Houches School - The Physics of Star Formation
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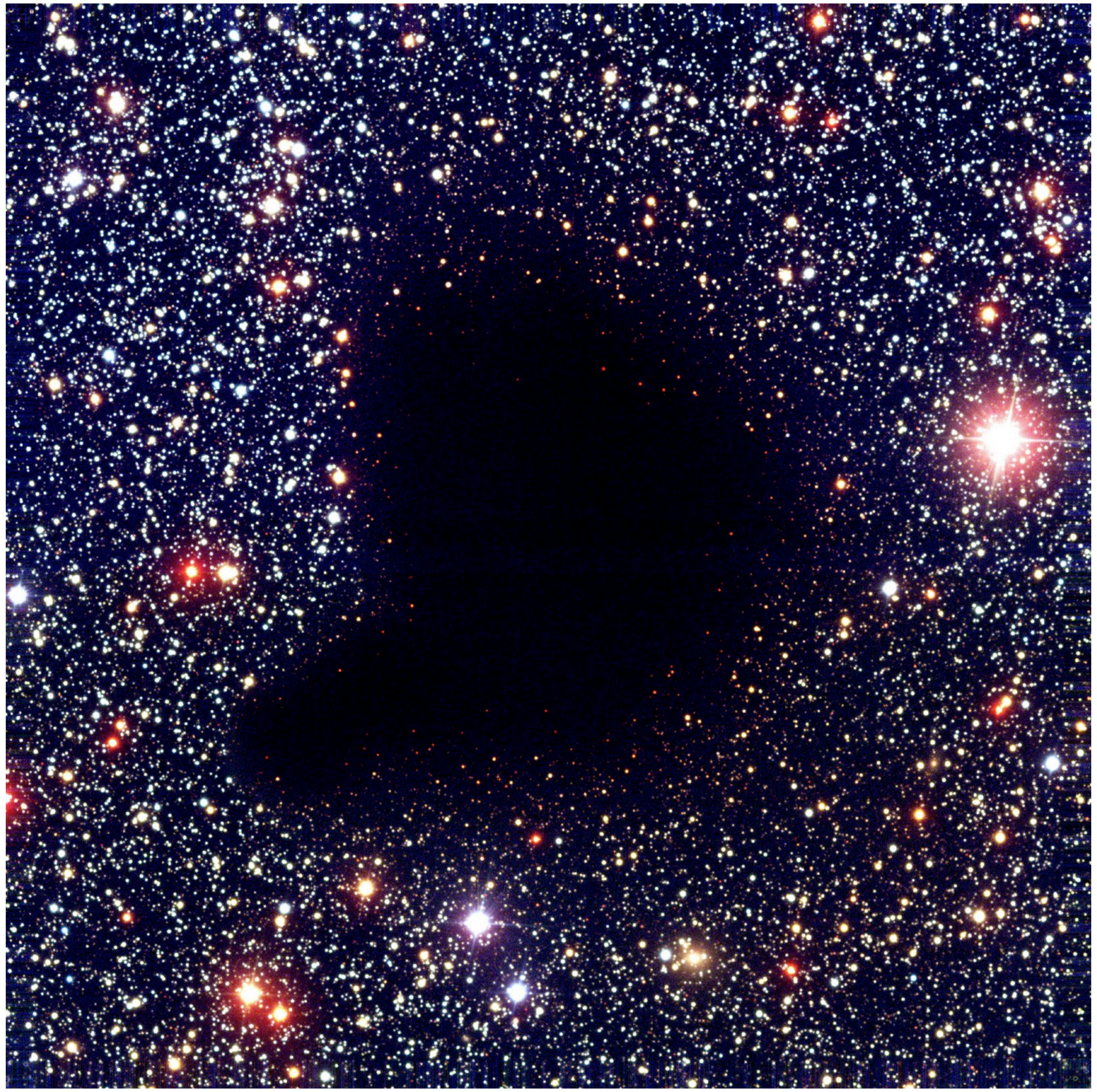
Dust dynamics and dust evolution during star formation

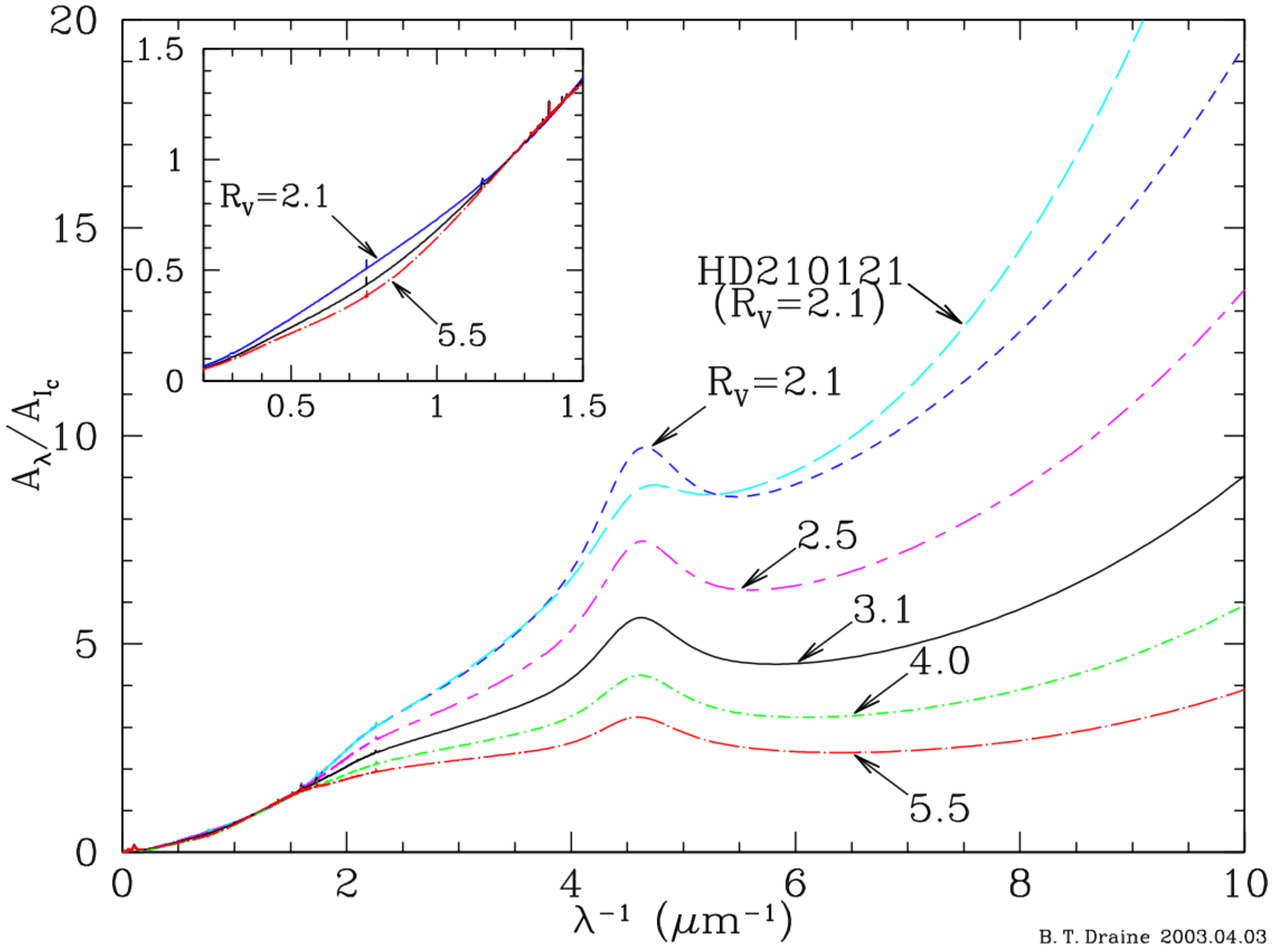
Dust extinction

Barnard (1907, 1910); Trumpler (1930)



Barnard 68, VLT
Visible and near-IR composite image
CREDITS: ESO

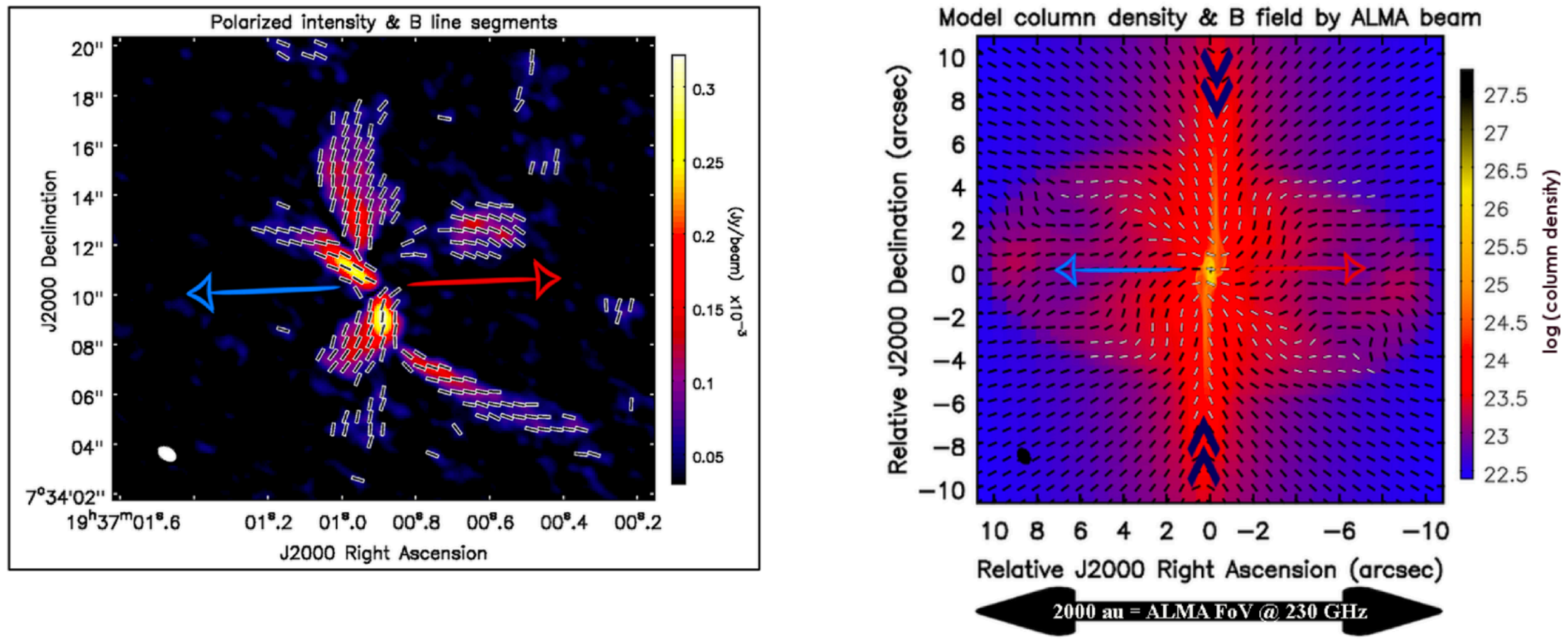
Absolute extinction as a function of the wavelength



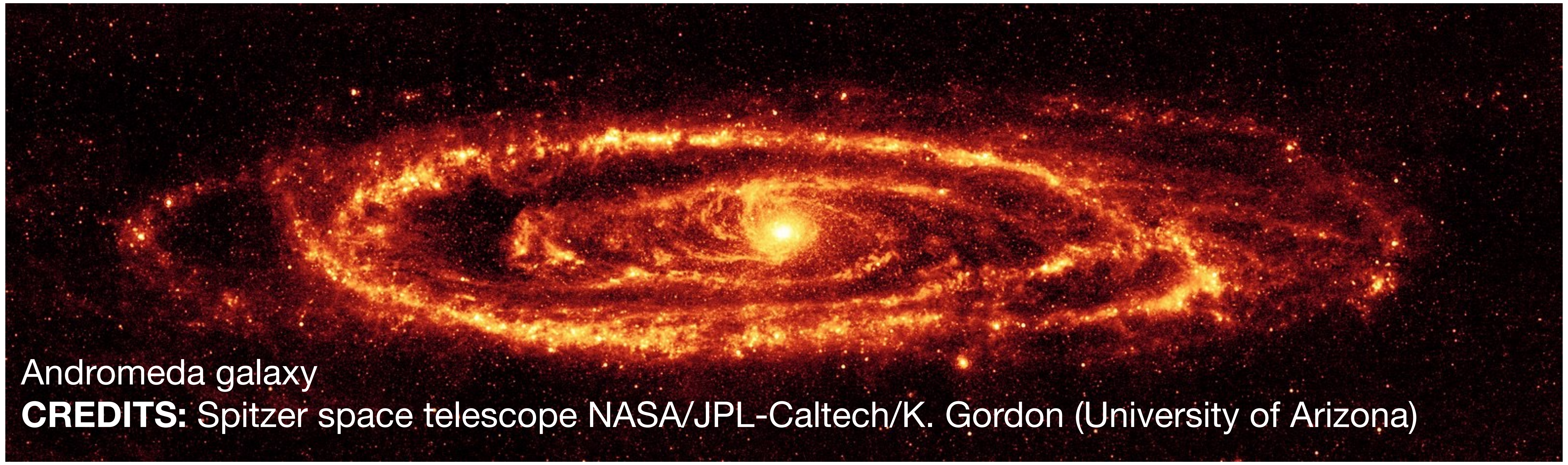
Fitzpatrick (1999); Draine (2003)

Dust : a very efficient probe of the ISM

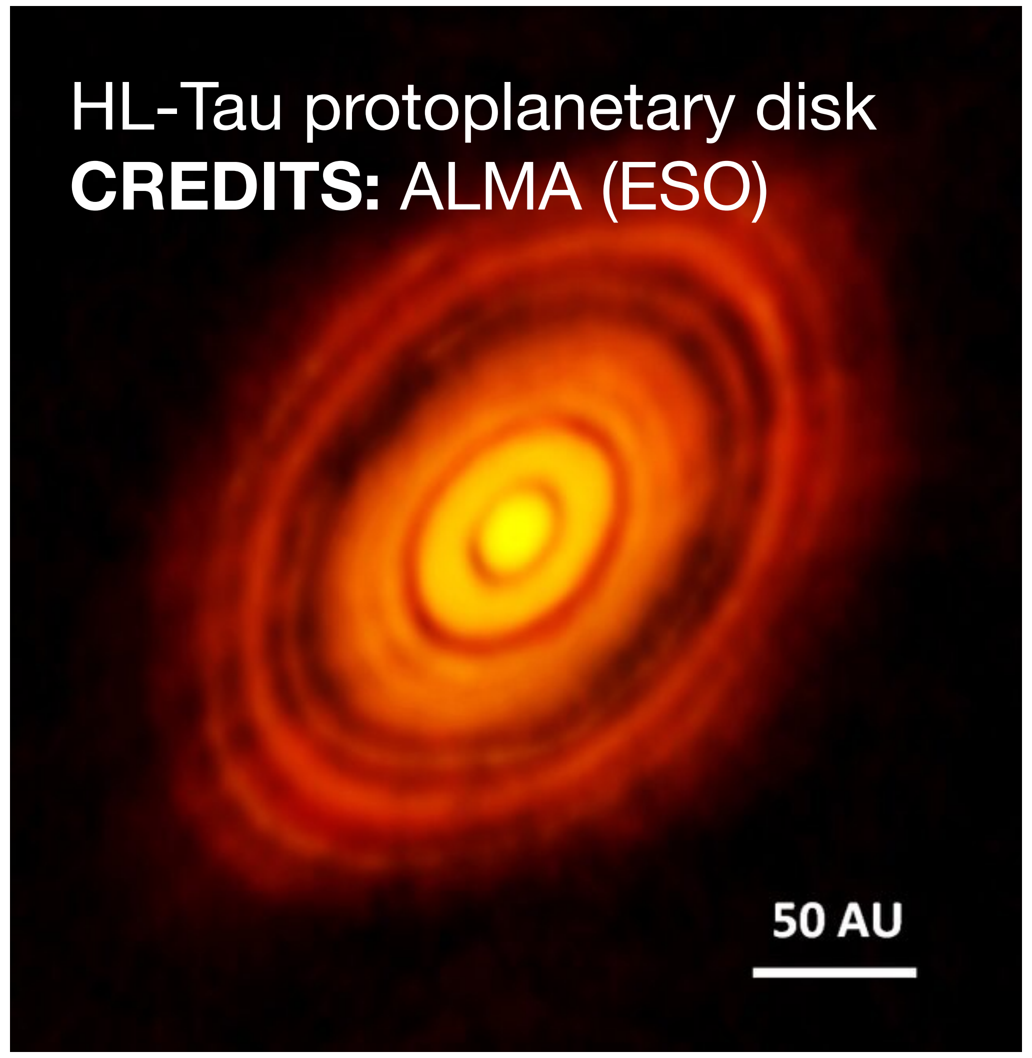
Carina nebula
CREDITS: James Webb Space Telescope(NASA/ESA/CSA/STScI)



B335 protostar (Maury et al., 2018)



Andromeda galaxy
CREDITS: Spitzer space telescope NASA/JPL-Caltech/K. Gordon (University of Arizona)



HL-Tau protoplanetary disk
CREDITS: ALMA (ESO)

Structure and composition of the dust

Dust grains :

Small solid particles or aggregates (few nm to few 0.1 microns) and up to even larger sizes in dense environments (mm? cm ? dm ?)
Intrinsic densities $1 - 3 \text{ g cm}^{-3}$ (Love et al. 1994)

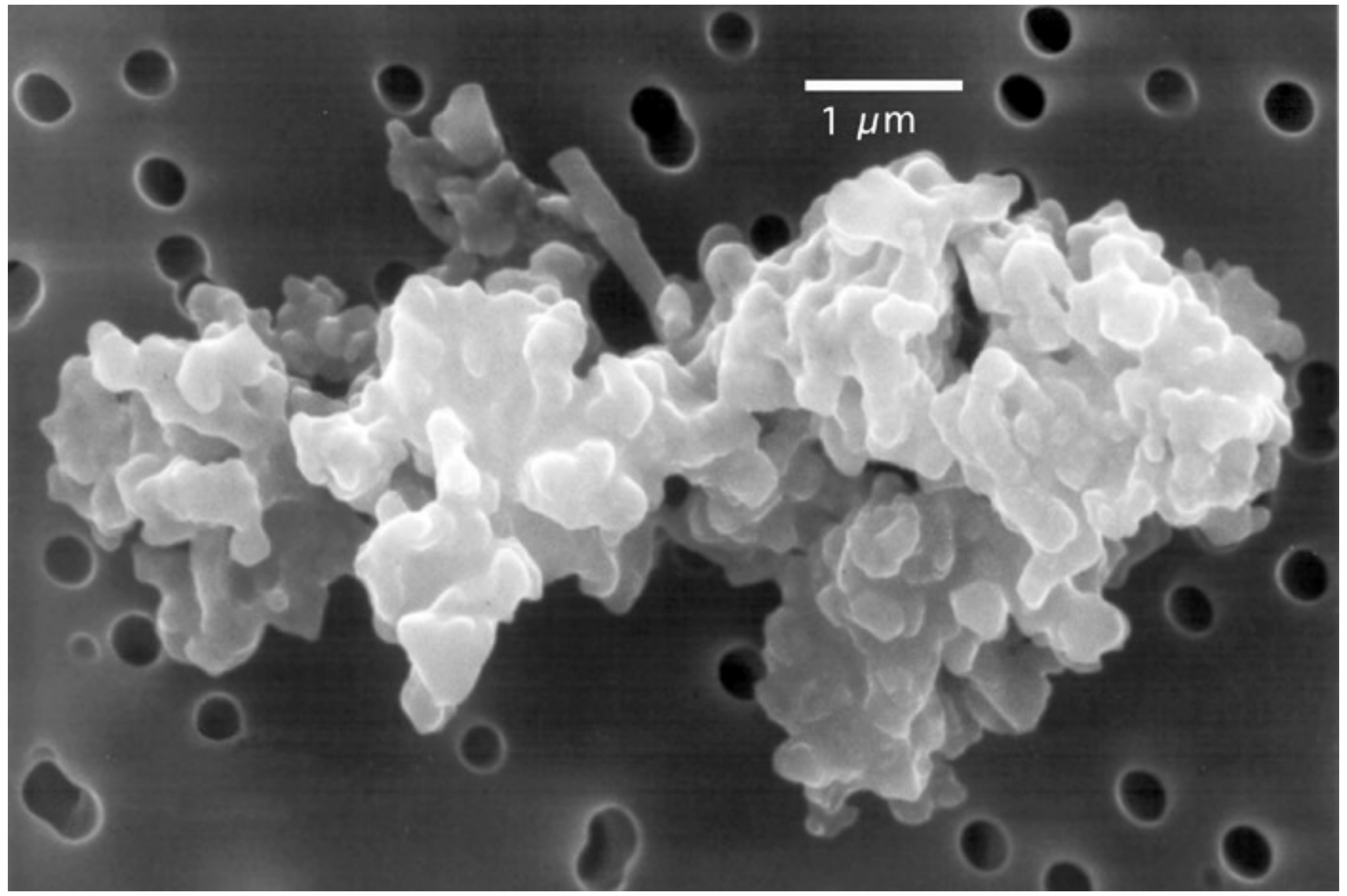
> They are much more massive than an H atom, even nm grains are ~ 8000 times more massive

Exact composition still discussed but :

Carbon
Silicates
Iron
Covered by ice mantles in cold environments

Important for later :

Grains can be charged (typically negatively at high density)
Draine & Sutin (1987); Guillet et al. (2007)



Interplanetary dust grain *(Jessberger et al., 2001)*

Dust-to-gas ratio

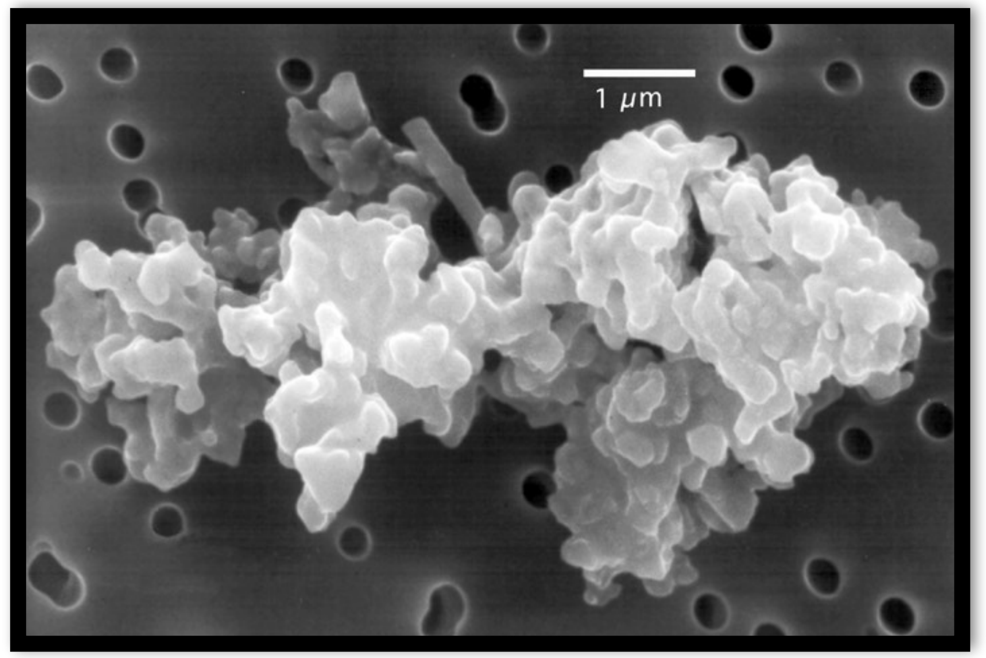
Dust density/gas density → $\frac{\rho_d}{\rho_g}$

~1 % in the ISM, **BUT** could vary locally :

- Local creation/destruction rate depend on the environment
- Dynamical sorting can happen

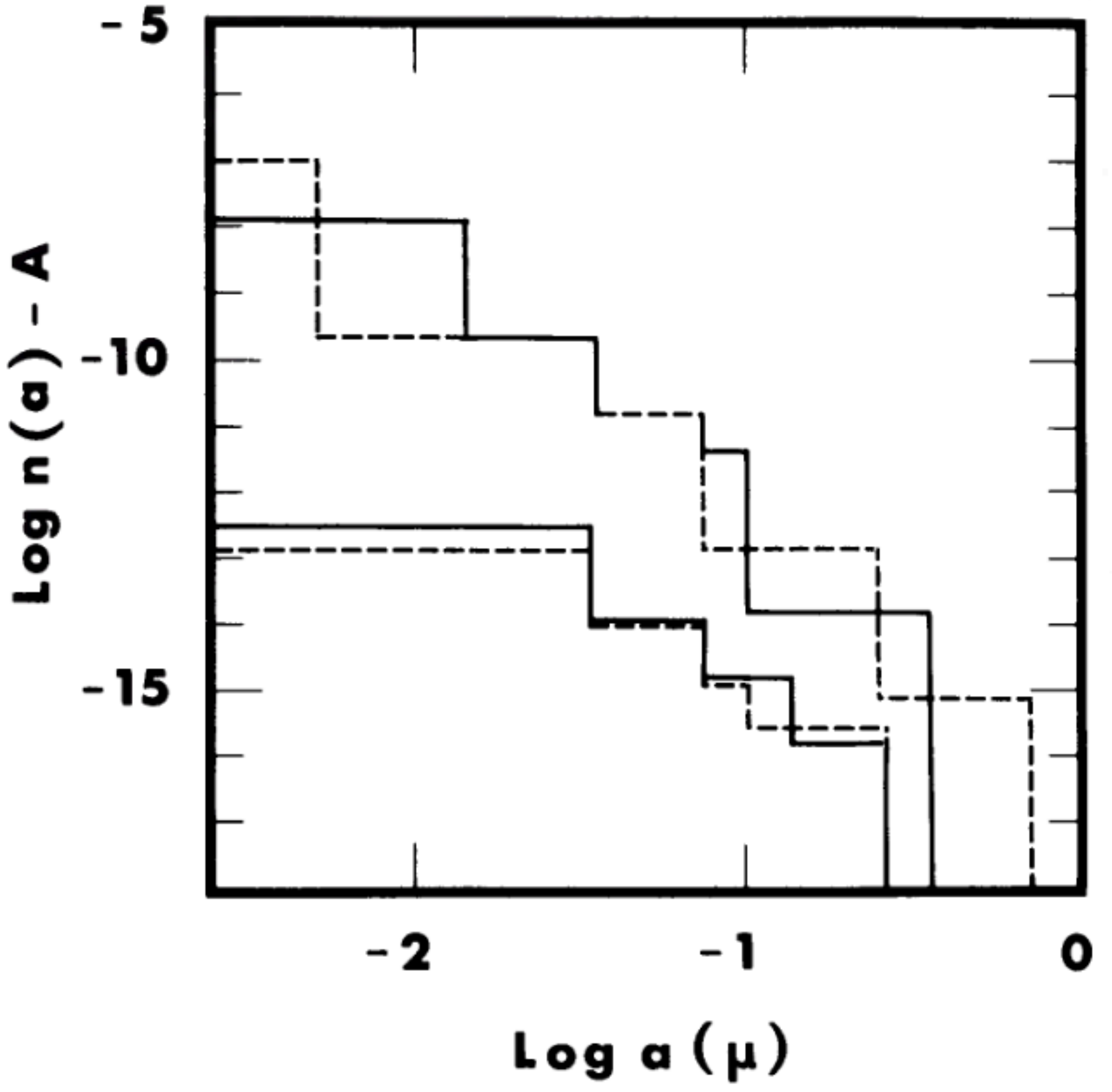
Notes :

- Different from the dust ratio ρ_d/ρ, or dust concentration ($\rho \equiv \rho_g + \rho_d +$ everything else)
- ρ_d is not the intrinsic grain density ρ_{grain} which is the density of a single dust grain (and which is much higher)

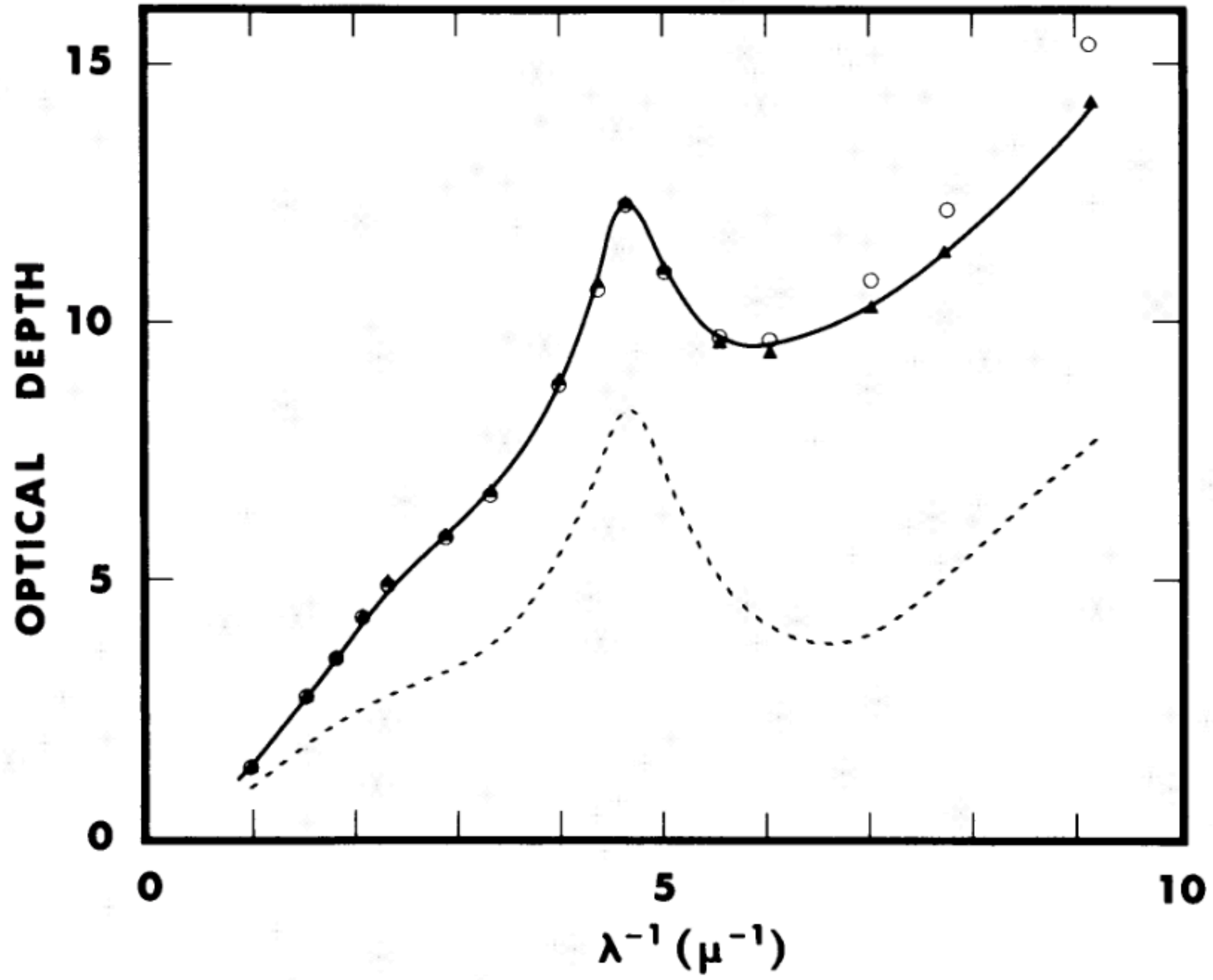
Dust  $\neq$ Grain 

MRN size distribution *Mathis, Rumpl, Nordsieck (1977)*

Grain size $a \in [5\text{nm}, 250\text{nm}]$, power law distribution $\frac{dn}{da} \equiv Ca^{\alpha}, \alpha = -3.5$



MRN distribution in the seminal paper



Fit of the extinction curve in the seminal paper

'Valid' in the diffuse ISM

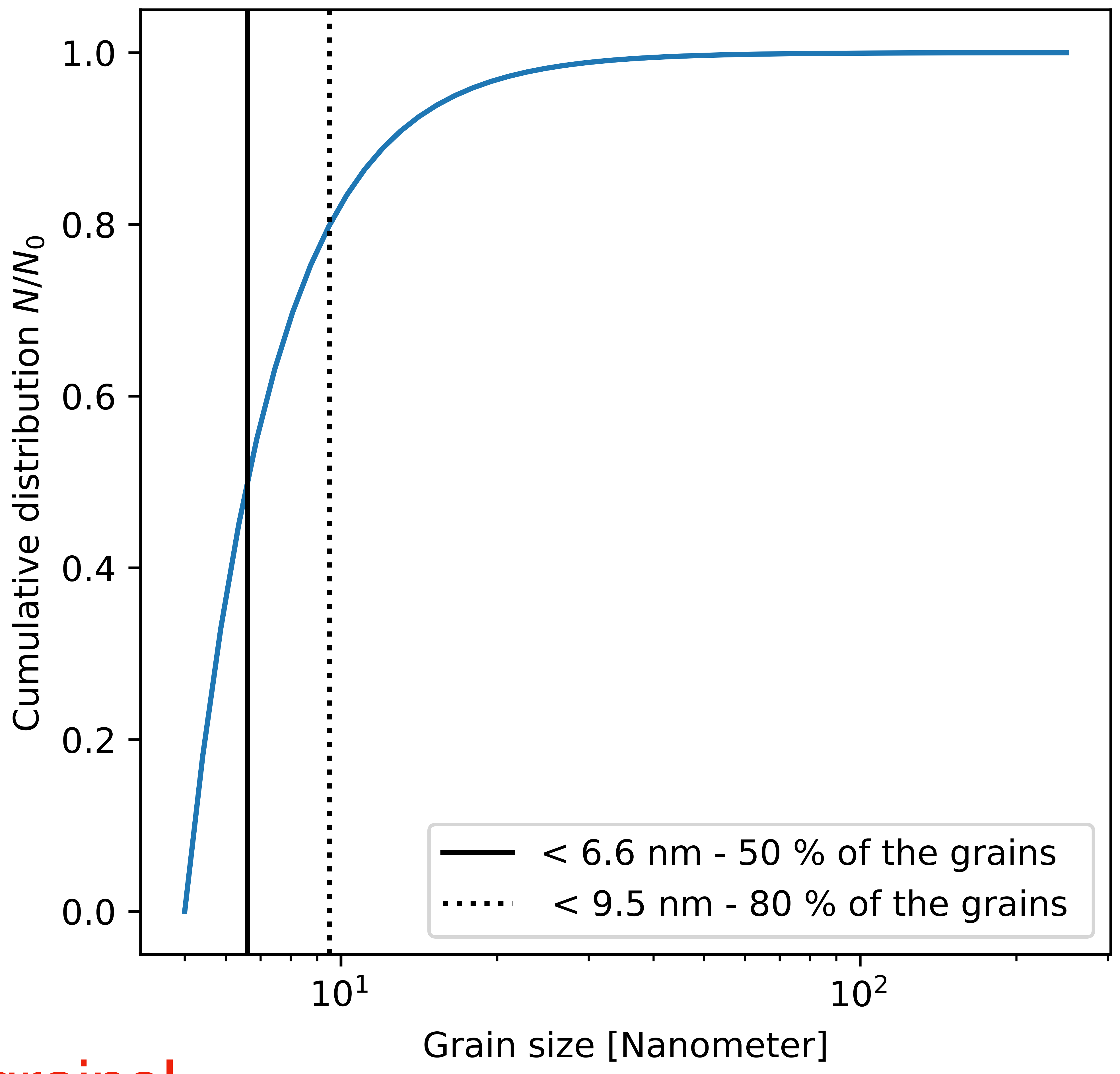
Note: this a simple distribution that has been updated/discussed/debated many times over (see Compiègne et al. 2011; Jones et al. 2013 or the class by Karine Demyk last week)

Some properties of the MRN

Abundance as a function of the size :

$$N(a < x) = C\int_{a_{\min}}^{x} a^{-3.5}da = N_0\frac{\left(a_{\min}^{-2.5} - x^{-2.5}\right)}{\left(a_{\min}^{-2.5} - a_{\max}^{-2.5}\right)}$$

$$N(y < a < x) = N_0\frac{\left(y^{-2.5} - x^{-2.5}\right)}{\left(a_{\min}^{-2.5} - a_{\max}^{-2.5}\right)}$$



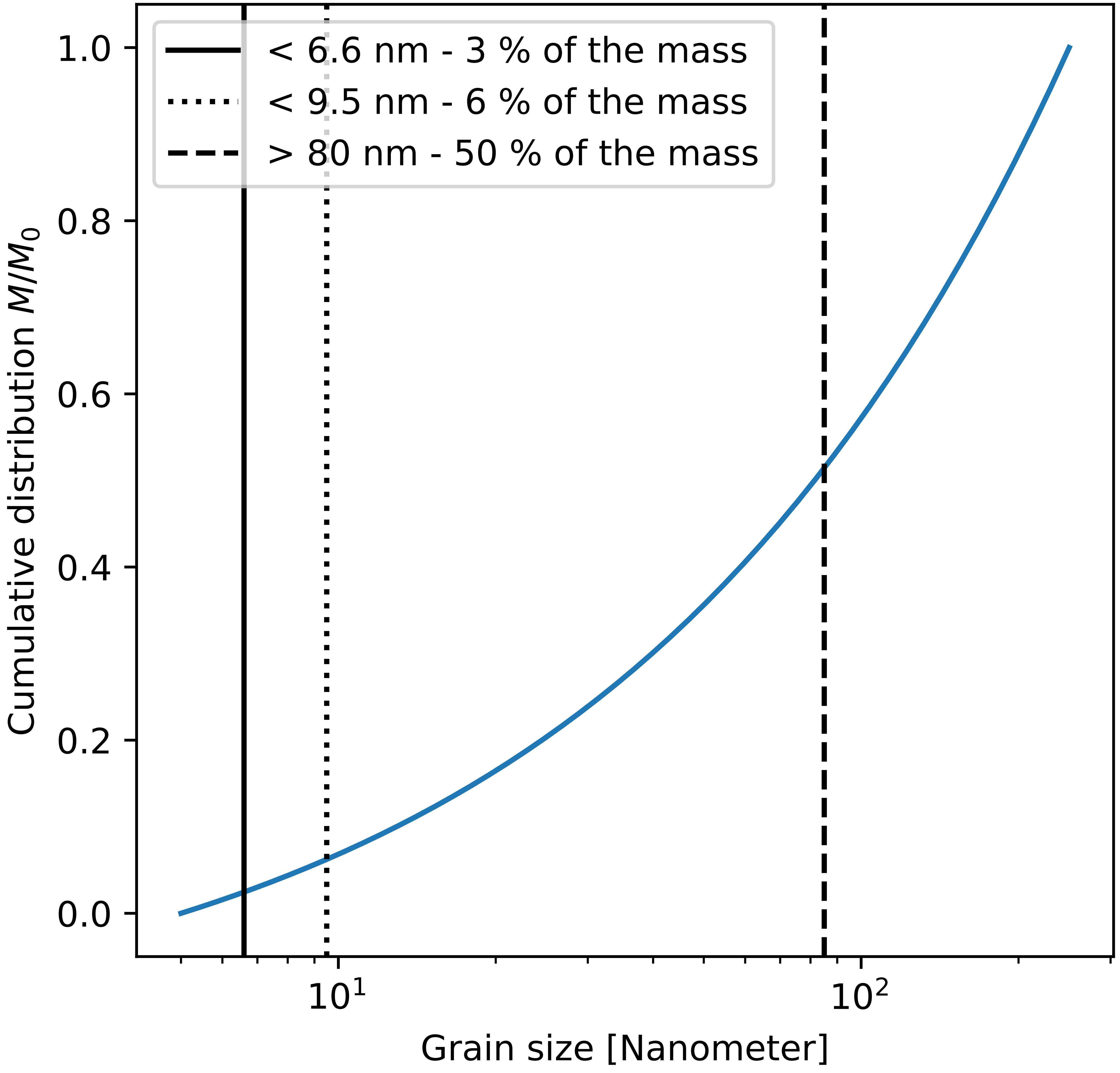
For the MRN : small grains vastly outnumber larger grains!

Some properties of the MRN

$$dm = m\frac{dn}{da}da \propto a^{3+\alpha}$$

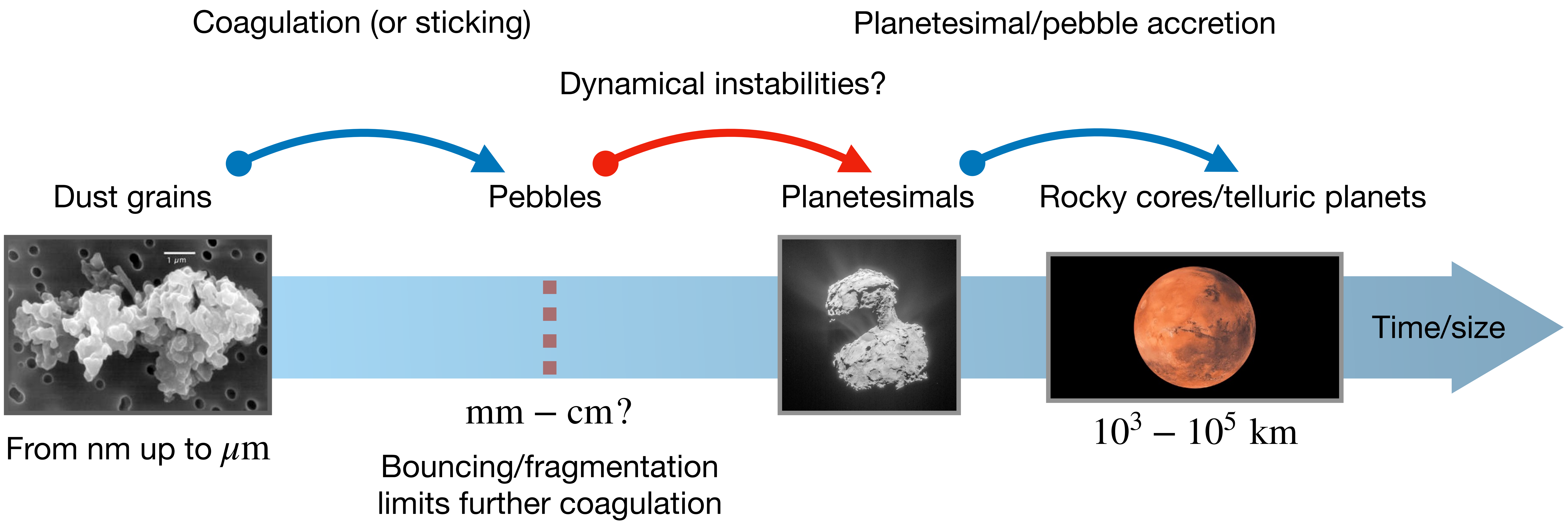
$$M(x) \propto \int_{a_{\min}}^{x} \frac{1}{\sqrt{a}}da$$

$$M(x) = M_0\frac{\sqrt{x}-\sqrt{a_{\min}}}{\sqrt{a_{\max}}-\sqrt{a_{\min}}}$$



For the MRN: the essential of the mass is contained in the larger grain

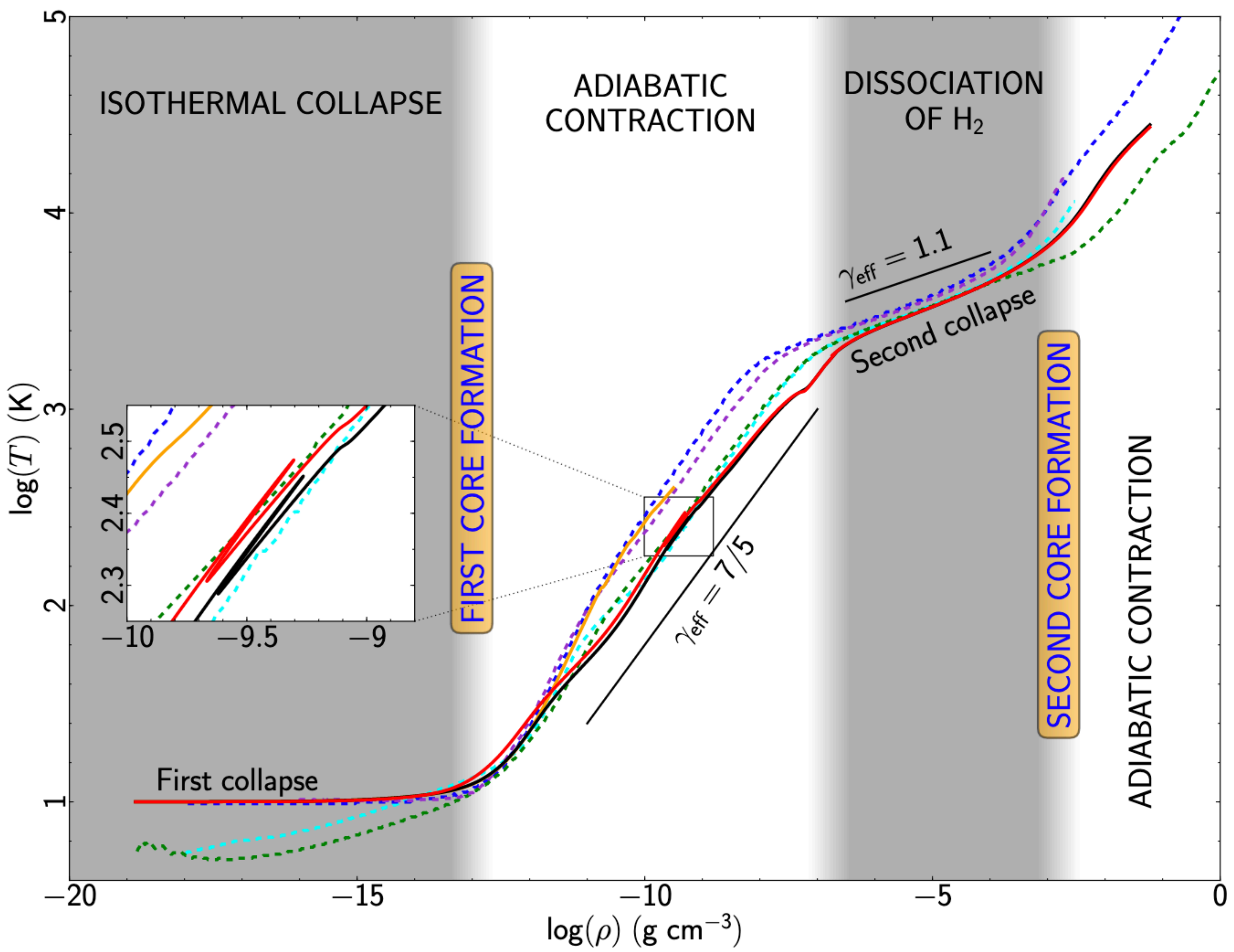
Planet formation : the rocky cores of planets form from the dust



Thermodynamics : Heating & Cooling of the ISM

- **Protostellar collapse :** see *(Larson 1969)*
  - IR photons emitted by the dust allow the collapse by radiating away the gravitational energy

  → cooling by dust emission
  - **Note :** The collapse stops (1rst Larson core) when the cloud becomes optically thick to the IR emitted by the dust because of dust opacity.

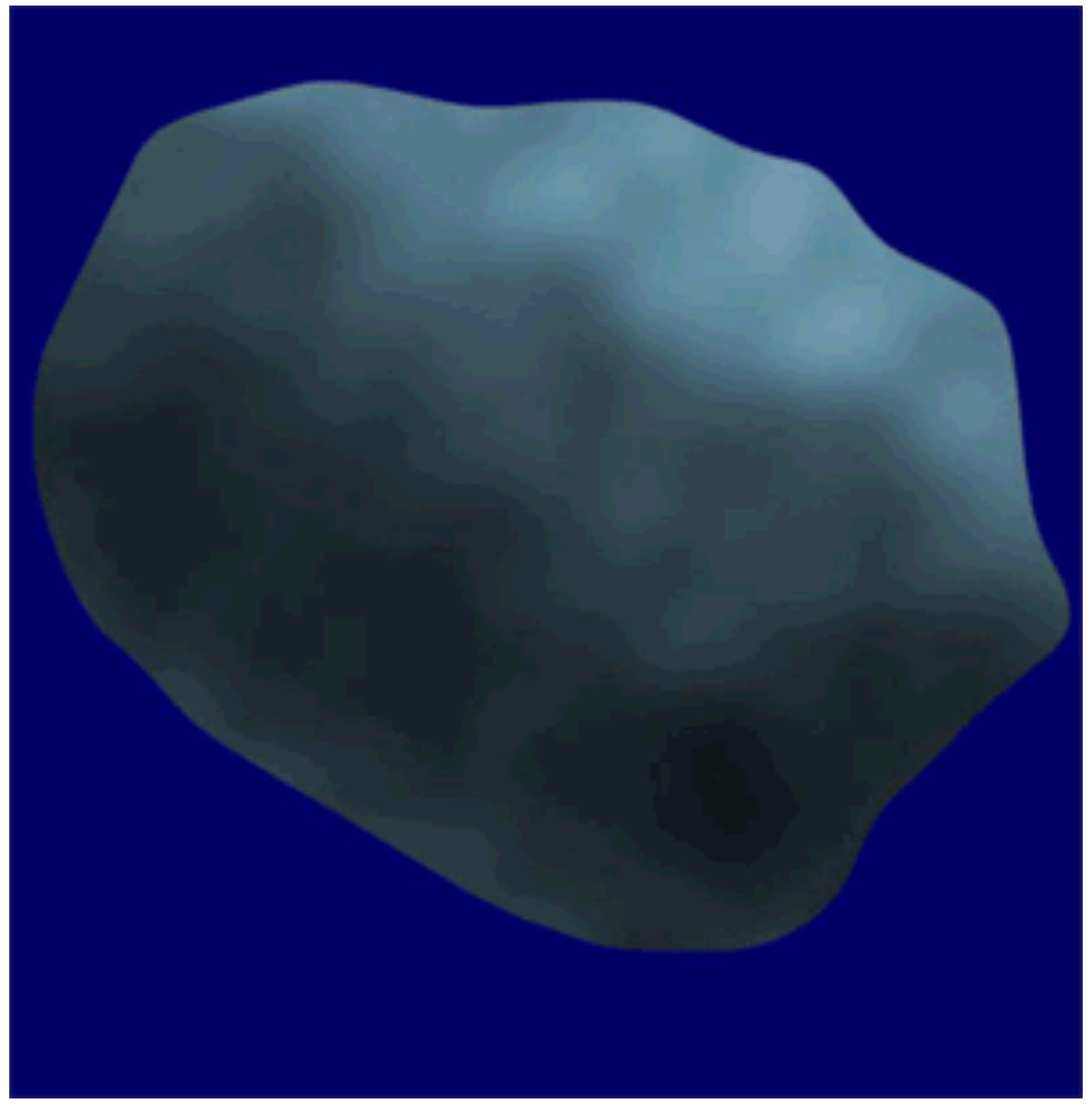


e.g. Vaytet et al. (2018)

- **Photodissociative regions (PDRs) :** see *Hollenbach et al. (1991)*
  - The photoelectric effect on small dust grains (an PaHs) dominates

Chemistry and charging : Grain surface is a catalyser for many reactions

- A lot of molecules form on grain surface by adsorption/desorption: Including H_2



See Simon Glover's class for more details on that

- Grains are very important for the charge budget of the ISM (see e.g. *Draine & Sutin 1987, Ivlev et al. 2015, Weingartner & Draine 2000*)
    1. They can collect negative or positive charges via collisions with electrons and ions
    2. They can get positively charged by the photoelectric effect
    3. The may recombine electron and ions on their surface at high density

→ This is very important for protostellar disk formation !

Dust dynamics

1. Drag force & drag regimes

2. Fluid description of the dust

3. Dust dynamics in various environments

4. Role of the grains in the MHD equations

Dust dynamics

1. Drag force & drag regimes

Drag force

Dust dynamics is largely controlled by gas-dust collisions

They yield a **drag force :** $\overrightarrow{F}_{\text{drag}} \equiv -m_{\text{grain}} \frac{\Delta \overrightarrow{v}}{t_{\text{s}}}$

- $\Delta \overrightarrow{v} \equiv \overrightarrow{v}_d - \overrightarrow{v}_g$ is the differential velocity between the gas and the dust
- Stopping time t_{s} : time for a dust grain to adjust to a change of velocity in the gas.

Stopping time: Epstein regime

Epstein (1924)

$$t_s = \frac{\rho_{grain} a_{\rm grain}}{\rho_{\rm g} w_{\rm th}} \text{ or } t_s = \sqrt{\frac{\pi\gamma}{8}} \frac{\rho_{grain} a_{\rm grain}}{\rho_{\rm g} c_{\rm s}}$$, $c_{\rm s}$ = gas sound speed

$a_{\rm grain}$ grain radius, $\rho_{\rm grain}$ grain intrinsic density

$\rho_{\rm g}$ gas density, $w_{\rm th} = \sqrt{\frac{8 k_B T_g}{\pi \mu m_H}}$ gas thermal speed

Epstein regime valid for :

- $m_{\rm grain} \gg m_H$: always quite safe even for nm grains.
- $a_{\rm grain} < \frac{4}{9}\lambda_g$, grains small compared to gas mean free path > unperturbed local Maxwellian distribution
- collisionless dust (compared with gas-grain collisions)
- $\Delta v \ll w_{\rm th}$

(also require spherical grains)

Stopping time: Kwok correction *Kwok (1975)*

$$t_s = \frac{\rho_{grain} a_{\rm grain}}{\rho_{\rm g} w_{\rm th}} \sqrt{1 + \frac{9}{128\pi}\mathcal{M}_d^2}, \quad \mathcal{M}_d \equiv \frac{\Delta v}{c_s}$$ dust Mach number in gas frame

Valid for:

- $m_{\rm grain} \gg m_H$
- $a_{\rm grain} < \frac{4}{9}\lambda_g$, grains small compared to gas mean free path > unperturbed local Maxwellian distribution
- collisionless dust (compared with gas-grain collisions)
- ~~$\Delta v \ll w_{\rm th}$~~

Note: the Kwok correction is not really the correct solution for all velocity regimes but rather an interpolation of the two extreme regimes.

The correct all regime expression is complicated and can be found in Laibe & Price (2012b)

Stopping time: Stokes regime

Probstein & Fassio (1969); Whipple (1972)

$$\overrightarrow{F}_{\rm drag} \equiv -\frac{1}{2} C_{\rm drag} \rho_{\rm g} \pi a^2_{\rm grain} |\Delta v| \Delta \overrightarrow{v}$$

$$C_{\rm drag} = \begin{cases} 24{\rm Re}^{-1} \text{ if } {\rm Rd} < 1, \\ 24{\rm Re}^{-0.6} \text{ if } 1 < {\rm Rd} < 800, \\ 0.44 \text{ if } 800 < {\rm Rd}. \end{cases}$$

${\rm Re} = \dfrac{2{\rm a}_{\rm grain} \Delta {\rm v}}{\nu}$ is the local Reynolds number at the vicinity of the grain

Stokes regime valid for $a_{\rm grain} > \dfrac{4}{9} \lambda_g$

Best regime for ISM grains ?

Gas mean free-path

$$\lambda_g = \frac{1}{n_g \sigma_{\mathrm{H}_2}}$$

- **Diffuse ISM (for H2 molecules) :**

$$\lambda_g = 5 \times 10^{16}\mathrm{cm} \left(\frac{n_g}{1\ \mathrm{cm}^{-3}} \right)^{-1} \left(\frac{\sigma_{H_2}}{2 \times 10^{15}\mathrm{cm}^2} \right)$$ ~ 340 au !

→ We are completely safe in the Epstein regime (Kwok correction might be needed).

- **Protoplanetary disks:** assuming $n_g \sim 10^{14}\ \mathrm{cm}^{-3}$, we find $\lambda_g \sim 2\ \mathrm{cm}$, when grain grow the Stokes regime might become relevant

Stokes number

Assuming a dynamical timescale $t_{\rm dyn}$

$$S_t \equiv \frac{t_s}{t_{\rm dyn}}$$

Regimes

$S_t \ll 1$, the dust adjusts quickly to the gas -> strong coupling regime

$\mathrm{St} \sim 1$ is intermediate. Strong variations of dust-to-gas ratio are expected

$S_t \gg 1$, the dust does not adjust to the gas -> weak coupling regime

Stokes number : in protoplanetary disks

$$\mathrm{St} = \sqrt{\frac{\pi\gamma}{8}}\frac{\rho_{grain}a_{\mathrm{grain}}}{\rho_{\mathrm{g}}c_{\mathrm{s}}}\Omega_K$$

It is customary to consider a mid-plane power law profile $\rho = \rho_0 \left(\frac{R}{R_0}\right)^{\alpha}$ and a disk scale height $H \equiv \left(\frac{H}{R}\right) R$ such as

$c_s = H\Omega_K$

Which yields

$$\mathrm{St} \sim 2.3 \times 10^{-2} \left(\frac{\rho_{\mathrm{grain}}}{2.3\ \mathrm{g/cc}}\right)\left(\frac{a_{\mathrm{grain}}}{1\ \mathrm{cm}}\right)\left(\frac{\rho_0}{10^{-10}\ \mathrm{g/cc}}\right)^{-1}\left(\frac{R}{1\ \mathrm{au}}\right)^{-1}$$, assuming $\alpha = -2$ and $\frac{H}{R} = 0.05$

Dust grains are relatively well coupled in pp. disks.

But lets keep in mind that this depends on

—> The disks properties (density, temperature)

—> The position in the disk -> At 50 au $\mathrm{St} \sim 1$ for this profile

Stokes number : during the protostellar collapse

$$\mathrm{St} = t_s/t_{\mathrm{ff}} = \sqrt{\frac{32G\gamma}{8}}\frac{\rho_{grain}a_{\mathrm{grain}}}{c_{\mathrm{s}}\sqrt{\mu_g m_H}}\frac{1}{\sqrt{n_g}}$$

reminder $t_{\mathrm{ff}} = \sqrt{\frac{3\pi}{32G\rho}}$

This yields $\mathrm{St} \sim 1 \times 10^{-2} \left(\frac{a_{\mathrm{grain}}}{1\ \mu\mathrm{m}}\right)\left(\frac{\rho_{\mathrm{grain}}}{2.3\mathrm{g/cc}}\right)\left(\frac{n_g}{10^5\ \mathrm{cm}^{-3}}\right)^{-1/2}$ with $\mu_g = 2.3,\ T = 10\ \mathrm{K}$

So ISM-like grains are relatively well coupled to the gas during the protostellar collapse (even at low densities)

But grains could be larger than this as (debated) evidence point out :

> 100 microns in protostellar enveloppes : Stokes ~ 1

Note that $\mathrm{St} \propto \frac{1}{\sqrt{n_g}}$ so, as the collapse proceed (unless they grow), grains become more and more coupled with the gas.

2. Fluid description of the dust

Multifluid equations *(Saffman 1962)*

$$\frac{\partial \rho_{\rm g}}{\partial t} + \nabla \cdot \rho_{\rm g} \vec{v}_{\rm g} = 0$$

Gas mass conservation

$$\frac{\partial \rho_{\rm d}}{\partial t} + \nabla \cdot \rho_{\rm d} \vec{v}_{\rm d} = 0$$

Dust mass conservation

$$\frac{\partial \rho_{\rm g} \vec{v}_{\rm g}}{\partial t} + \nabla (\rho_{\rm g} \vec{v}_{\rm g} \otimes \vec{v}_{\rm g} + P_{\rm g} \mathbb{I}) = \rho_{\rm g} \vec{f} + \frac{\rho_d}{t_s} \overrightarrow{\Delta v}$$

Gas momentum conservation

$$\frac{\partial \rho_{\rm d} \vec{v}_{\rm d}}{\partial t} + \nabla \rho_{\rm d} \vec{v}_{\rm d} \otimes \vec{v}_{\rm d} = \rho_{\rm d} \vec{f} - \frac{\rho_d}{t_s} \overrightarrow{\Delta v}$$

Dust momentum conservation

$$\frac{\partial E_{\rm g}}{\partial t} + \nabla \cdot (E_{\rm g} + P_{\rm g}) \vec{v}_{\rm g} = \frac{\rho_d}{t_s} \overrightarrow{\Delta v} \cdot \overrightarrow{\Delta v}$$

Gas energy conservation

Dust is pressureless, no energy equation needed, but that also mean fluid approximation only valid if (St <1) and if the fluid approximation is valid for the gas.

A useful formalism: the monofluid

Can we reformulate the multi-fluid equations using one fluid ?

- One density $\rho \equiv \rho_{\mathrm{g}} + \rho_{\mathrm{d}}$
- One advection velocity $\overrightarrow{v} \equiv \dfrac{\rho_{\mathrm{d}}\overrightarrow{v}_{\mathrm{d}} + \rho_{\mathrm{g}}\overrightarrow{v}_{\mathrm{g}}}{\rho}$
- Several phases (gas and each dust sizes) $\epsilon \equiv \rho_{\mathrm{d}}/\rho$ and $\Delta\overrightarrow{v} \equiv \overrightarrow{v}_{\mathrm{d}} - \overrightarrow{v}_{\mathrm{g}}$

Note: Same methods employed to derive magnetohydrodynamics (MHD) : ions, electrons and neutrals are all part of a monofluid.

Laibe & Price 2014 (a,b,c) See also *Price & Laibe 2015; Lin & Youdin 2017; Hutchison et al., 2018 and Lebreuilly et al., 2019*

Deriving the monofluid equations

$$\frac{\partial \rho_g}{\partial t} + \nabla \cdot \rho_g \vec{v}_g = 0$$

$$\frac{\partial \rho_d}{\partial t} + \nabla \cdot \rho_d \vec{v}_d = 0$$

$$\longrightarrow \quad \frac{\partial \rho_g}{\partial t} + \frac{\partial \rho_d}{\partial t} + \nabla \cdot \rho_g \vec{v}_g + \nabla \cdot \rho_d \vec{v}_d = 0$$

$$\downarrow$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \vec{v} = 0$$

Remember : $\vec{v} \equiv \frac{\rho_d \vec{v}_d + \rho_g \vec{v}_g}{\rho}$

$$\frac{\partial \rho_d}{\partial t} + \nabla \cdot \rho_d \vec{v}_d = 0 \quad \longrightarrow \quad \frac{\partial \rho \epsilon}{\partial t} + \nabla \cdot \left[\rho \epsilon \left(\vec{v} + (1 - \epsilon) \overrightarrow{\Delta v} \right) \right] = 0$$

Laibe & Price 2014 (a,b,c)

Full monofluid equations

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -\rho(\vec{\nabla}\cdot\vec{v}),$$

$$\frac{\mathrm{d}\vec{v}}{\mathrm{d}t} = -\frac{\nabla P_{\mathrm{g}}}{\rho} + \vec{f} - \frac{1}{\rho}\nabla\left(\epsilon(1-\epsilon)\rho\overrightarrow{\Delta v}\otimes\overrightarrow{\Delta v}\right),$$

$$\frac{\mathrm{d}\epsilon}{\mathrm{d}t} = -\frac{1}{\rho}\vec{\nabla}\cdot\left(\epsilon(1-\epsilon)\rho\overrightarrow{\Delta v}\right)$$

$$\frac{\mathrm{d}\overrightarrow{\Delta v}}{\mathrm{d}t} = \frac{\nabla P_{\mathrm{g}}}{(1-\epsilon)\rho} - \frac{\overrightarrow{\Delta v}}{t_{\mathrm{s}}} - (\overrightarrow{\Delta v}\cdot\vec{\nabla})\vec{v} + \frac{1}{2}\nabla\left((2\epsilon-1)\overrightarrow{\Delta v}\cdot\overrightarrow{\Delta v}\right)$$

$$+(1-\epsilon)\overrightarrow{\Delta v}\times(\nabla\times(1-\epsilon)\overrightarrow{\Delta v}) - \epsilon\overrightarrow{\Delta v}\times(\nabla\times\epsilon\overrightarrow{\Delta v}),$$

$$\frac{\mathrm{d}e_{\mathrm{g}}}{\mathrm{d}t} = -\frac{P_{\mathrm{g}}}{\rho(1-\epsilon)}\nabla\cdot\left(\vec{v}-\epsilon\Delta\vec{v}\right) + (\epsilon\Delta\vec{v}\cdot\nabla)e_{\mathrm{g}} + \epsilon\frac{\overrightarrow{\Delta v}\cdot\overrightarrow{\Delta v}}{t_{\mathrm{s}}}$$

Laibe & Price 2014 (a,b,c); Lebreuilly et al. 2019

Monofluid: small grains approximation

Diffusion approximation + terminal velocity approximation (valid for small Stokes)

Second order in St — First order in St/Decaying in a stopping time

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -\rho(\vec{\nabla}\cdot\vec{v}),$$

$$\frac{\mathrm{d}\vec{v}}{\mathrm{d}t} = -\frac{\nabla P_{\mathrm{g}}}{\rho} + \vec{f} - \frac{1}{\rho}\nabla\cdot\left(\epsilon(1-\epsilon)\rho\overrightarrow{\Delta v}\otimes\overrightarrow{\Delta v}\right),$$

$$\frac{\mathrm{d}\epsilon}{\mathrm{d}t} = -\frac{1}{\rho}\vec{\nabla}\cdot\left(\epsilon(1-\epsilon)\rho\overrightarrow{\Delta v}\right)$$

$$\frac{\mathrm{d}\overrightarrow{\Delta v}}{\mathrm{d}t} = \frac{\nabla P_{\mathrm{g}}}{(1-\epsilon)\rho} - \frac{\overrightarrow{\Delta v}}{t_{\mathrm{s}}} - (\overrightarrow{\Delta v}\cdot\vec{\nabla})\vec{v} + \frac{1}{2}\nabla\left((2\epsilon-1)\overrightarrow{\Delta v}\cdot\overrightarrow{\Delta v}\right)$$
$$+(1-\epsilon)\overrightarrow{\Delta v}\times(\nabla\times(1-\epsilon)\overrightarrow{\Delta v}) - \epsilon\overrightarrow{\Delta v}\times(\nabla\times\epsilon\overrightarrow{\Delta v}),$$

$$\frac{\mathrm{d}e_{\mathrm{g}}}{\mathrm{d}t} = -\frac{P_{\mathrm{g}}}{\rho(1-\epsilon)}\nabla\cdot\left(\vec{v} - \epsilon\Delta\vec{v}\right) + (\epsilon\Delta\vec{v}\cdot\nabla)e_{\mathrm{g}} + \epsilon\frac{\overrightarrow{\Delta v}\cdot\overrightarrow{\Delta v}}{t_{\mathrm{s}}}$$

→

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -\rho(\vec{\nabla}\cdot\vec{v}),$$

$$\frac{\mathrm{d}\vec{v}}{\mathrm{d}t} = -\frac{\nabla P_{\mathrm{g}}}{\rho} + \vec{f},$$

$$\frac{\mathrm{d}\epsilon}{\mathrm{d}t} = -\frac{1}{\rho}\vec{\nabla}\cdot\left(\epsilon t_{\mathrm{s}}\nabla P_{\mathrm{g}}\right)$$

$$\frac{\mathrm{d}e_{\mathrm{g}}}{\mathrm{d}t} = -\frac{P_{\mathrm{g}}}{\rho(1-\epsilon)}\nabla\cdot\vec{v}$$

Laibe & Price 2014 (a,b,c); Lebreuilly et al. 2019

Monofluid: terminal velocity approximation

Lets spend a moment on the velocity equation:

$$\frac{\mathrm{d}\overrightarrow{\Delta v}}{\mathrm{d}t} = \frac{\nabla P_{\mathrm{g}}}{(1-\epsilon)\rho} - \frac{\overrightarrow{\Delta v}}{t_{\mathrm{s}}} - (\overrightarrow{\Delta v}\cdot\vec{\nabla})\vec{v} + \frac{1}{2}\nabla\left((2\epsilon-1)\overrightarrow{\Delta v}\cdot\overrightarrow{\Delta v}\right) + (1-\epsilon)\overrightarrow{\Delta v}\times(\nabla\times(1-\epsilon)\overrightarrow{\Delta v}) - \epsilon\overrightarrow{\Delta v}\times(\nabla\times\epsilon\overrightarrow{\Delta v})$$

This yields $\overrightarrow{\Delta v} = \frac{t_{\mathrm{s}}\nabla P_{\mathrm{g}}}{(1-\epsilon)\rho}$, the differential velocity is controlled by the pressure force

—-> dust drifts toward pressure maxima

Or more generally the difference in force budget between the gas and the dust :

$$\overrightarrow{\Delta v} = \frac{t_{\mathrm{s}}\left(\vec{f}_{\mathrm{d}} - \vec{f}_{\mathrm{g}}\right)}{(1-\epsilon)\rho}$$

Laibe & Price 2014 (a,b,c)

Monofluid: small grains approximation

Or terminal velocity approximation + diffusion approximation (valid for small Stokes)

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -\rho(\vec{\nabla} \cdot \vec{v}),$$

$$\frac{\mathrm{d}\vec{v}}{\mathrm{d}t} = -\frac{\nabla P_{\mathrm{g}}}{\rho} + \vec{f},$$

$$\frac{\mathrm{d}\epsilon}{\mathrm{d}t} = -\frac{1}{\rho}\vec{\nabla} \cdot \left(\epsilon t_{\mathrm{s}} \nabla P_{\mathrm{g}}\right)$$

$$\frac{\mathrm{d}e_{\mathrm{g}}}{\mathrm{d}t} = -\frac{P_{\mathrm{g}}}{\rho(1-\epsilon)}\nabla \cdot \vec{v}$$

Laibe & Price 2014 (a,b,c)

Dust as a fluid: generalisation to multiple dust sizes

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -\rho(\nabla \cdot \vec{v}),$$

$$\frac{\mathrm{d}\vec{v}}{\mathrm{d}t} = -\frac{\nabla P_{\mathrm{g}}}{\rho} + \vec{f},$$

$$\frac{\mathrm{d}\epsilon_k}{\mathrm{d}t} = -\frac{1}{\rho}\nabla \cdot \left(\epsilon_k T_{\mathrm{s},k} \nabla P_{\mathrm{g}}\right), \forall k \in [1, \mathcal{N}],$$

$$\frac{\mathrm{d}e_{\mathrm{g}}}{\mathrm{d}t} = -\frac{P_{\mathrm{g}}}{\rho(1-\mathscr{E})}\nabla \cdot \vec{v} + \left(\mathscr{E}\mathscr{T}_{\mathrm{s}} \frac{\nabla P_{\mathrm{g}}}{(1-\mathscr{E})\rho} \cdot \nabla\right) e_{\mathrm{g}},$$

With $\mathscr{E} \equiv \sum_{l=1}^{\mathcal{N}} \epsilon_l$, $T_{\mathrm{s},k} \equiv \frac{t_{\mathrm{s},k}}{1-\epsilon_k} - \sum_{l=1}^{\mathcal{N}} \frac{\epsilon_l}{1-\epsilon_l} t_{\mathrm{s},l}$ and $\mathscr{T}_{\mathrm{s}} \equiv \frac{1}{\mathscr{E}} \sum_{l=1}^{\mathcal{N}} \epsilon_l T_{\mathrm{s},l}$

Note that here we discretised the continuous dust size distribution into $\mathcal{N}$ distinct dust species (also called dust bins).

Laibe & Price 2014 (a,b,c)

3. Dust dynamics in various environments

Settling in protoplanetary disks

e.g. Hoyle 1960; Kusaka et al. 1970; Cameron 1973; Adachi et al. 1976; Handbury & Williams 1977; Coradini et al. 1980

Let's assume a disk at hydrostatic equilibrium : (remember Giuseppe Lodato's talk)

The gas disk is at vertical equilibrium because the vertical gravity is supported by the thermal pressure

$$\frac{\partial P_g}{\partial z} = \rho_g f_{\mathrm{grav},z} = -\rho_g \frac{\mathrm{G}M_\star z}{(R^2+z^2)^{3/2}} = -\rho_g \Omega_K^2 z \left(1+\frac{z^2}{R^2}\right)^{-3/2}$$

But the dust is not supported and is therefore not at equilibrium.

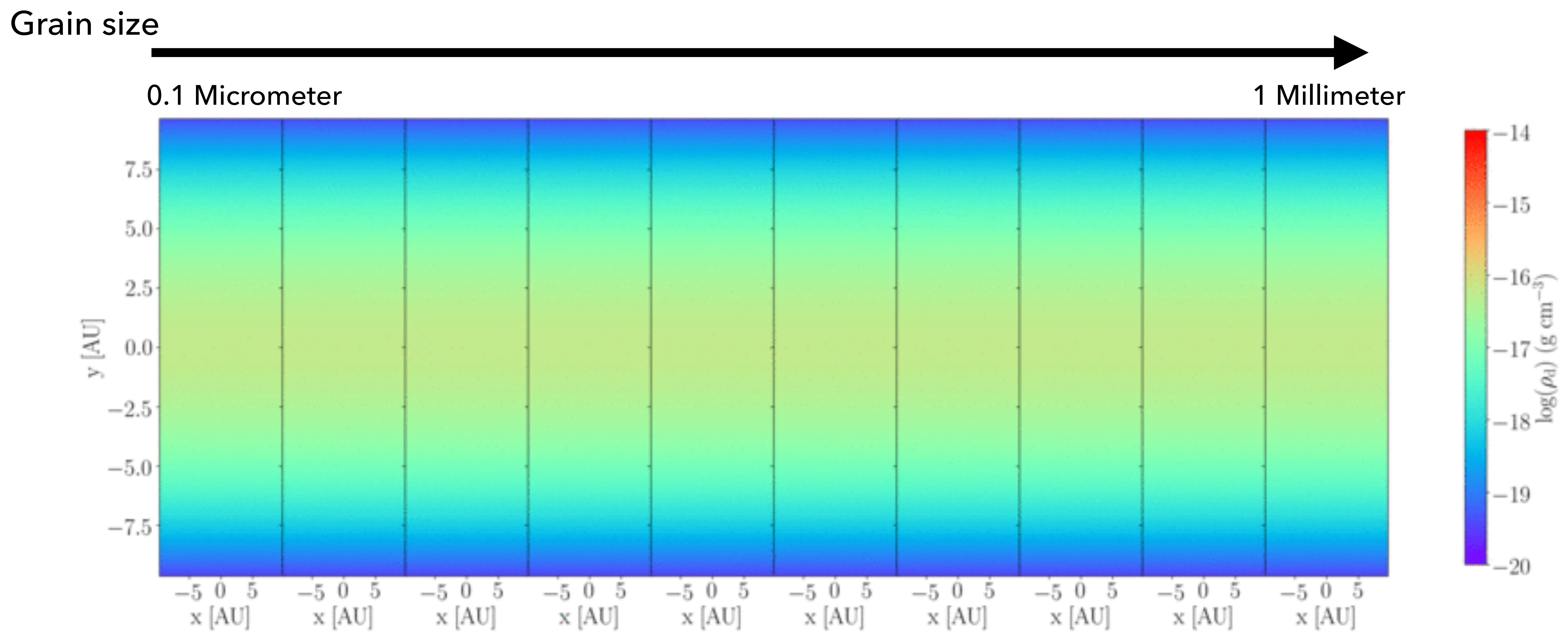
$\overrightarrow{\Delta v} = t_s \frac{\nabla P_g}{\rho_g}$ translates to $v_{d,z} = \frac{t_s}{\rho_g}\frac{\partial P_g}{\partial z}$ since $v_{g,z} = 0$.

Assuming a thin disk

We can easily show that $v_{d,z} = -t_s \Omega_K^2 \frac{z}{\left(1+\frac{z^2}{R^2}\right)^{3/2}}$, or $v_{d,z} = -\mathrm{St}\Omega_K \frac{z}{\left(1+\frac{z^2}{R^2}\right)^{3/2}} \approx -\mathrm{St}\Omega_K z$

Dust grains settle toward the mid-plane increasingly faster with an increasing Stokes number.

Settling in protoplanetary disks



Settling timescale (for small St): $\dfrac{1}{\mathrm{St}\Omega_K}$, for all St : $\dfrac{1}{(\mathrm{St}+\mathrm{St}^{-1})\Omega_K}$

Simulation (with RAMSES) from *Lebreuilly et al. (2019)*

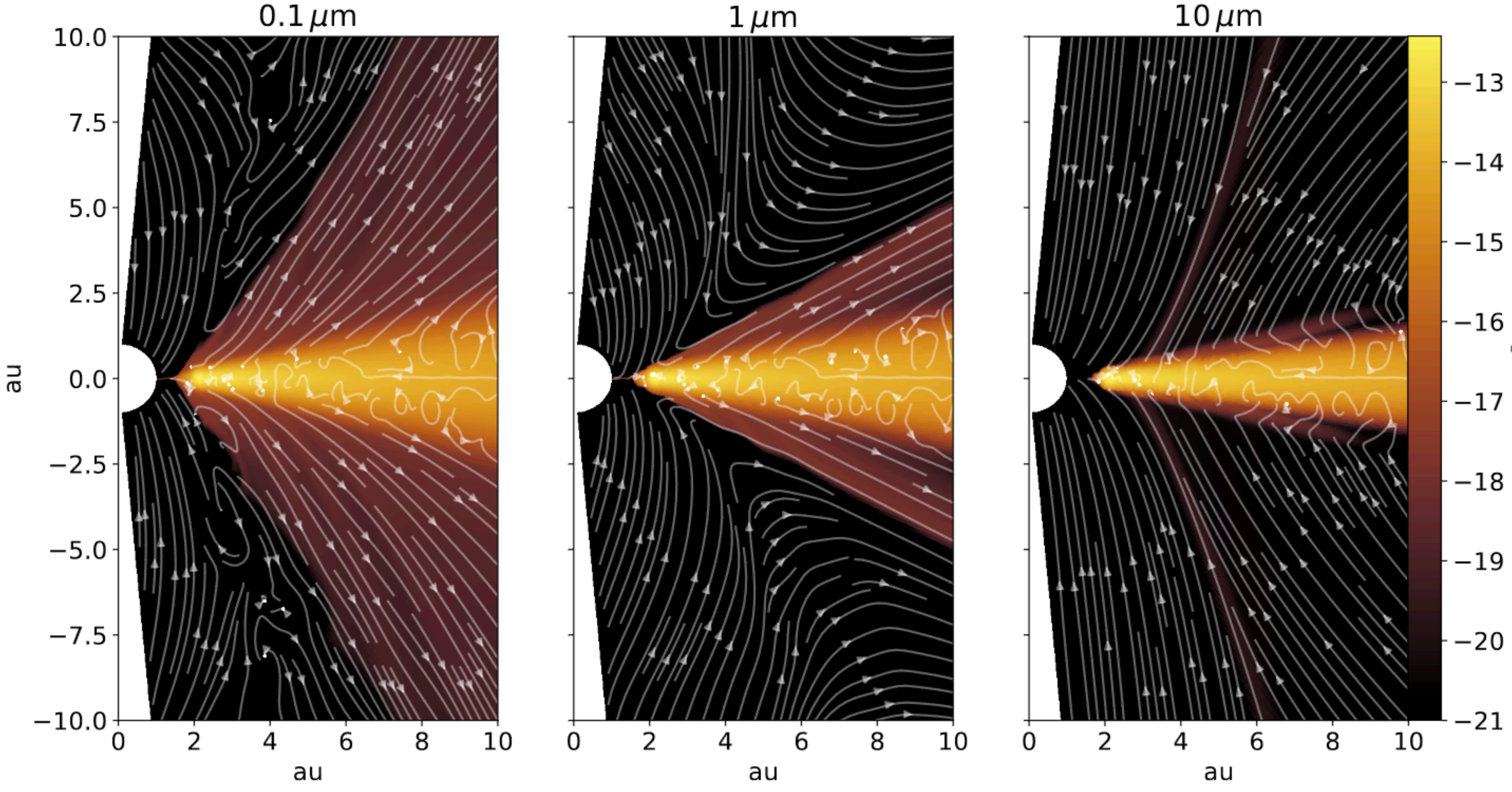
Settling in protoplanetary disks

In principle : dust should settle infinitely no matter the size

In practice : grains can be lifted by

- **Disk winds, protostellar outflows, jets :**
  - e.g. *Riols et al. (2019), Lebreuilly et al. (2020), Tsukamoto et al. (2021)*
- **Turbulence in the disk :**
  - *Fromang & Papaloizou (2006); Carballido (2011)*

Dust settling in a wind-driven disk



Rodenkirch & Dullemond (2022)

Radial drift in protoplanetary disks

Whipple (1973); Weidenschilling (1977); Nakagawa et al. (1986)

Let's assume a disk at hydrostatic equilibrium (This time, we look at the radial equilibrium).

We still have : $\Delta \vec{v_d} = t_s \frac{\nabla P}{\rho_g}$

Assuming no back-reaction from the dust and negligible gas radial velocities

In the radial direction : $\Delta v_{d,R} = v_{d,R} = t_s \frac{1}{\rho_g} \frac{\partial P}{\partial R}$

Assuming $\frac{H}{r} = \mathrm{Cst}$, and a density profile such as $\rho = \rho_0 \left(\frac{R}{R_0} \right)^{\alpha}$, we get $\frac{1}{\rho_g} \frac{\partial P}{\partial R} = \alpha \left(\frac{H}{R} \right)^2 \Omega_K^2 R$

This gives : $v_{d,R} = \alpha \left(\frac{H}{R} \right)^2 \mathrm{St}\, \Omega_K R < 0$ (as alpha is negative), dust drifts inward as it gives AM to the gas (which, in principle, moves outward as a consequence).

Radial drift timescale $\sim \frac{1}{|\alpha| \left(\frac{H}{R} \right)^2 \mathrm{St}\, \Omega_K} \gg t_{\mathrm{settling}}$, radial drift is a much slower process than settling

Note : we used the terminal velocity approximation so this solution is valid for low St

The issue with the radial drift

More general expression is $v_{d,R} = \alpha \left(\frac{H}{R}\right)^2 \frac{1}{\mathrm{St} + \mathrm{St}^{-1}} \Omega_K R$

Dust radial drift is still fast enough so that grains of St =1 drift to the stars very rapidly (in ~ 100 orbits)

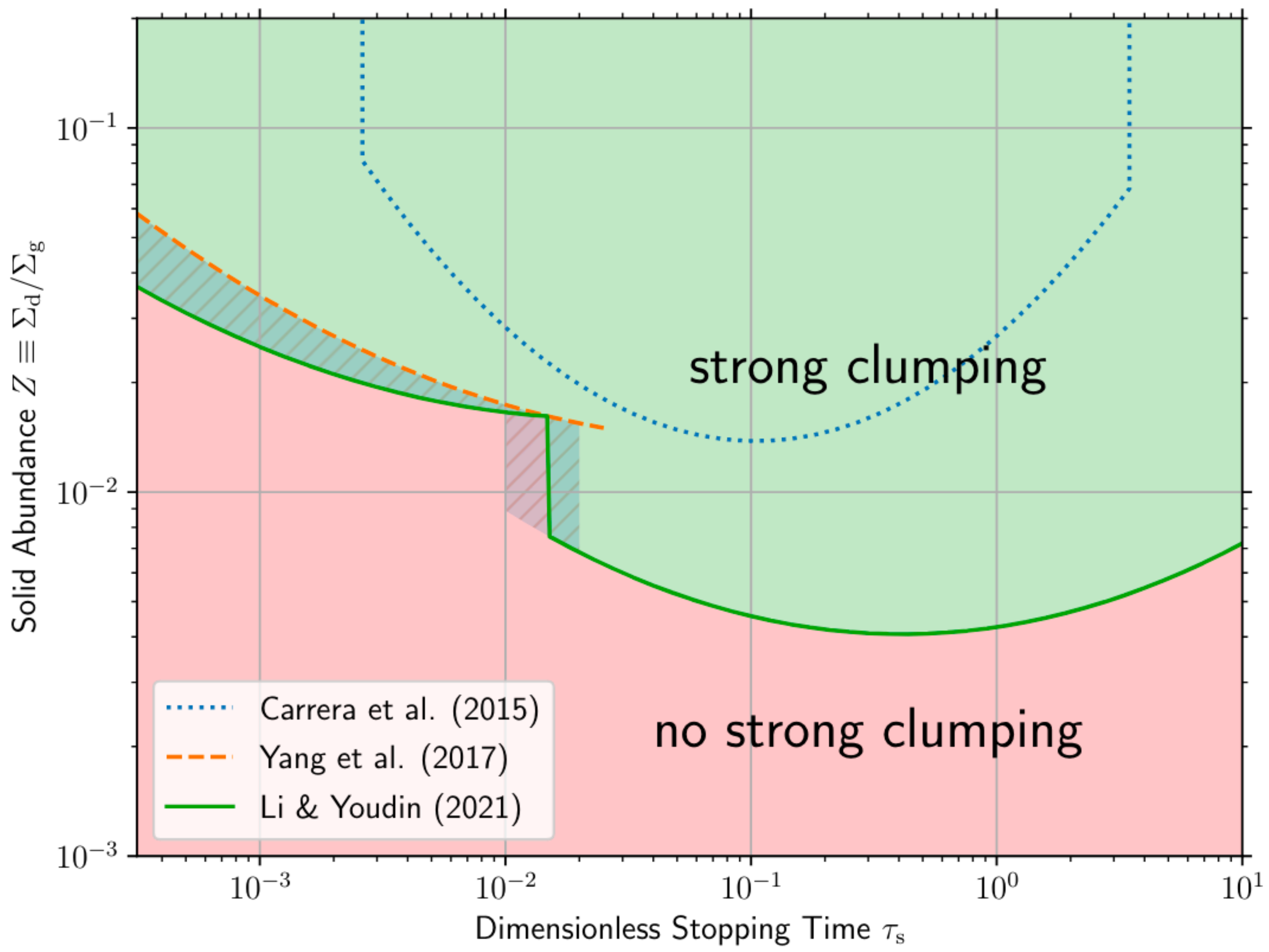
—–> this is called the **radial drift barrier**

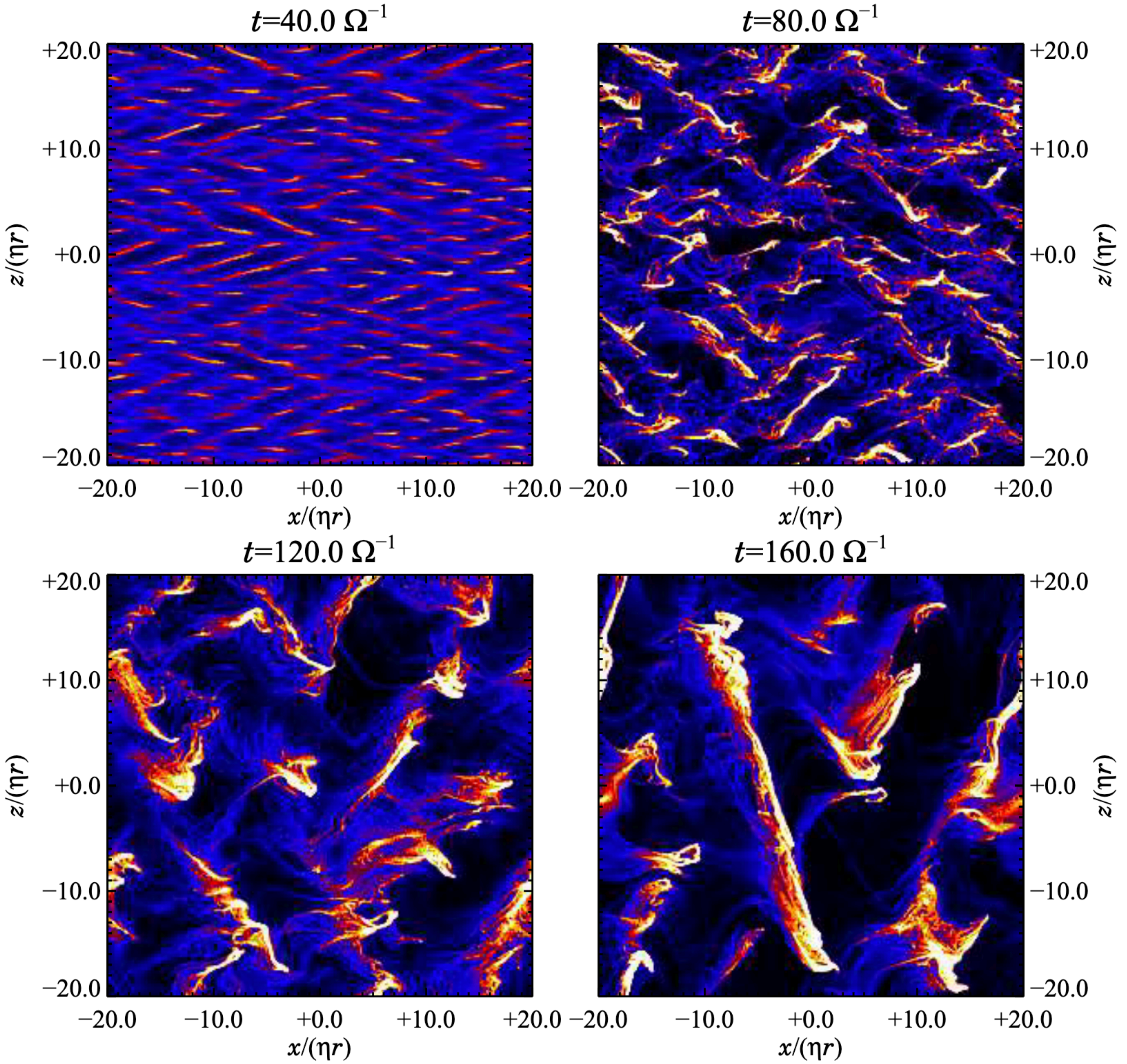
But then how do we form planets ?

Streaming instability

Youdin & Goodman (2005)

See Lesur et al. 2023 (PP7 review)

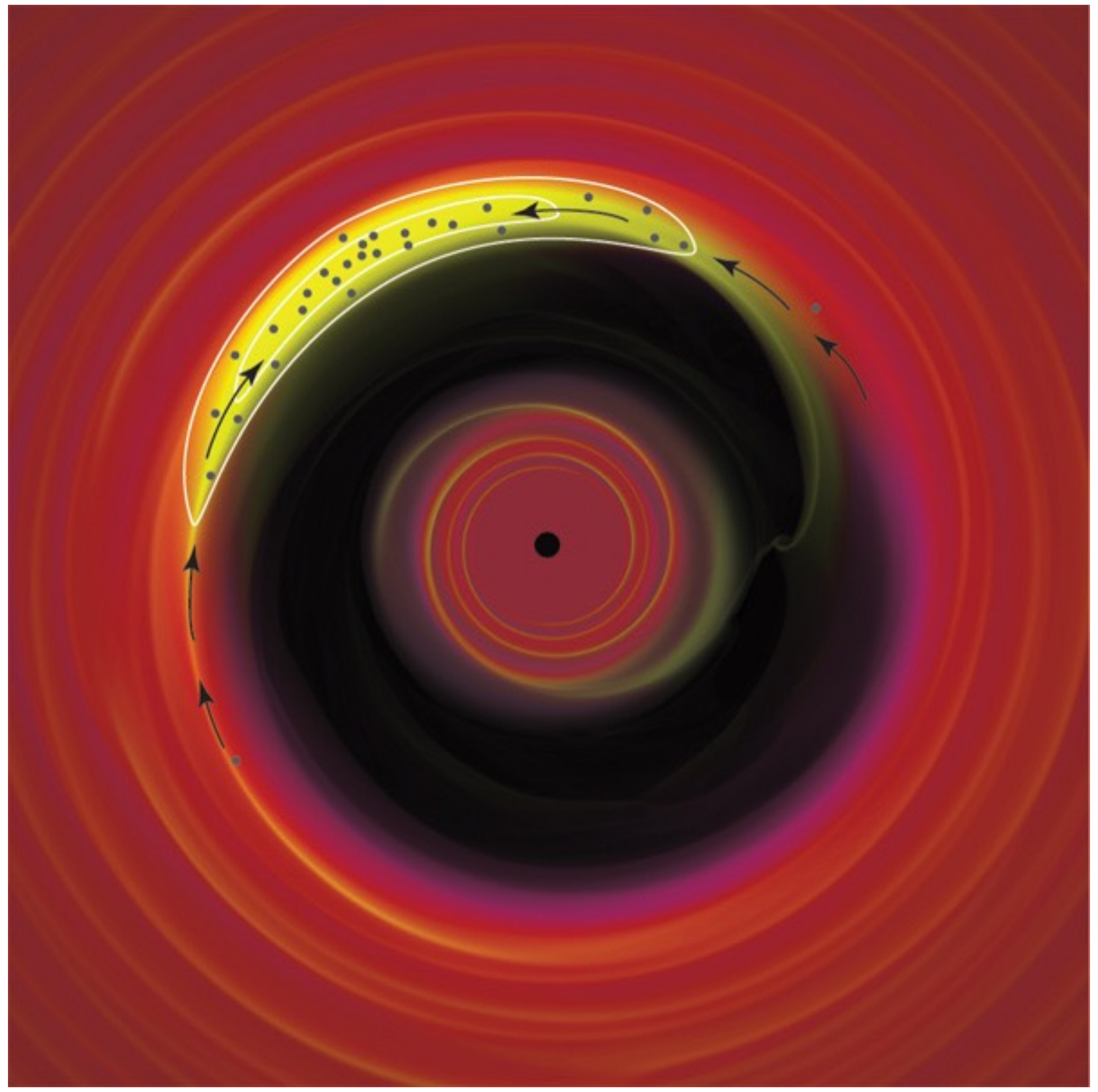




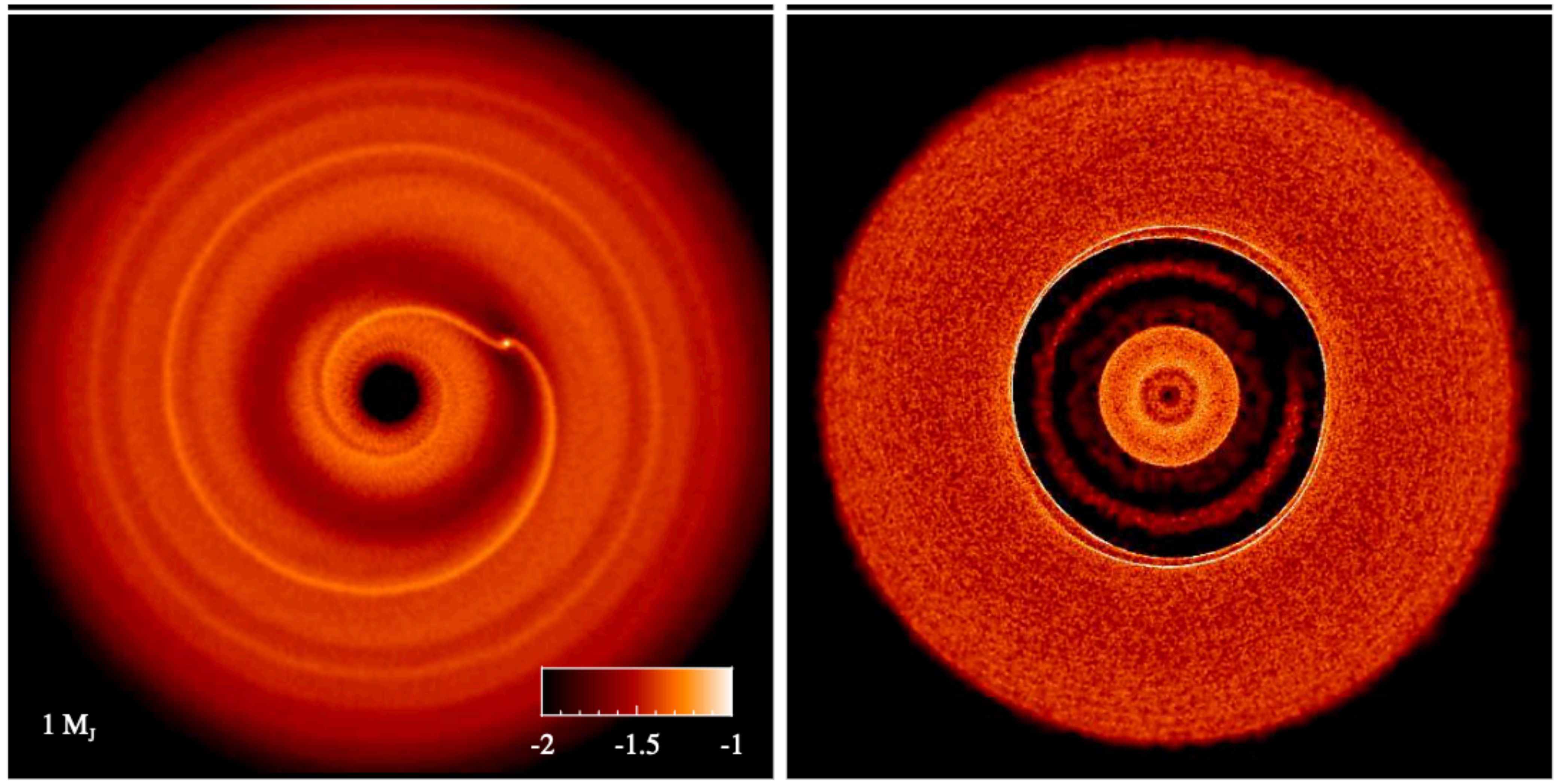
Johansen & Youdin (2007)

See also (among others) Jacquet et al. 2011; Squire and Hopkins 2018a
Zhuravlev 2019; Jaupart and Laibe 2020; Pan 2020
Squire and Hopkins 2020; Krapp et al. (2019); Zhu and Yang (2021); Paardekooper et al. (2020, 2021)

Sub-structures : Other ways to trap dust grains



Vortices :
Lovascio et al. (2019; 2022) Fu et al. (2014); Crnkovic-Rubsamen et al. (2015); Surville & Mayer (2019)



A gap opening planet: but chicken and egg problem

e.g. Dipierro et al. (2016)

Other examples :

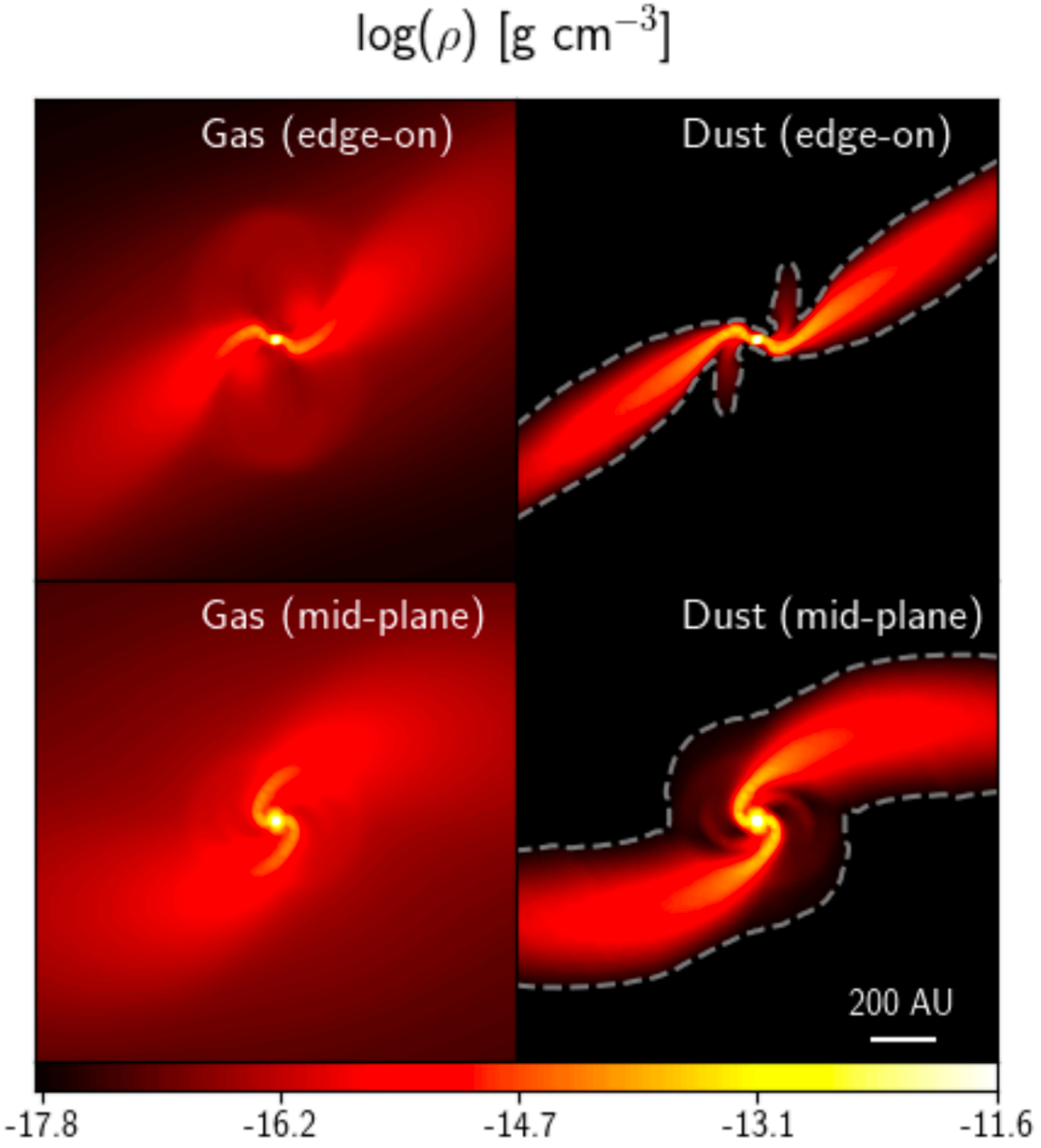
—> Pressure bumps (Pinilla et al. 2012, Taki et al. 2016)

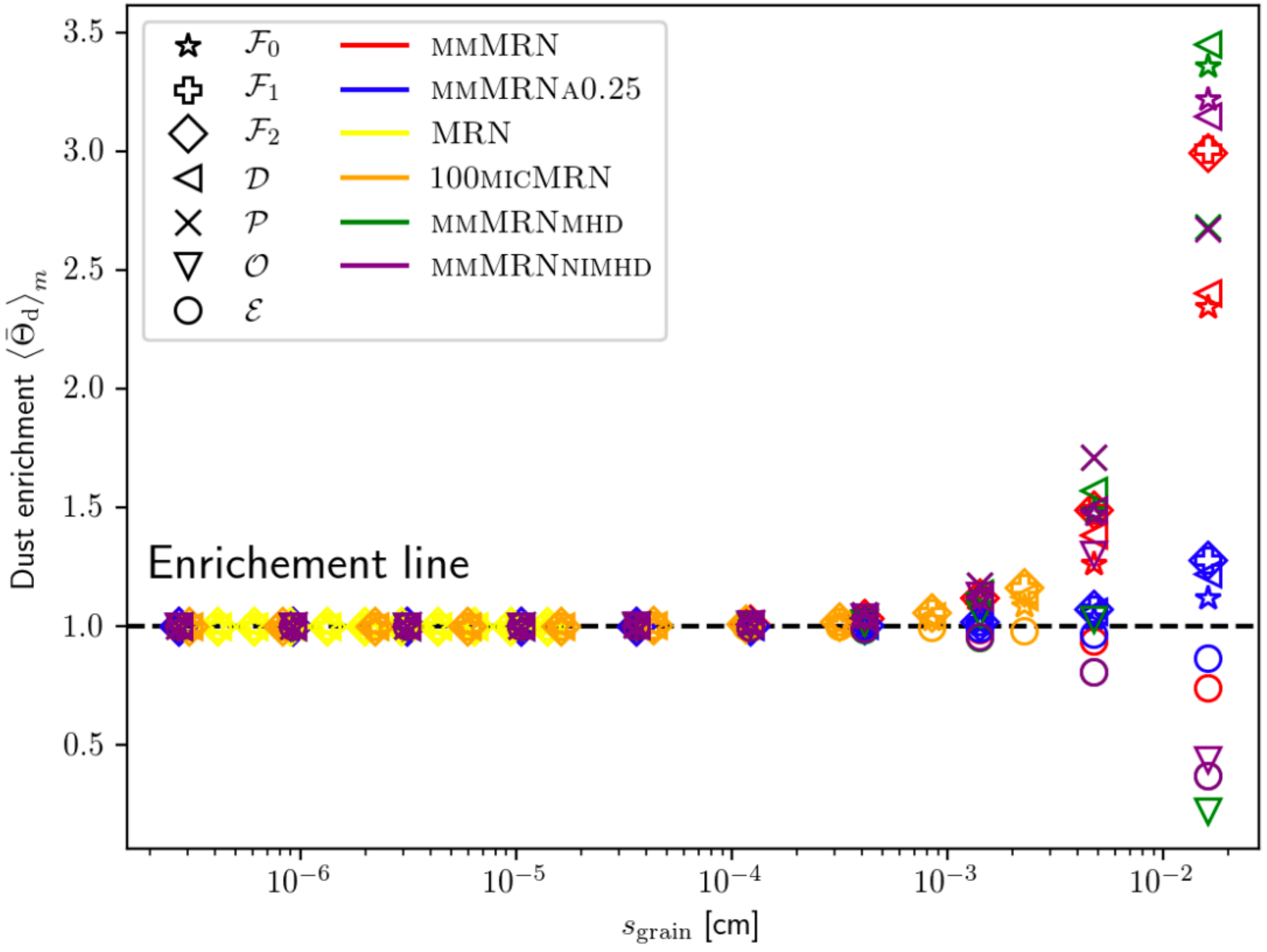
—> Spirals, for e.g. driven by accretion (*Bae et al. 2015*)

—> GI (Dipierro et al. 2015a; Elbakyan et al. 2020; Vorobyov et al., 2019a, b; 2024)

—> Wind driven sub-structures (*Riols et al. 2019*)

Dust dynamics during the protostellar collapse





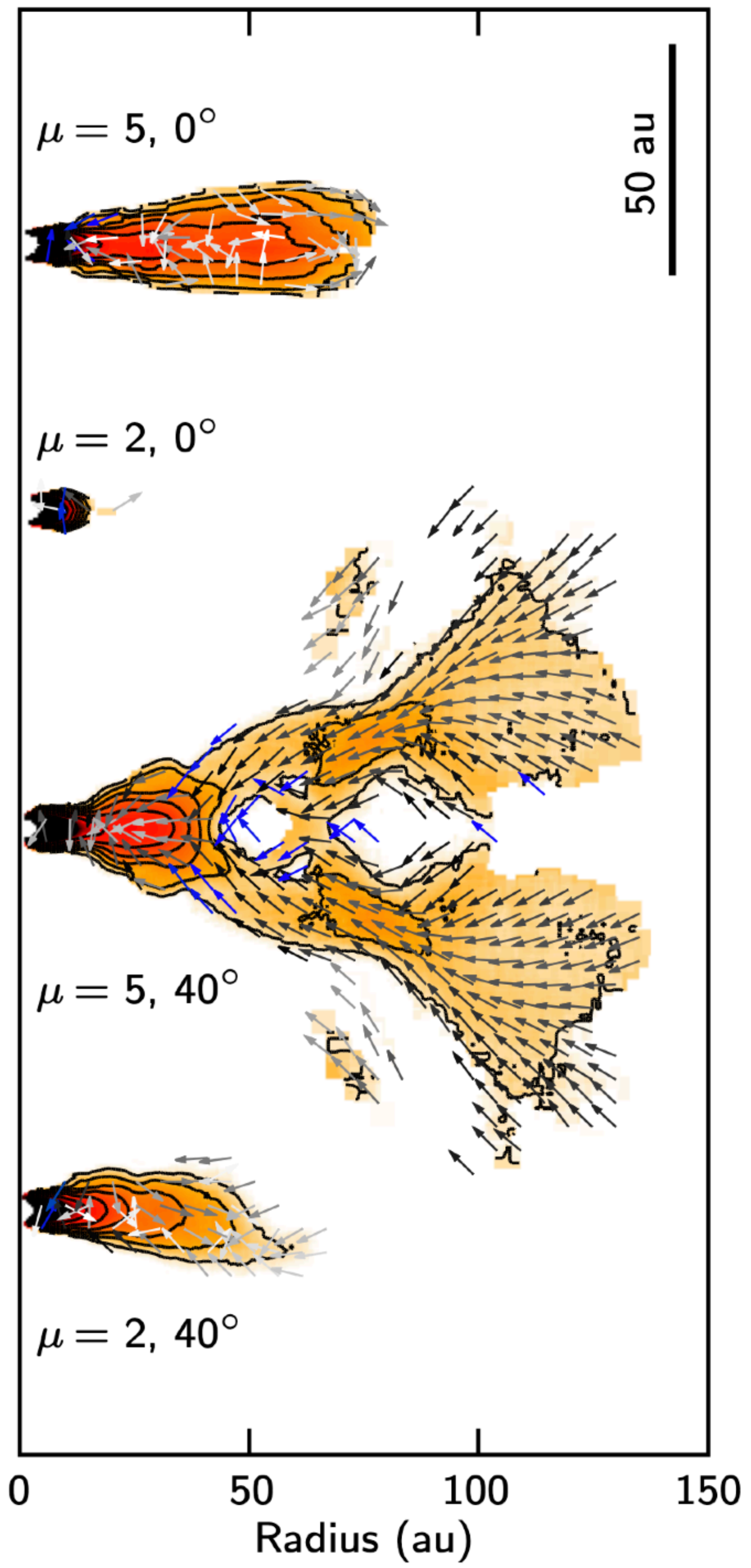
Enrichment ϵ/ϵ_0 in the disk increases exponentially with the grain size (or Stokes number)

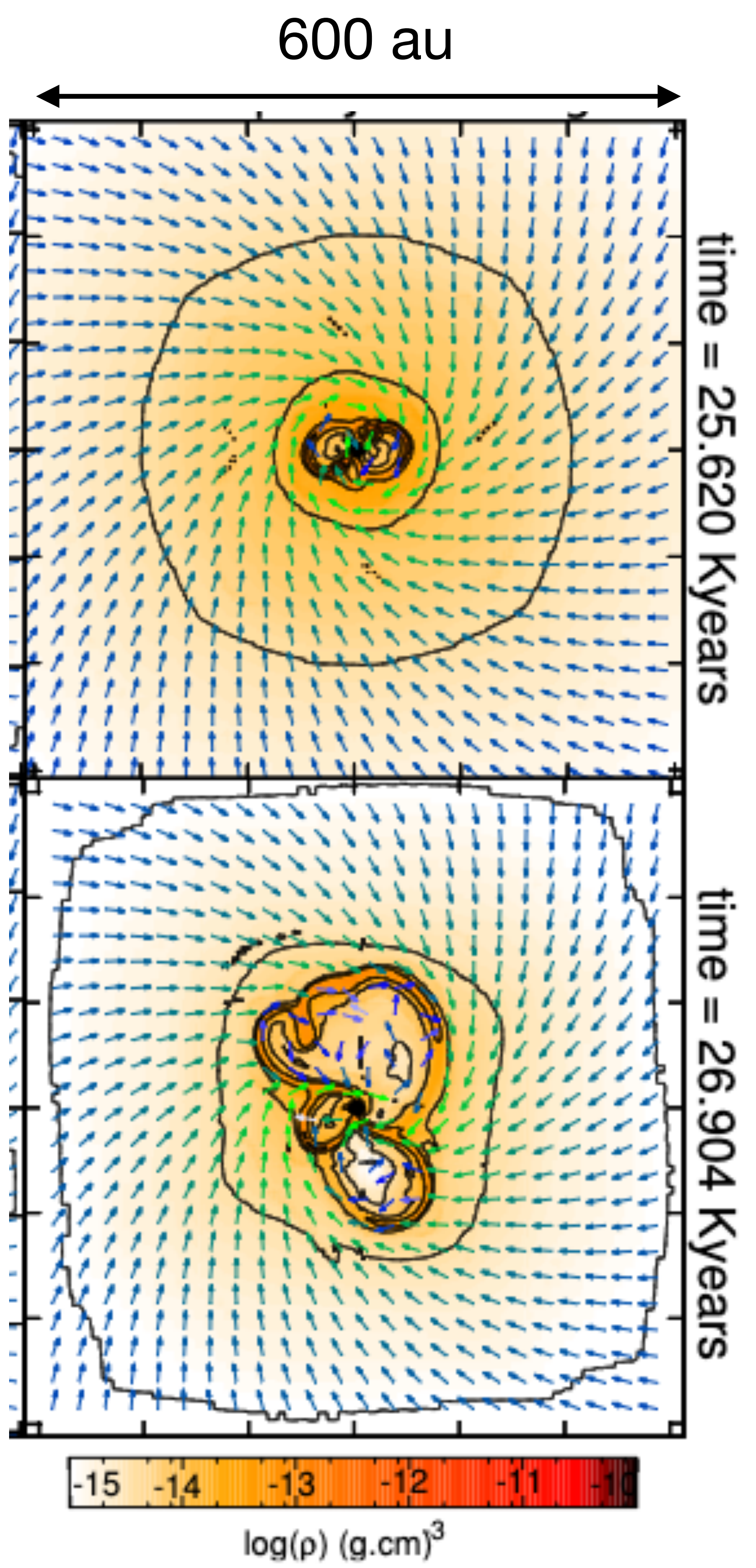
Lebreuilly et al. (2020) see also *Bate & Loren-Aguilar (2017); Tsukamoto et al. (2021); Cridland et al. (2022); Koga et al. (2022;2023)*

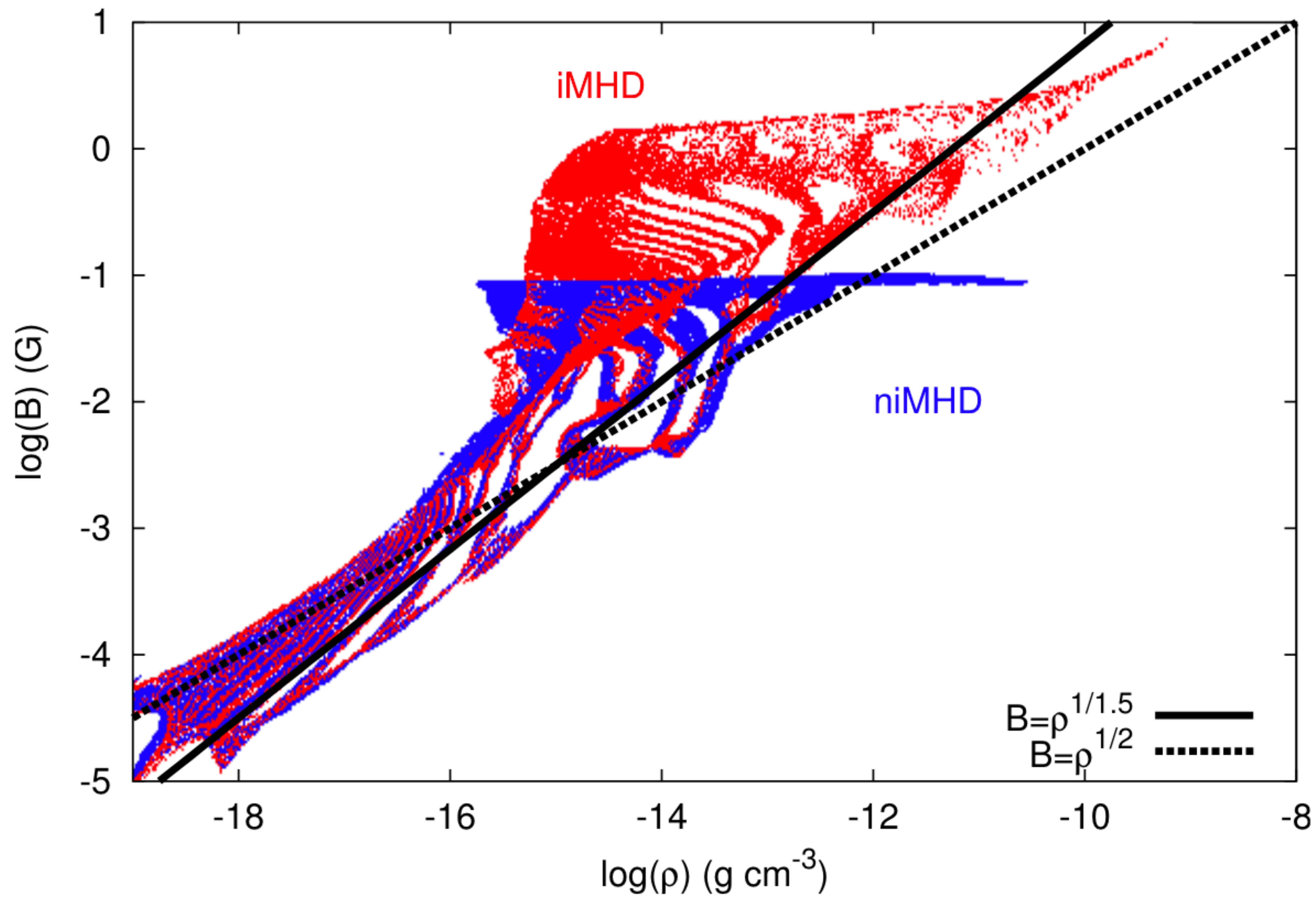
Role of grains in the MHD equations

Role of grains in the MHD equations

Non-ideal MHD is a promising solution (but not the only one) to solve the magnetic braking catastrophe (and explain disk formation) and the magnetic flux problem







Masson et al. 2016

See also (among many others) : Hennebelle & Fromang 2008; Mellon & Li 2009; Krasnopolsky et al. 2011, Tomida et al. (2013,2015); Machida et al. 2015, Tsukamoto et al. 2015, Wurster et al. (2016, 2018), Marchand et al. (2018, 2019, 2020)

Role of grains in the MHD equations

‘Unfortunately’ grains are charged and therefore feel magnetic and electric fields (and vice versa)

$$\frac{\partial \rho_{\rm d} \vec{v}_{\rm d}}{\partial t} + \nabla \rho_{\rm d} \vec{v}_{\rm d} \otimes \vec{v}_{\rm d} = -\frac{\rho_d}{t_s}\overrightarrow{\Delta v} + \frac{Z_d e \rho_d}{m_d}\left(\vec{E} + \frac{\vec{v}_d}{c} \times \vec{B}\right)$$

We can simplify that by assuming very small grains and neglecting the dust inertia

$$\vec{0} = -\frac{1}{t_s}\overrightarrow{\Delta v} + \frac{Z_d e}{m_d c}\left(c\vec{E} + \vec{v}_d \times \vec{B}\right), \text{ or } \vec{0} = -\frac{1}{t_s}\overrightarrow{\Delta v} + \frac{1}{t_{\rm gyr}}\left(\frac{c\vec{E}}{|\vec{B}|} + \vec{v}_d \times \vec{b}\right)$$

Useful references : *Kunz & Mouschovias (2009, 2010); Lesur (2020)*

To go beyond the neglected inertia : *Hennebelle & Lebreuilly (2023)*

Role of grains in the MHD equations

We can do a Lorentz transform such as $\overrightarrow{E} + \frac{\overrightarrow{v}}{c} \times \overrightarrow{B} = \overrightarrow{E_b}$

Which allows to turn this equation

$$\overrightarrow{0} = -\frac{1}{t_s}\overrightarrow{\Delta v} + \frac{1}{t_{\text{gyr}}}\left(\frac{c\overrightarrow{E}}{|\overrightarrow{B}|} + \overrightarrow{v}_d \times \overrightarrow{b}\right)$$

Into this one

$$\overrightarrow{\Delta v} - \Gamma_d \Delta \overrightarrow{v}_d \times \overrightarrow{b} = \frac{c\Gamma_d}{|\overrightarrow{B}|}\overrightarrow{E}_b$$

Γ_d the Hall factor is the ratio between the stopping time and the gyration time

Which can be inverted to give $\Delta\overrightarrow{v} = \frac{c}{|\overrightarrow{B}|}\left(\frac{\Gamma_d^2}{1+\Gamma_d^2}\overrightarrow{E}_b \times \overrightarrow{b} + \frac{\Gamma_d}{1+\Gamma_d^2}\overrightarrow{E}_{b,\perp} + \Gamma_d \overrightarrow{E}_{b,\parallel}\right)$

Role of grains in the MHD equations

Almost there !

From the differential velocity generalised to an arbitrary charged species i (ions, electrons and of course grains) $\Delta \overrightarrow{v}_i = \frac{c}{|\overrightarrow{B}|}\left(\frac{\Gamma_i^2}{1+\Gamma_i^2}\overrightarrow{E}_b \times \overrightarrow{b} + \frac{\Gamma_i}{1+\Gamma_i^2}\overrightarrow{E}_{b,\perp} + \Gamma_i \overrightarrow{E}_{b,\|}\right)$

The definition of the electric current (neglecting the displacement current)

$$\overrightarrow{J} = \frac{c}{4\pi}\nabla \times B = \sum_i n_i Z_i e \overrightarrow{v_i} = (\sum_i n_i Z_i e)\overrightarrow{v_g} + \sum_i n_i Z_i e \Delta v_i$$

And the local electroneutrality : $\sum_i n_i Z_i e = 0$

We obtain something quite ugly

$$\overrightarrow{J} = \sum_i \left(\frac{c}{|\overrightarrow{B}|} n_i Z_i e \frac{\Gamma_i^2}{1+\Gamma_i^2}\right)\overrightarrow{E}_b \times \overrightarrow{b} + \sum_i \left(\frac{c}{|\overrightarrow{B}|} n_i Z_i e \frac{\Gamma_i}{1+\Gamma_i^2}\right)\overrightarrow{E}_{b,\perp} + \sum_i \left(\frac{c}{|\overrightarrow{B}|} n_i Z_i e \Gamma_i\right)\overrightarrow{E}_{b,\|}$$

Role of grains in the MHD equations

Which we can "simplify" by grouping some terms together $\overrightarrow{J} = \sigma_H \overrightarrow{E}_b \times \overrightarrow{b} + \sigma_p \overrightarrow{E}_{b,\perp} + \sigma_o \overrightarrow{E}_{b,\parallel} = \frac{c}{4\pi}\nabla \times \overrightarrow{B}$

Inverting this relation (is tedious but possible) leads to the Ohm's law:

$$c\overrightarrow{E}_b = \eta_o \nabla \times \overrightarrow{B} - \frac{\eta_A}{|\overrightarrow{B}|^2}\left(\left(\nabla \times \overrightarrow{B}\right) \times \overrightarrow{B}\right) \times \overrightarrow{B} + \frac{\eta_H}{|\overrightarrow{B}|^2}\left(\nabla \times \overrightarrow{B}\right) \times \overrightarrow{B}$$

Where we defined the 3 non-ideal MHD resistivities:

- The Ohmic resistivity : $\eta_o = \frac{c^2}{4\pi}\frac{1}{\sigma_o}$
- The ambipolar resistivity : $\eta_a = \frac{c^2}{4\pi}\frac{\sigma_H}{\sigma_H^2 + \sigma_p^2}$
- The Hall resistivity : $\eta_H = \frac{c^2}{4\pi}\left(\frac{\sigma_p}{\sigma_H^2 + \sigma_p^2} - \frac{1}{\sigma_o}\right)$

Role of grains in the MHD equations

Recalling $\vec{E} + \frac{\vec{v}}{c} \times \vec{B} = \vec{E_b}$ and the induction equation $\frac{\partial \vec{B}}{\partial t} = -c\nabla \times \vec{E}$

We obtain the induction equation in the non-ideal MHD regime :

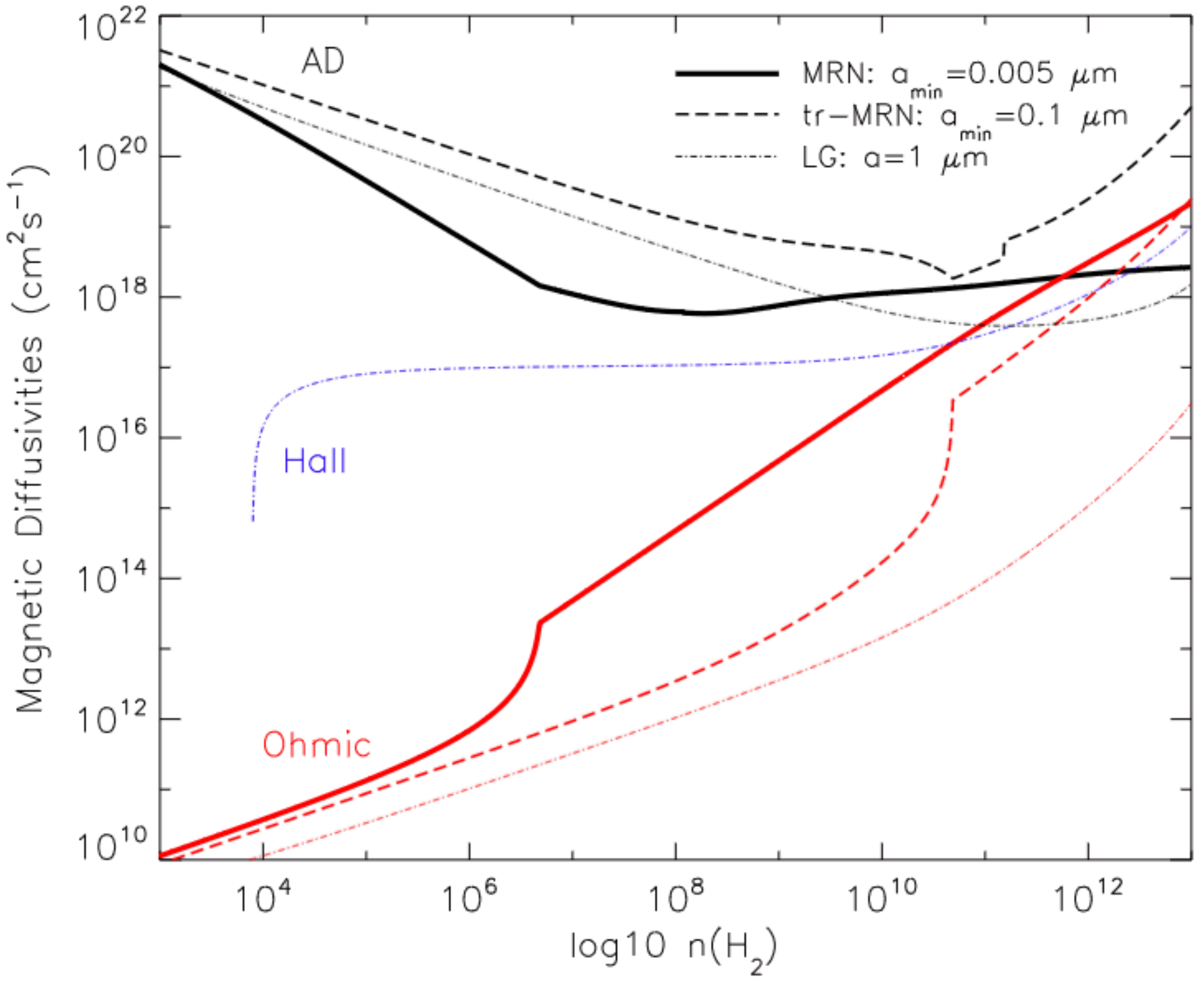
$$\frac{\partial \vec{B}}{\partial t} + \underbrace{\nabla \times \left(\vec{v} \times \vec{B}\right)}_{\text{Advection}} = \underbrace{\nabla \times \left[\eta_o \nabla \times \vec{B}\right]}_{\text{Ohmic dissipation}} \underbrace{- \nabla \times \left[\frac{\eta_A}{|\vec{B}|^2}\left(\left(\nabla \times \vec{B}\right) \times \vec{B}\right) \times \vec{B}\right]}_{\text{Ambipolar "diffusion" Or ion-neutral drift}} + \underbrace{\nabla \times \left[\frac{\eta_H}{|\vec{B}|^2}\left(\nabla \times \vec{B}\right) \times \vec{B}\right]}_{\text{Hall effect}}$$

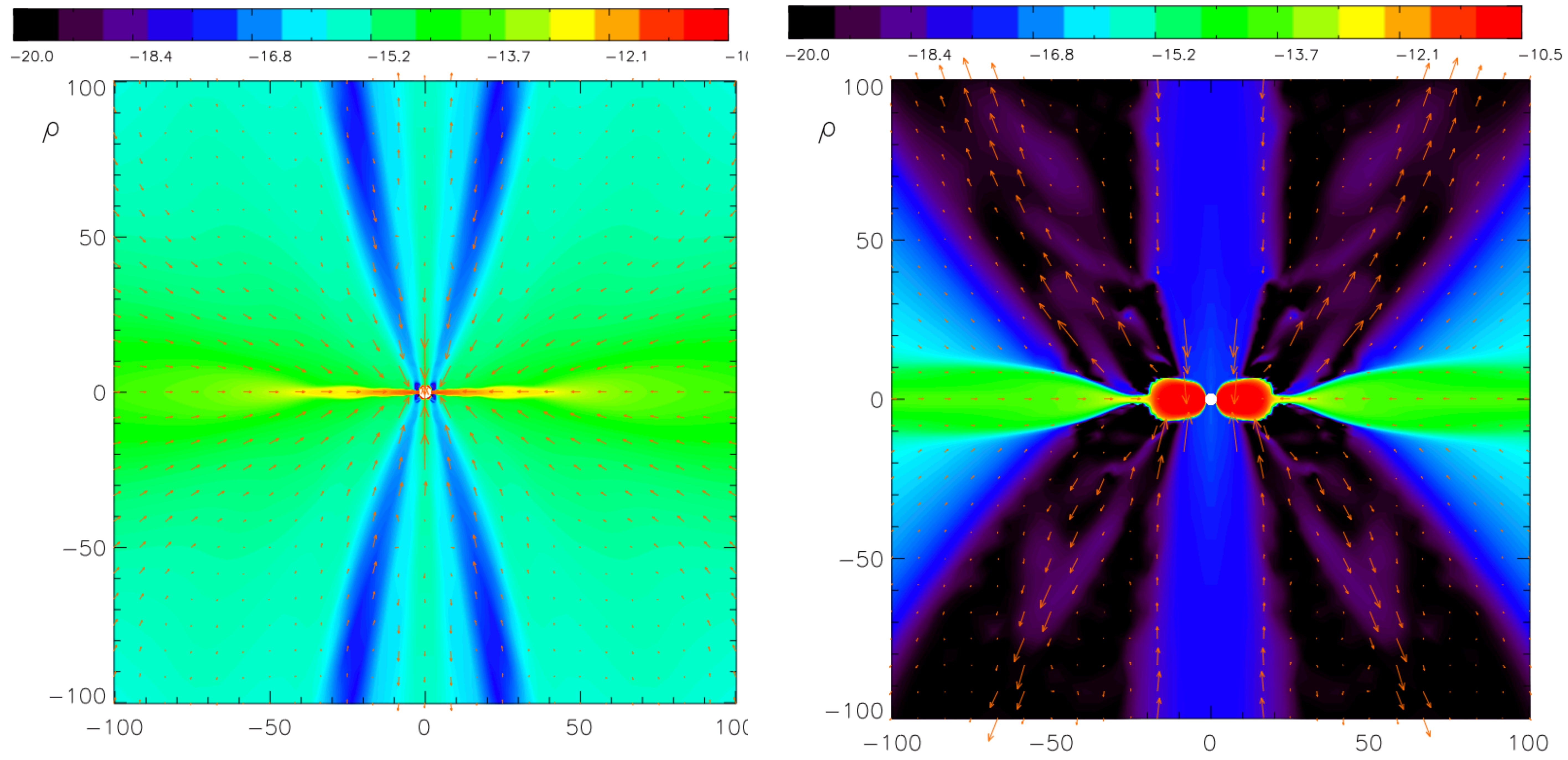
Roughly speaking, large resistivities induce weaker magnetic braking - the Hall term is a bit special.

The resistivities depends on the abundance of charged particles, their charge, their Hall factor, the magnetic field strength

Role of grains in the MHD equations

Changing the dust size distribution (by for example removing the very small grains) can have a dramatic influence on the resistivities and therefore on the disk formation





Zhao, Caselli et al. (2016) :

See also Marchand et al. (2020)

Conclusions I

- Dust dynamics is important and can be very different from the gas during star formation and disk evolution.

- The dynamics of grains is controlled by the Stokes number which depend on the grain and gas properties.

- The dust dynamics in disks leads to the radial drift barrier that must be solved to understand for planet formation

- Charged dust dynamics is relevant in the context of disk formation as dust grains influence heavily the evolution of the angular momentum through the magnetic resistivities.