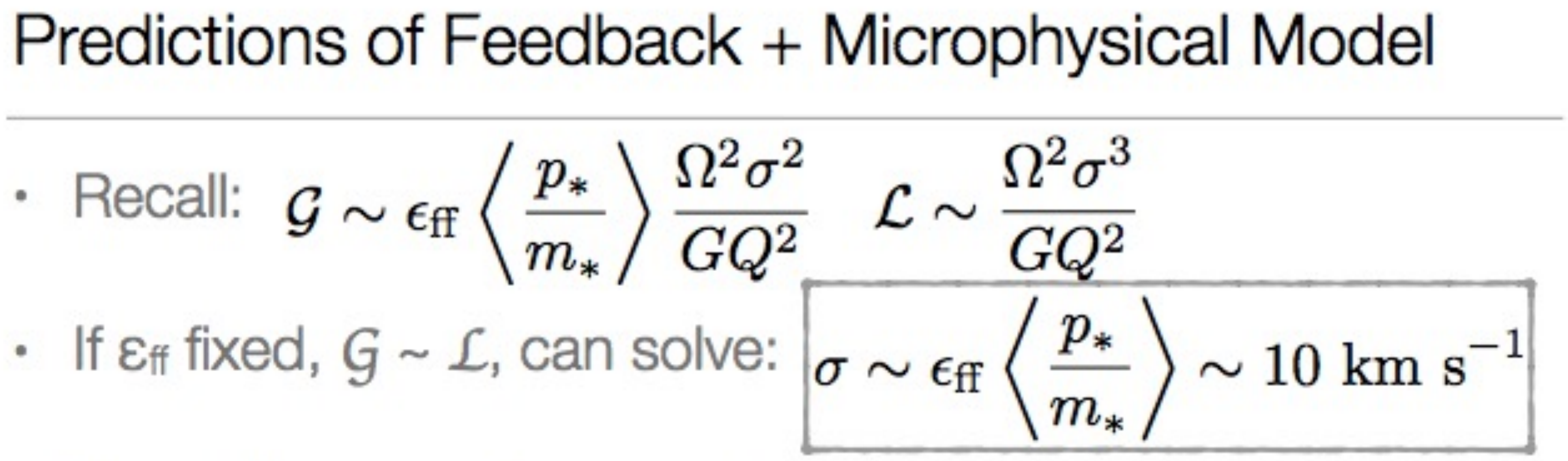
For example. Shetty & Ostriker 11 explains KS relation/ISM/Feedback

Predictions of Feedback + Microphysical Model

- Recall: $\mathcal{G} \sim \epsilon_{\rm ff} \left\langle \frac{p_*}{m_*} \right\rangle \frac{\Omega^2 \sigma^2}{G Q^2}$ $\mathcal{L} \sim \frac{\Omega^2 \sigma^3}{G Q^2}$
- If $\epsilon_{\rm ff}$ fixed, $\mathcal{G} \sim \mathcal{L}$, can solve: $\boxed{\sigma \sim \epsilon_{\rm ff} \left\langle \frac{p_*}{m_*} \right\rangle \sim 10 \text{ km s}^{-1}}$
- Simulations confirm result



Feedback alone fails to predict velocity dispersion diversity!

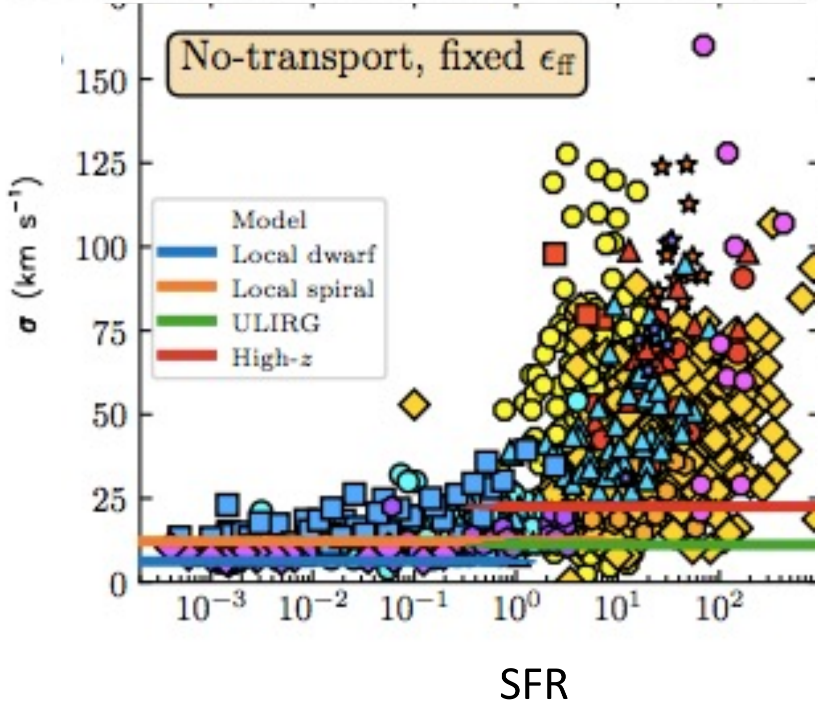


Predictions of Feedback + Microphysical Model

- Recall: $\mathcal{G} \sim \epsilon_{\rm ff} \left\langle \frac{p_*}{m_*} \right\rangle \frac{\Omega^2 \sigma^2}{GQ^2}$ $\quad \mathcal{L} \sim \frac{\Omega^2 \sigma^3}{GQ^2}$
- If $\epsilon_{\rm ff}$ fixed, $\mathcal{G} \sim \mathcal{L}$, can solve: $\boxed{\sigma \sim \epsilon_{\rm ff} \left\langle \frac{p_*}{m_*} \right\rangle \sim 10 \text{ km s}^{-1}}$

Provides a fixed velocity dispersion.

Feedback alone can not explain velocity dispersions in excess of 10-40km/s



Building a global model: physical intuition

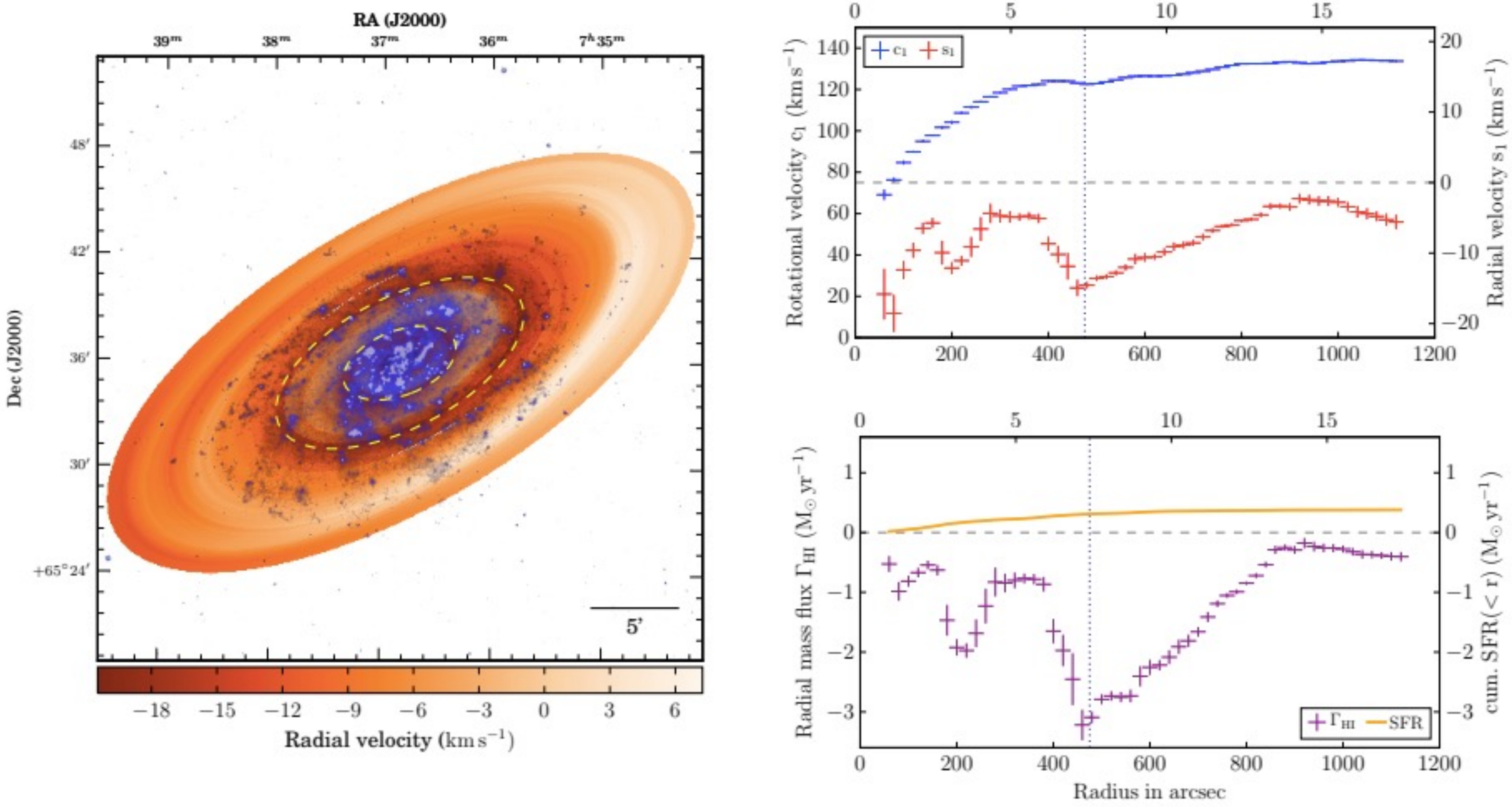
$$\left(\frac{dE}{dA}\right)_{\rm turb} \approx \frac{3}{2}\Sigma_{\rm g}\sigma_{\rm g}^2 = 3.1\times 10^9\,\Sigma_{{\rm g},10}\sigma_{{\rm g},10}^2\ {\rm erg\ cm^{-2}},$$

$$\begin{aligned}\left(\frac{dE}{dA}\right)_{\rm sf} &\approx \dot{\Sigma}_*\left\langle\frac{p_*}{m_*}\right\rangle\sigma_{\rm g}\frac{r}{v_\phi}\\ &= 3.1\times 10^9\,\dot{\Sigma}_{*,-3}\sigma_{{\rm g},10}r_{10}v_{\phi,200}^{-1}\ {\rm erg\ cm^{-2}},\end{aligned}$$

$$\left(\frac{dE}{dA}\right)_{\rm inflow} \approx \frac{\dot{M}_{\rm in}v_\phi}{2\pi r} = 6.5\times 10^9\,\dot{M}_{{\rm in},1}v_{\phi,200}r_{10}^{-1}\ {\rm erg\ cm^{-2}},$$

Mass inflow can be an important energy source in galaxies…as important as feedback!
Can be driven by gravitational disk instability ….

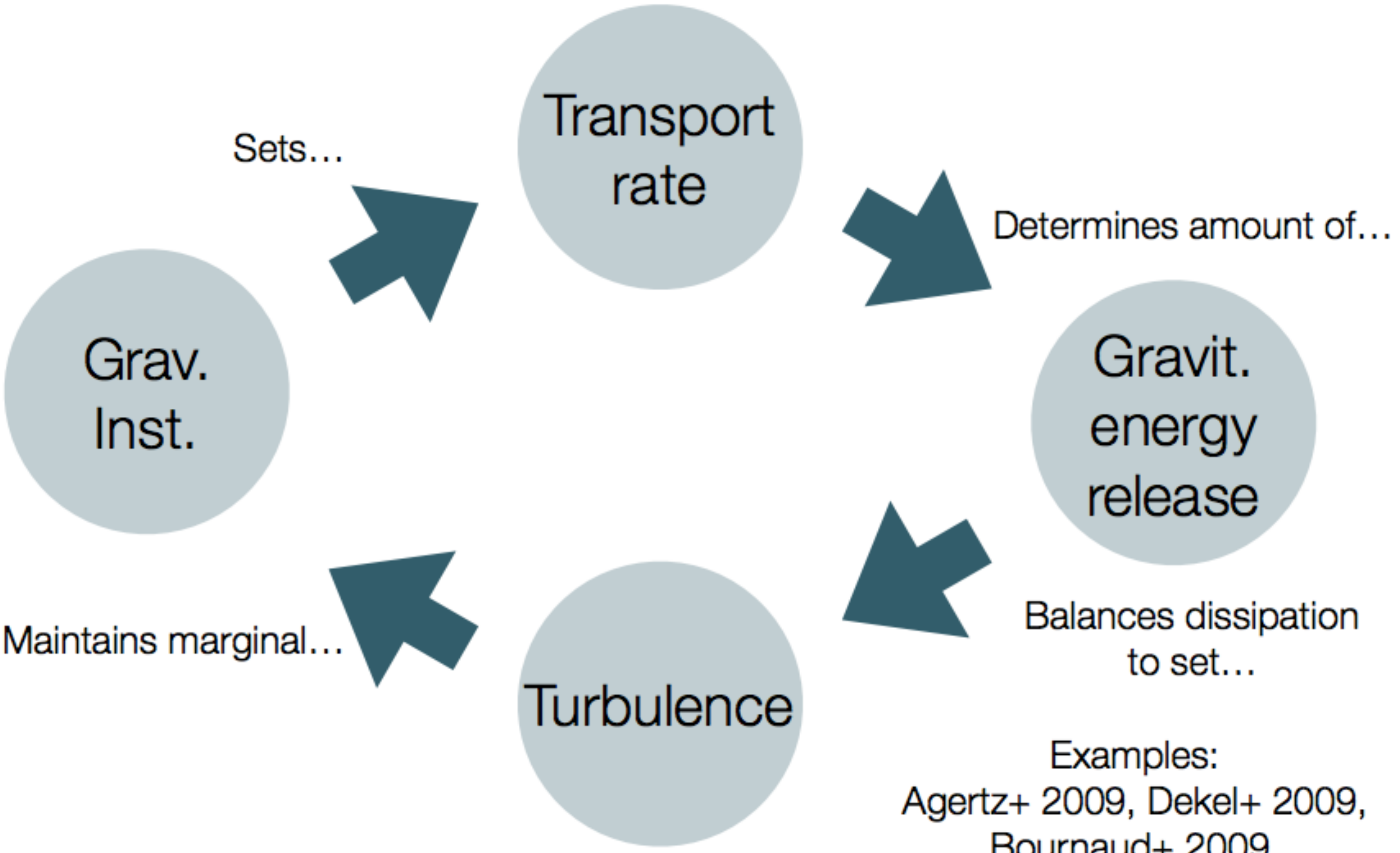
Radial transport: observations



Direct Detection of
Radial Transport

Schmidt+ 2016:
Left: NGC 2403 radial velocity w/
GALEX FUV overlaid
Right: radial velocity and mass flux

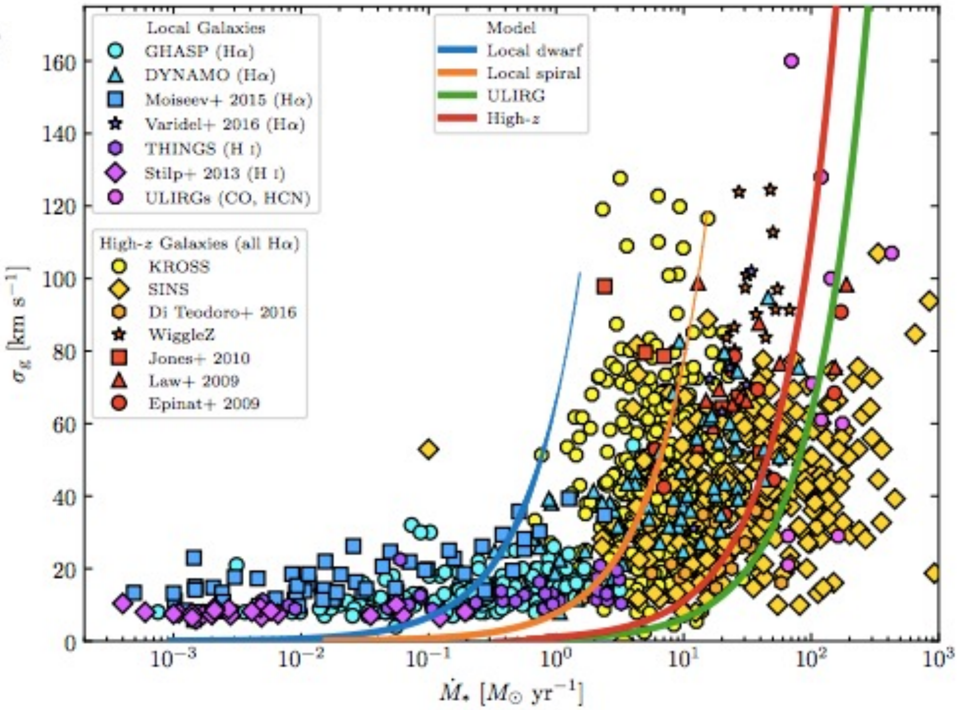
GI + Transport + Microphysical Law



Examples:
Agertz+ 2009, Dekel+ 2009,
Bournaud+ 2009,
Krumholz+ 2010, Forbes+ 2012

Velocity Dispersion From Transport

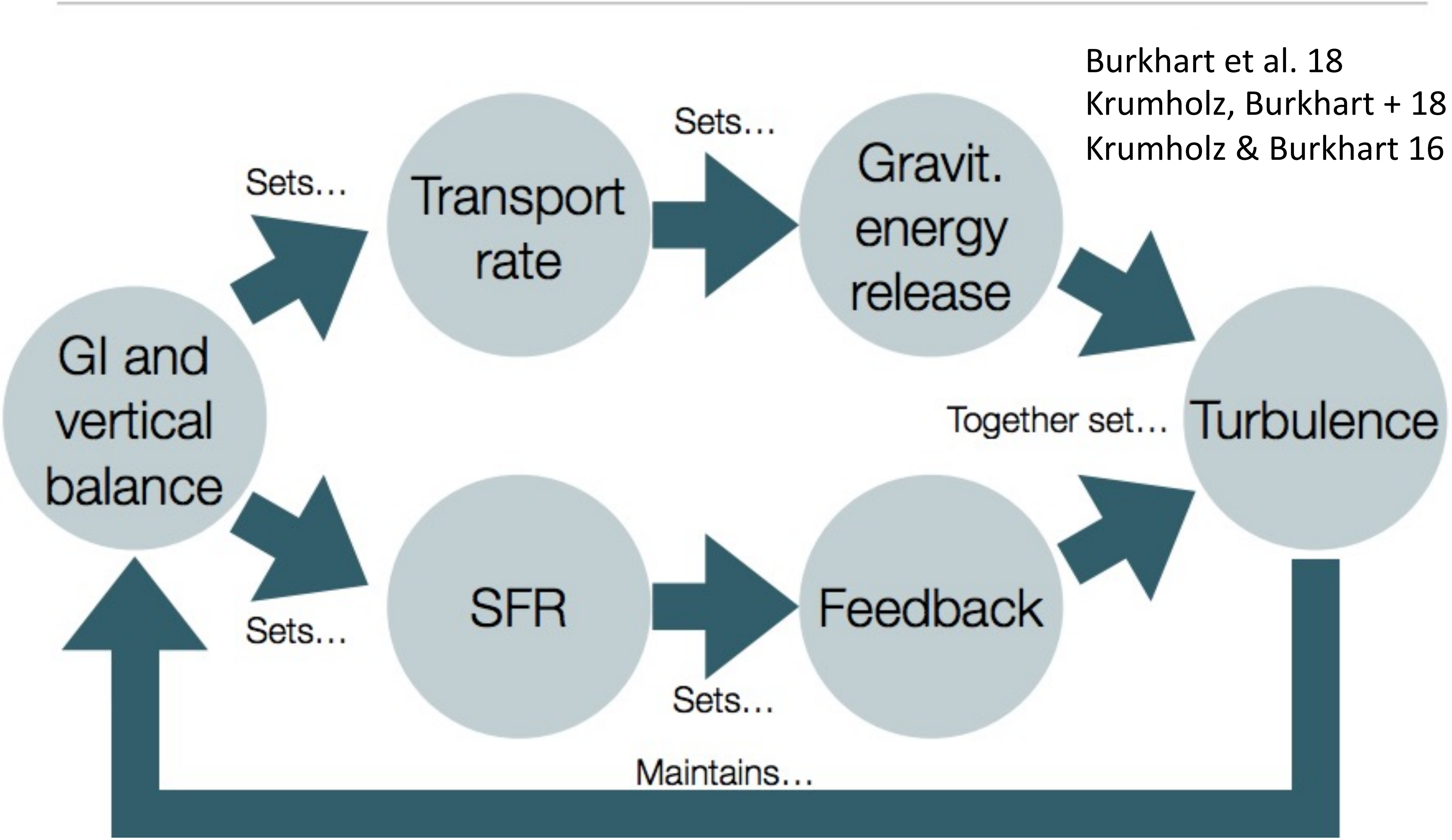
- Energy release by transport: $\mathcal{G}_{\rm trans} \sim \dot{M}\Omega^2/2\pi$
- If $\mathcal{G}_{\rm trans} \sim \mathcal{L}$, can solve: $\mathcal{L} \sim \dfrac{\Omega^2\sigma^3}{GQ^2} \Longrightarrow \dot{M} \sim \sigma^3/G$
- SFR from microphysical SF law applied to surface density profile set by GI
- Gets SF law right by construction, but doesn't get right σ in low redshift galaxies



Conclusion: We still must have a gain terms from feedback to set the velocity dispersion floor

We require feedback + mass transport to match observations

Feedback + GI + Transport + Micro SF



GI and vertical balance

Sets…

Transport rate

Sets…

Gravit. energy release

Sets…

SFR

Sets…

Feedback

Together set…

Turbulence

Maintains…

Burkhart et al. 18
Krumholz, Burkhart + 18
Krumholz & Burkhart 16

Transport Plus Feedback Model

- Energy balance w/transport: $\mathcal{G} + \mathcal{G}_{\mathrm{trans}} - \mathcal{L} = 0$
- Solution: $\dot{M}_{\mathrm{trans}} \sim \frac{\sigma^3}{GQ^2}\left(1 - \frac{\sigma_{\mathrm{sf}}}{\sigma}\right)$

$$\sigma_{\mathrm{sf}} \sim \epsilon_{\mathrm{ff}} \left\langle \frac{p_*}{m_*} \right\rangle \sim 10 \text{ km s}^{-1}$$

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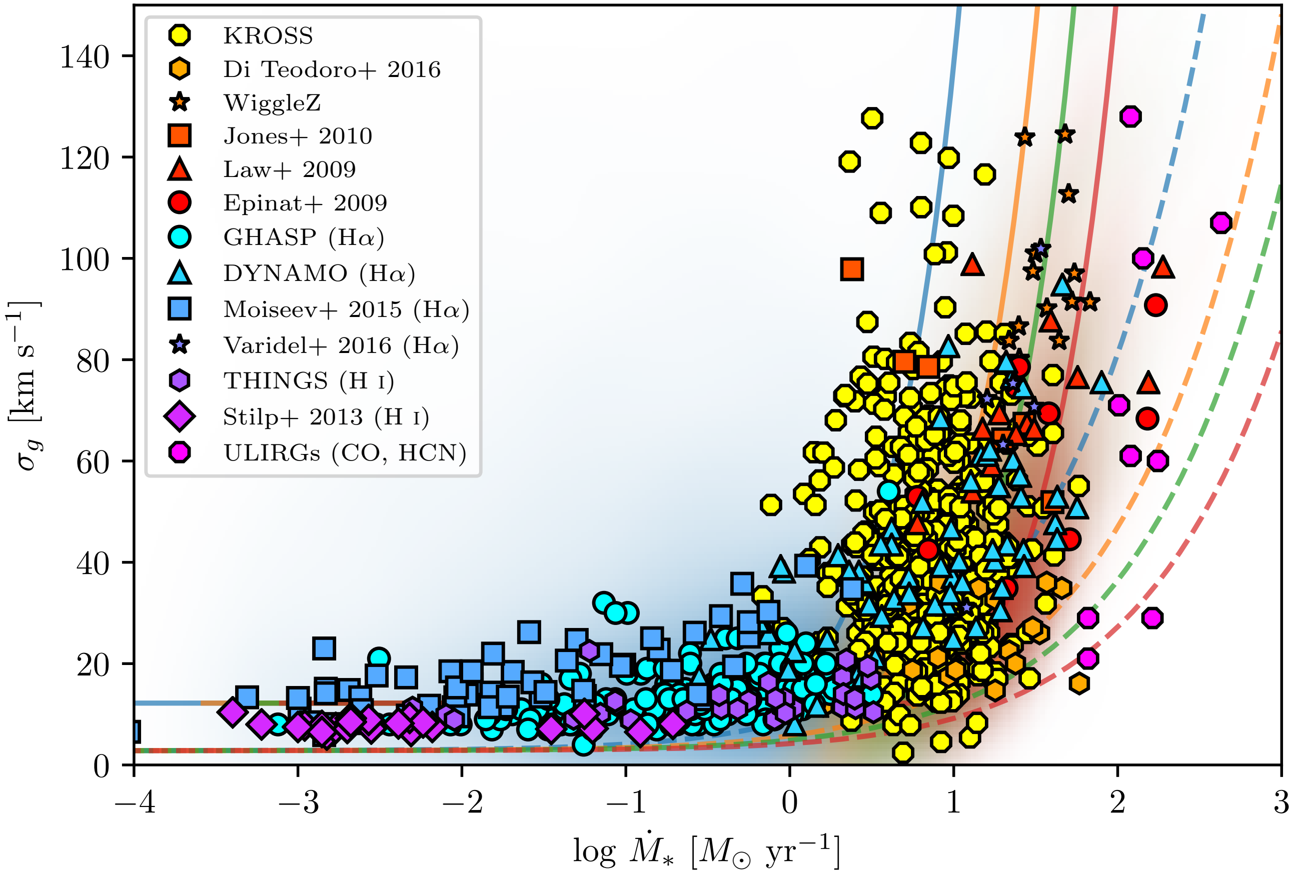
$$\sigma_{\mathrm{sf}} \sim \epsilon_{\mathrm{ff}} \left\langle \frac{p_*}{m_*} \right\rangle \sim 10 \mathrm{~km~s}^{-1}$$

- Physical interpretation: SF produces $\sigma = \sigma_{\mathrm{sf}}$ ~10 km s^{-1} because $\varepsilon_{\mathrm{ff}}$ ~ 0.01; if σ needed for Q ~ 1 is larger, mass transport provides energy to make up the deficit

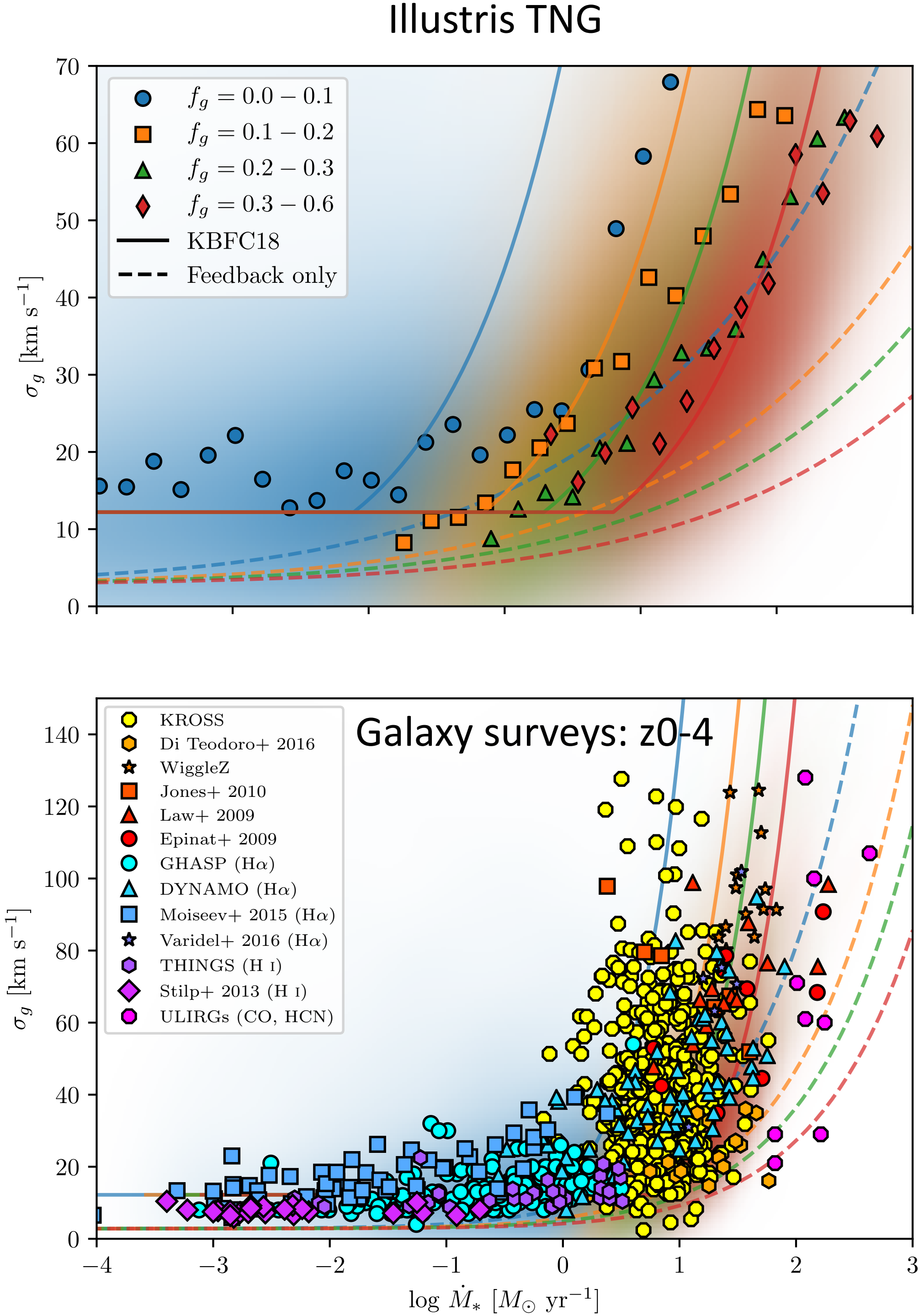
- Fraction of energy provided by SF is σ_{sf} / σ

The correlation of velocity dispersion & SFR

Solid lines: feedback+transport model (Krumholz, Burkhart, Forbes & Crocker 2018)
Dashed lines: feedback model (Faucher-Giguere et al. 2013)



The correlation of velocity dispersion & SFR



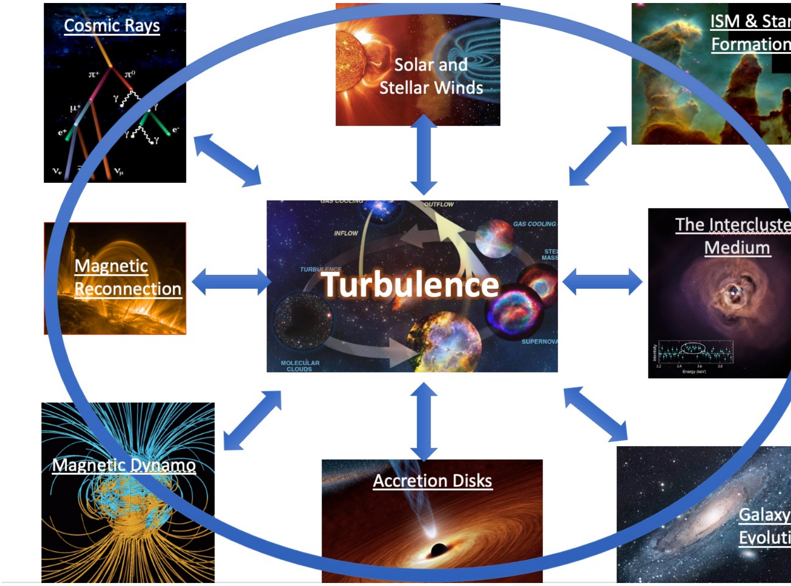
IllustrisTNG: color coded by gas fraction

Blue: low gas fraction (mostly low redshift)

Orange, green, red: increasing gas fraction

Simulations reproduce observed SFR-gas velocity dispersion relationship and agree with the model which includes feedback+transport.

Open Questions



What is the correct picture of MHD turbulence (incompressible/compressible) ? Does it matter for astrophysics? It DOES matter for solar wind…

-What is the relevant scale and density of star formation? Is there a critical scale/density for collapse? Can this be seen in statistics: structure function/pdf analysis?

-Feedback is clearly important for the SFE: how does this extend to high z, including IMF changes?

-Is SFR set by cosmic accretion (bathtub) vs local processes (i.e. local disk instabilities producing GMCs or thermal instability)…?

-Large scales HI sub-critical/sub-Alfvenic…what is the role of magnetic field in dynamics of collapse?

Most likely nature gives many modes of star formation. Either/or picture is good for getting grants but maybe not the right thinking.

www.mhdturbulence.com